EGU General Assembly 2013
April 11, 2013, Vienna

In-situ estimate of submesoscale horizontal eddy diffusion coefficients across a front

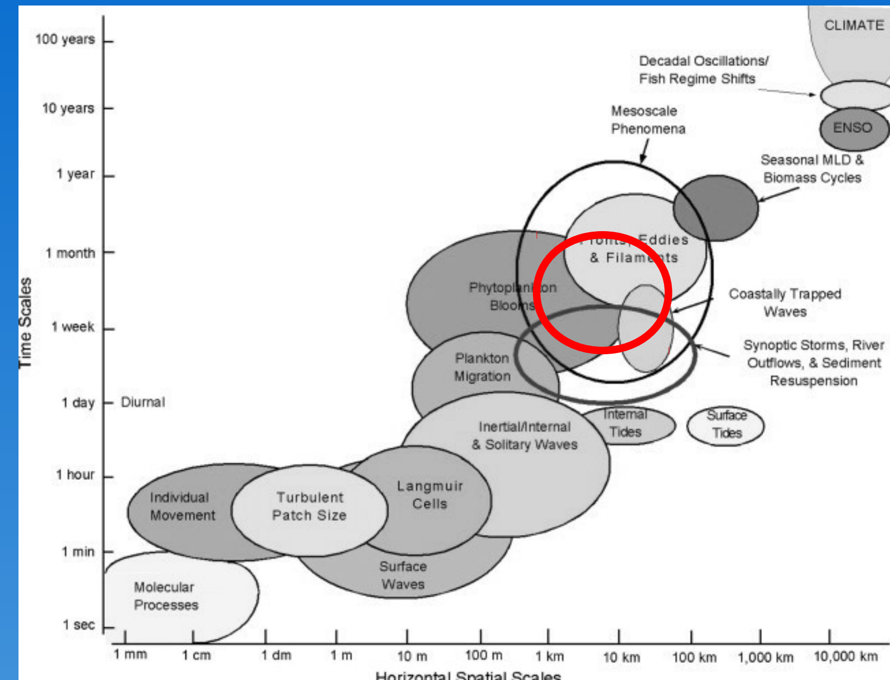
F. Nencioli [1], F. d'Ovidio [2], A.M. Doglioli [1], A.A. Petrenko [1]

(1) Aix-Marseille Université, Mediterranean Institute of Oceanography (MIO), 13288, Marseille, Cedex 9, France ; Université du Sud Toulon-Var; CNRS-INSU/IRD UM 110

(2) Laboratoire d'Océanographie et du Climat: Experimentation et Approches Numeriques, IPSL, Paris, France

Submesoscale structures

(from http://oceanservice.noaa.gov/)



(from Dickey et al., J. Mar. Syst,.2003)

➔ **Fronts, jets and eddies:** **Horizontal scales ~(100m - 10km)**
Time scales ~(days - week)

➔ **Key role for :** **Energy transfer**
Horizontal and vertical transport
Biogeochemical cycles

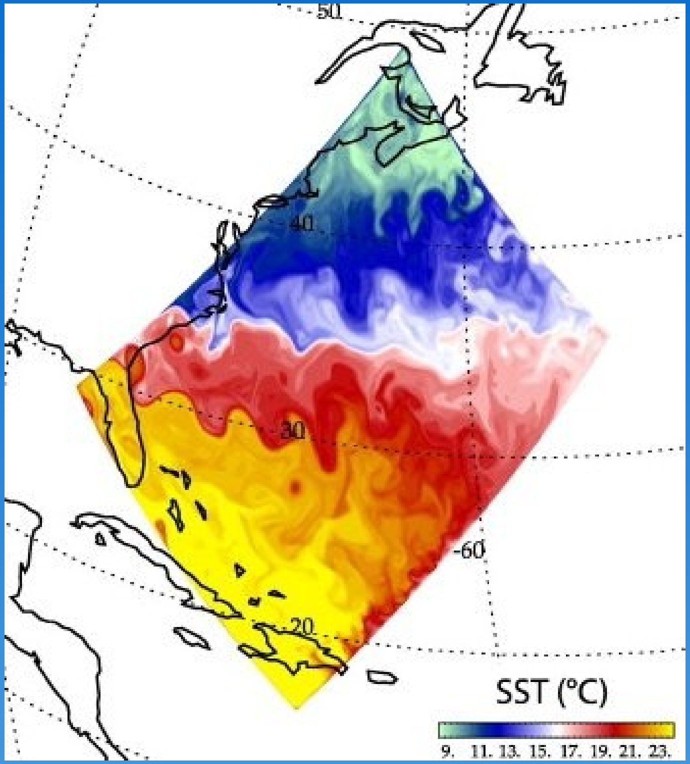
Submesoscale structures

➔ **Numerical model studies**

Favored by:

Advances in computational power

Development of regional models



(from Levy et al., Ocean Model., 2012)

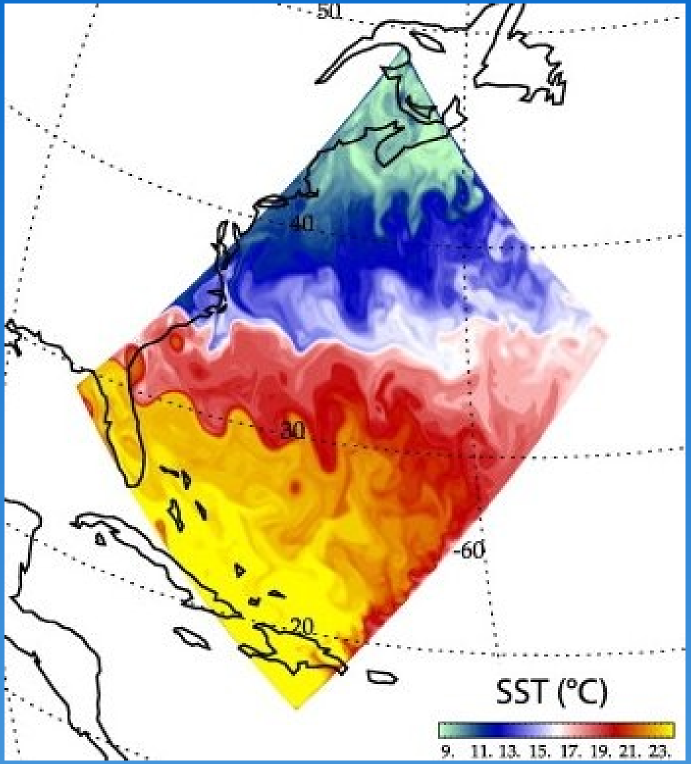
Submesoscale structures

➔ **Numerical model studies**

Favored by:

Advances in computational power

Development of regional models



(from Levy et al., Ocean Model., 2012)

➔ **In-situ observations**

Challenging due to small spatial and temporal scales

⇩

Limited estimates of key physical parameters

Submesoscale structures

➔ **Numerical model studies**

Favored by:

Advances in computational power

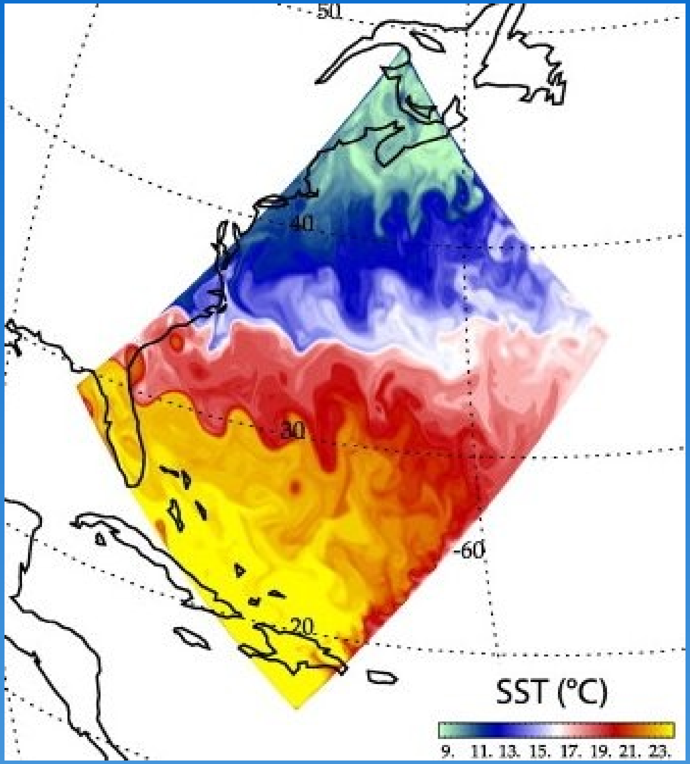
Development of regional models

➔ **In-situ observations**

Challenging due to small spatial and temporal scales

⇩

Limited estimates of key physical parameters



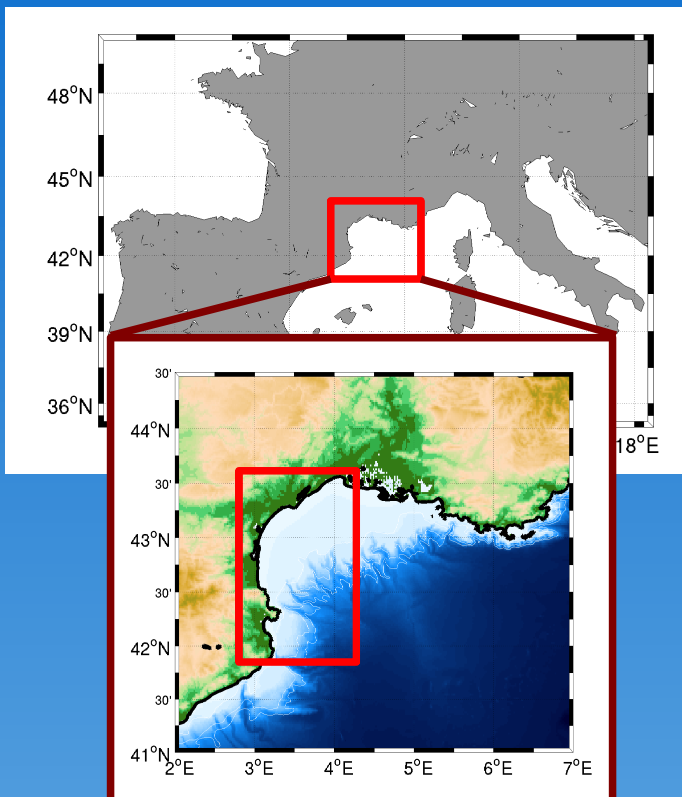
(from Levy et al., Ocean Model., 2012)

Focus of this study

New approach to estimate horizontal eddy diffusion coefficients (Kh) from in-situ observations

Lagrangian Transport EXperiment

➔ Latex10 campaign *(September 1-24, 2010)*



- **Western part of Gulf of Lion**
  (NW Mediterranean)

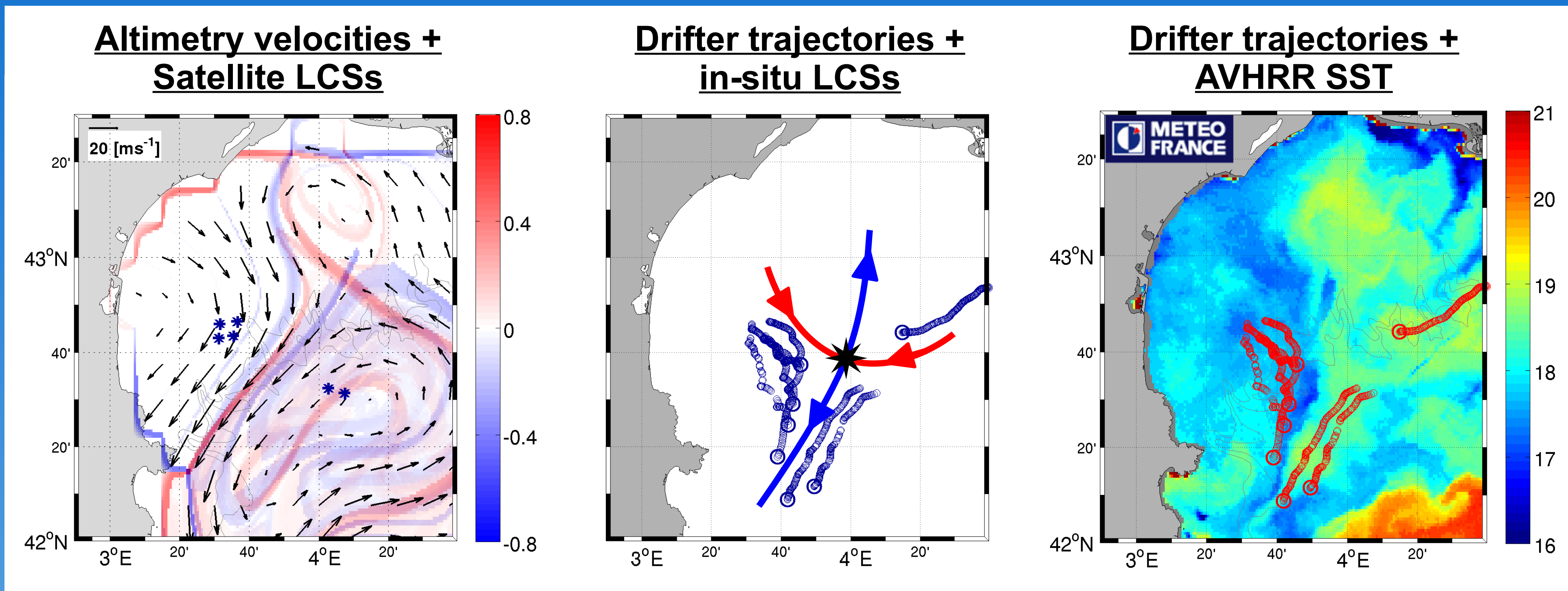
- **<u>Main goals</u> : (Sub)mesoscale dynamics**
  Cross-shelf exchanges

- **Adaptive sampling strategy based on :**
  - **- Satellite data**
  - **- Ship-based current measurements**
  - **- Iterative Lagrangian drifter releases**

In-situ LCSs

Example:

➔ **Drifter array deployment "Lyap01"** *(Sep. 15, 2010)*

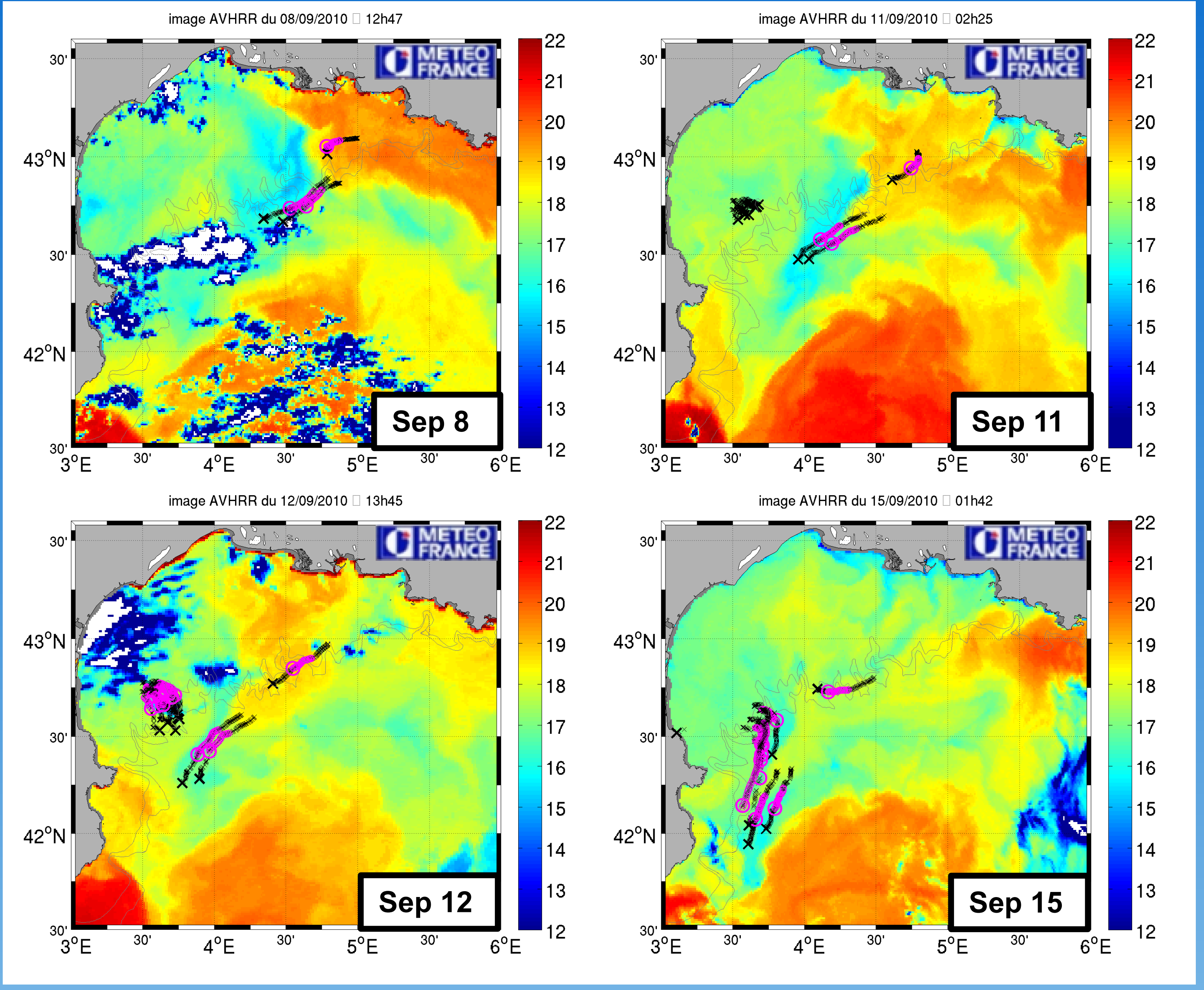


(from Nencioli et al., GRL, 2011)

- **Identified in-situ Lagrangian Coherent Structures (LCSs)**
- **Evidenced inaccuracy of altimetry in coastal region**
- **LCS associated with a frontal structure**

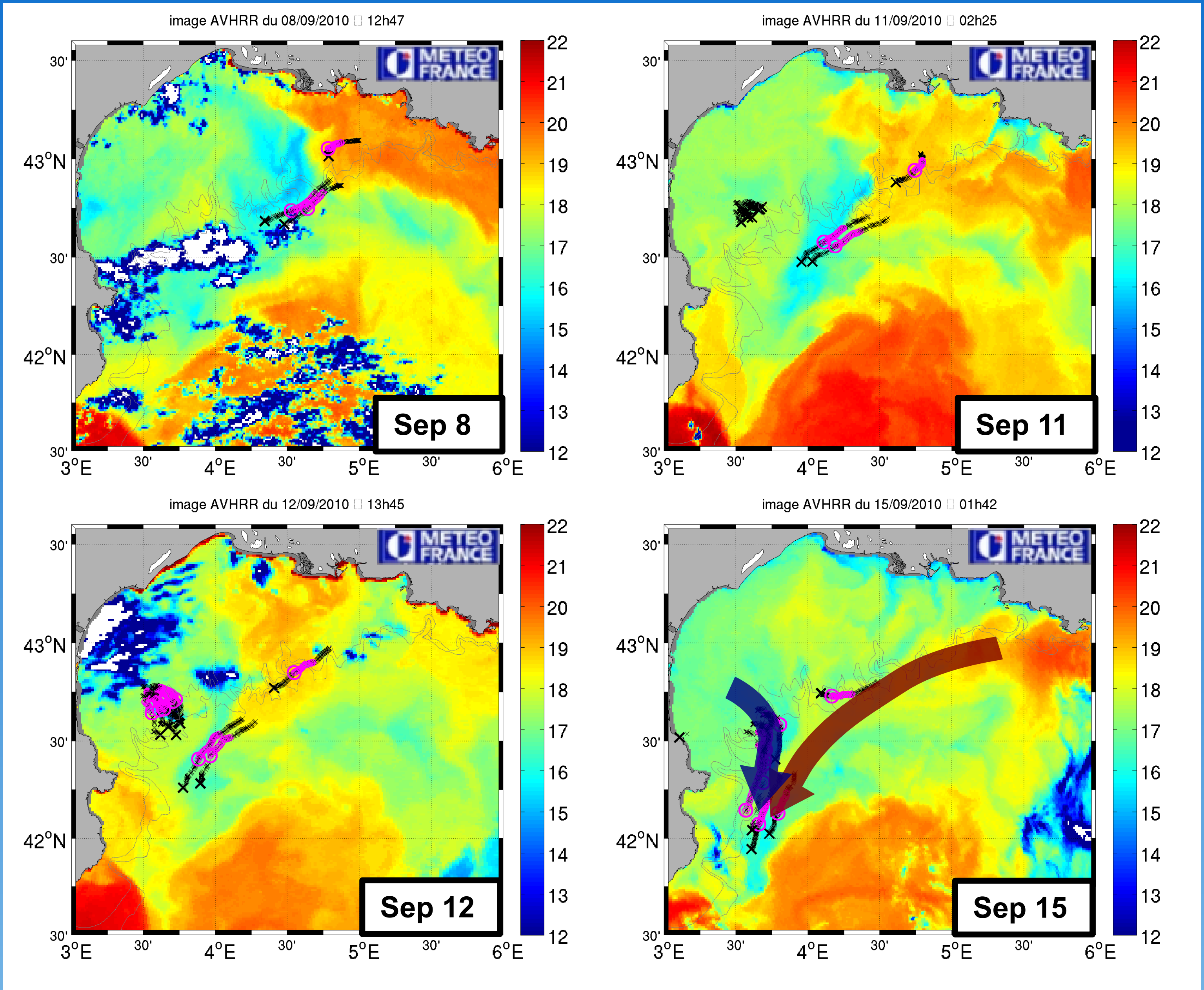
Front development

➔ **AVHRR SST + 3-day drifter trajectories** *(Sep. 8 to 15, 2010)*



Front development

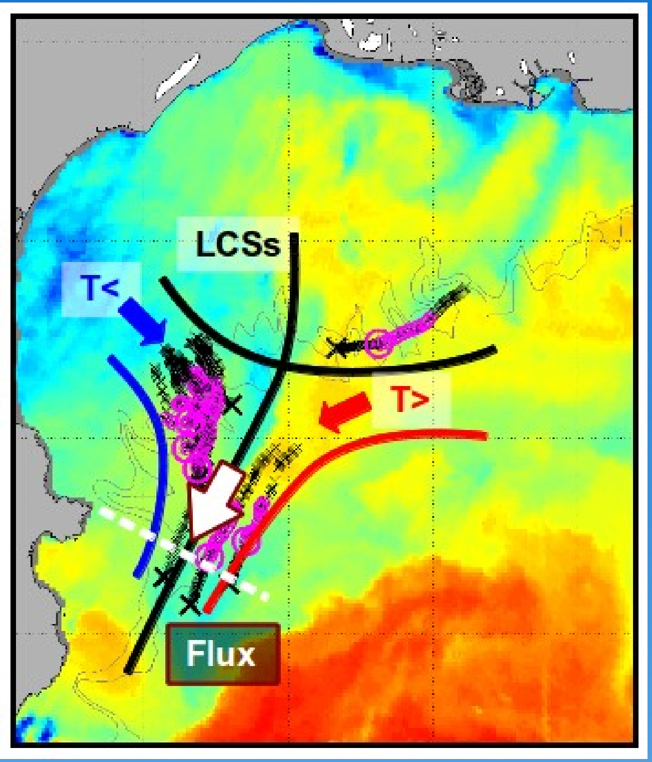
➔ AVHRR SST + 3-day drifter trajectories *(Sep. 8 to 15, 2010)*



Convergence of warmer (eastern outer shelf) and colder (western inner shelf) water masses

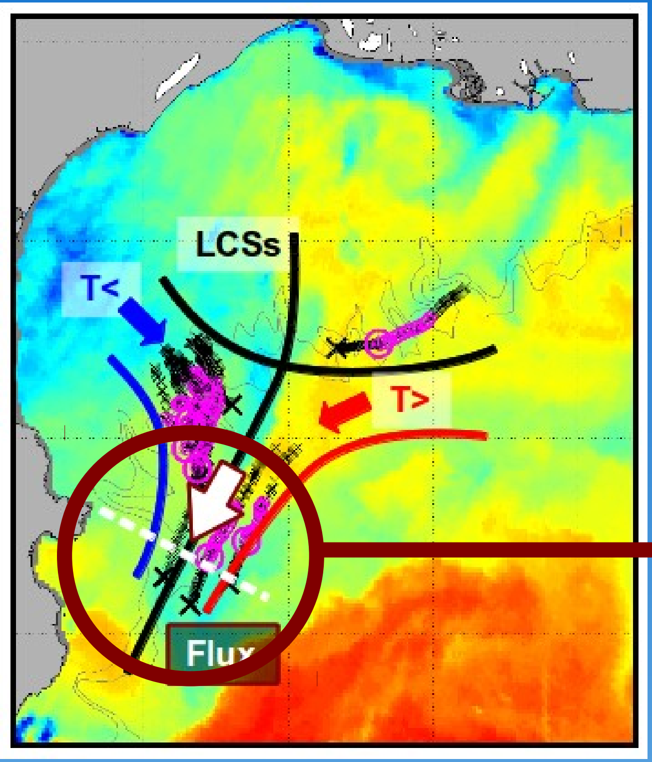
In-situ approach

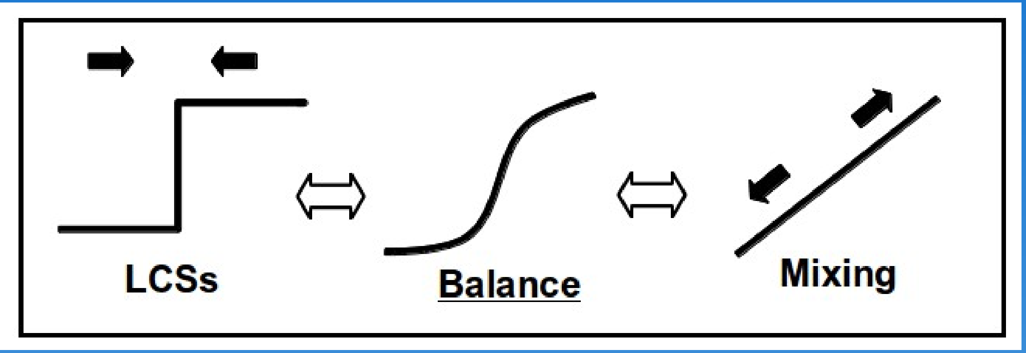
➔ **Colder and warmer water masses converging along attractive LCS**



In-situ approach

➔ **Colder and warmer water masses converging along attractive LCS**

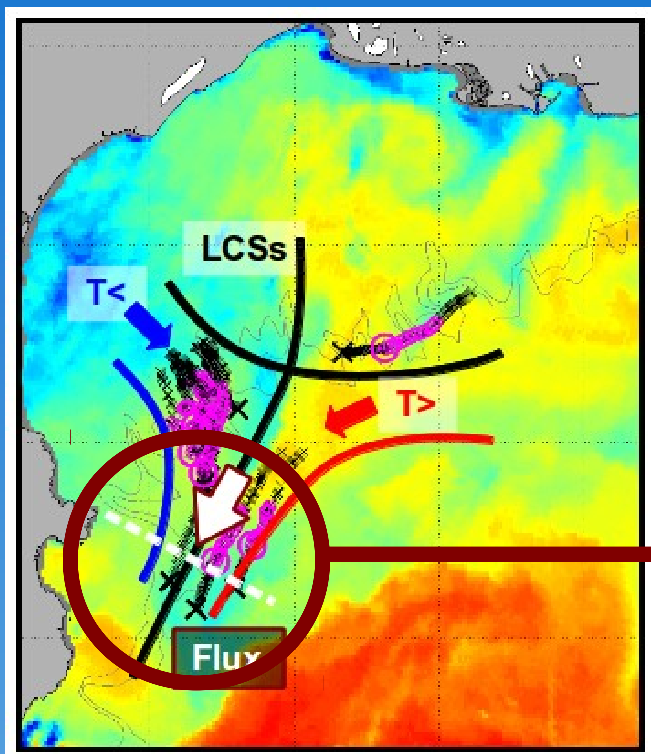


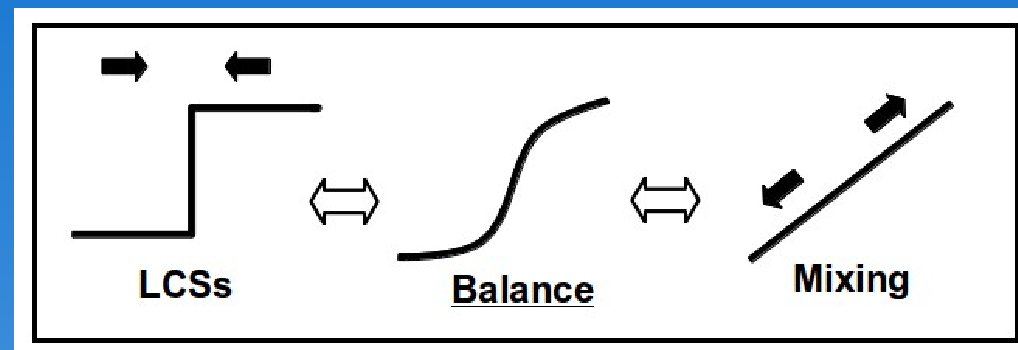


- **Shape of T and S profile across the front results from balance between convergence and horizontal mixing**

In-situ approach

➔ **Colder and warmer water masses converging along attractive LCS**





- **Shape of T and S profile across the front results from balance between convergence and horizontal mixing**

$$\frac{\partial T}{\partial t} + u(x)\frac{\partial T}{\partial x} = K_H \frac{\partial^2 T}{\partial x^2}$$

- **Analytical solution to 1D advection diffusion equation for a tracer *T***

➔ **Analogous to Flament et al. 1985, Ledwell et al. 1998 (from satellite)**

Advection-diffusion equation

1D equation for a tracer *T*

$$\frac{\partial T}{\partial t} + u(x)\frac{\partial T}{\partial x} = K_H \frac{\partial^2 T}{\partial x^2}$$

Advection-diffusion equation

1D equation for a tracer *T*

$$\cancel{\frac{\partial T}{\partial t}} + \underbrace{u(x)}_{-\gamma(x-\mu)}\frac{\partial T}{\partial x} = K_H \frac{\partial^2 T}{\partial x^2}$$

$$-\gamma(x-\mu)\frac{dT}{dx} = K_H \frac{d^2 T}{dx^2}$$

Assumptions:

- Front is at equilibrium (steady state)
- x is the across LCS direction

Advection-diffusion equation

1D equation for a tracer *T*

$$\cancel{\frac{\partial T}{\partial t}} + u(x)\frac{\partial T}{\partial x} = K_H \frac{\partial^2 T}{\partial x^2}$$

$$-\gamma(x-\mu)\frac{dT}{dx} = K_H \frac{d^2 T}{dx^2}$$

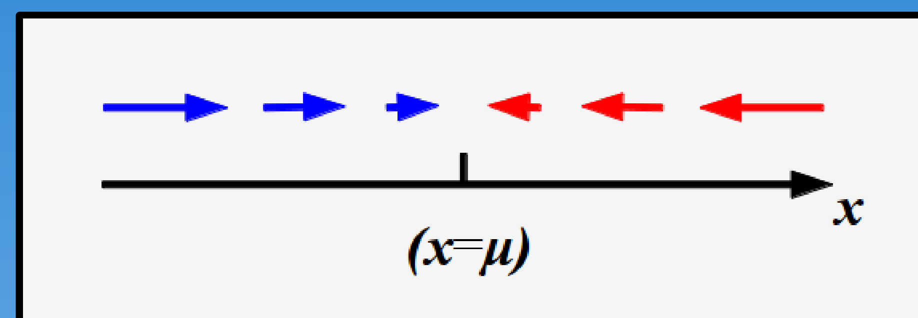
Assumptions:

- Front is at equilibrium (steady state)
- x is the across LCS direction

with

γ : Strain rate (Lyapunov exponent)

μ : Position of LCS axis



Advection-diffusion equation

1D equation for a tracer *T*

$$\cancel{\frac{\partial T}{\partial t}} + u(x)\frac{\partial T}{\partial x} = K_H \frac{\partial^2 T}{\partial x^2}$$

$$-\gamma(x-\mu)\frac{dT}{dx} = K_H \frac{d^2 T}{dx^2}$$

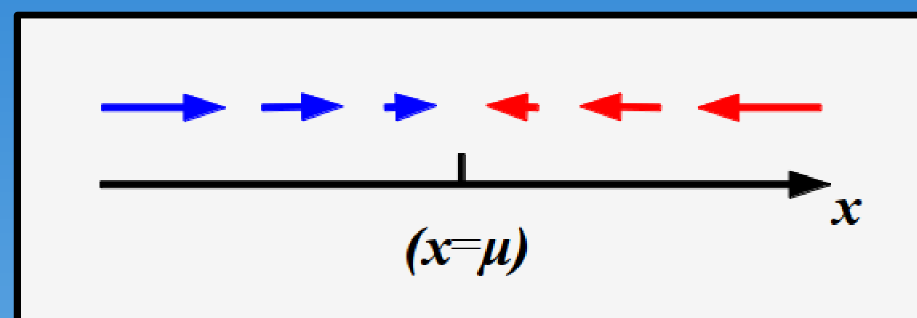
with

γ : Strain rate (Lyapunov exponent)

μ : Position of LCS axis

Assumptions:

- Front is at equilibrium (steady state)
- x is the across LCS direction



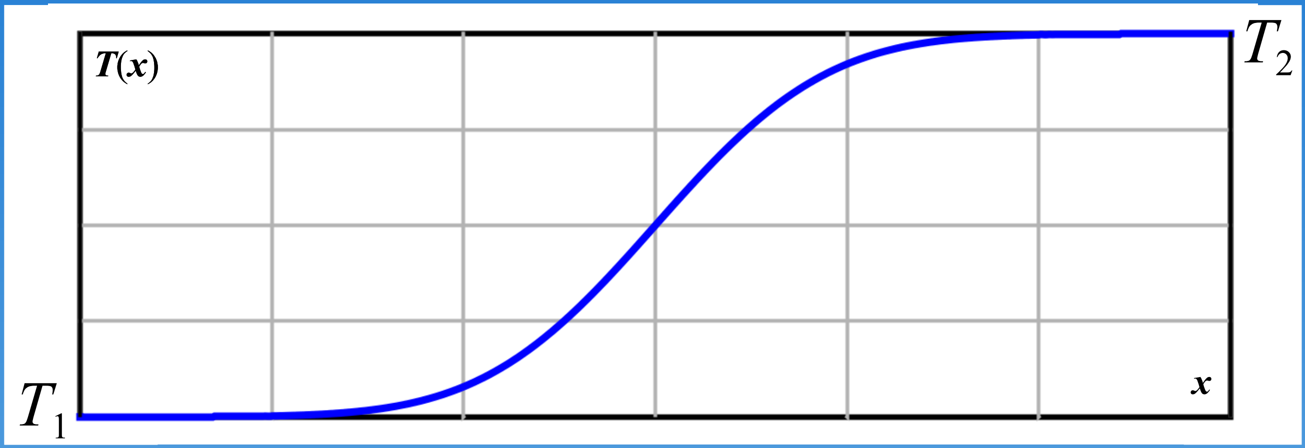
Boundary Conditions

$$T(x=-\infty) = T_1;$$
$$T(x=+\infty) = T_2;$$

$$\boxed{T(x) = \frac{T_2+T_1}{2} + \frac{T_2-T_1}{2}\,\mathrm{erf}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{\gamma}{K_H}}\,(x-\mu)\right)}$$

Analytical solution

$$T(x) = \frac{T_2 + T_1}{2} + \frac{T_2 - T_1}{2} \operatorname{erf}\left(\frac{1}{\sqrt{2}} \sqrt{\frac{\gamma}{K_H}} (x - \mu)\right)$$

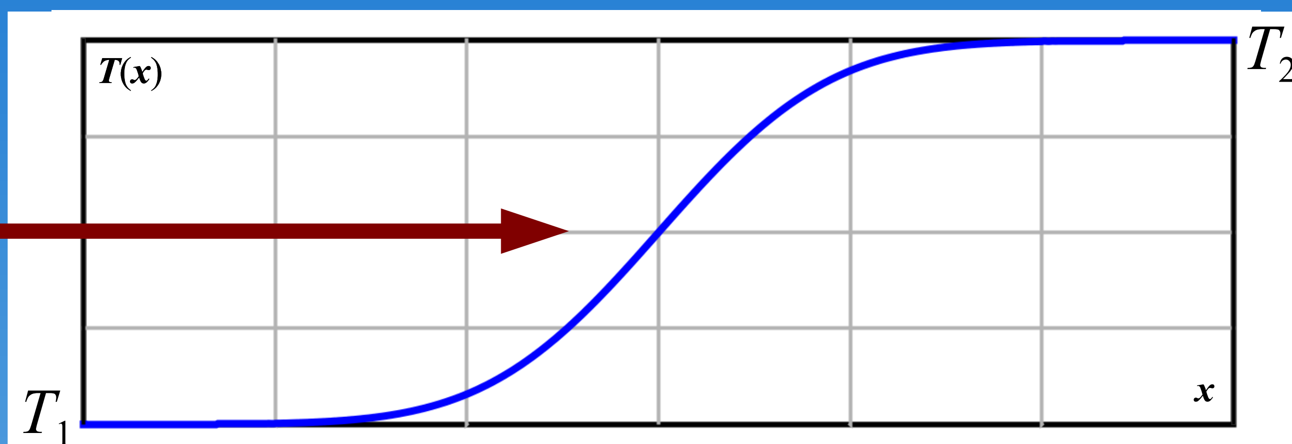


$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \mathrm{d}t$$

Analytical solution

$$T(x) = \underbrace{\frac{T_2 + T_1}{2}}_{\mathrm{C1}} + \frac{T_2 - T_1}{2}\,\mathrm{erf}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{\gamma}{K_H}}\,(x-\mu)\right)$$

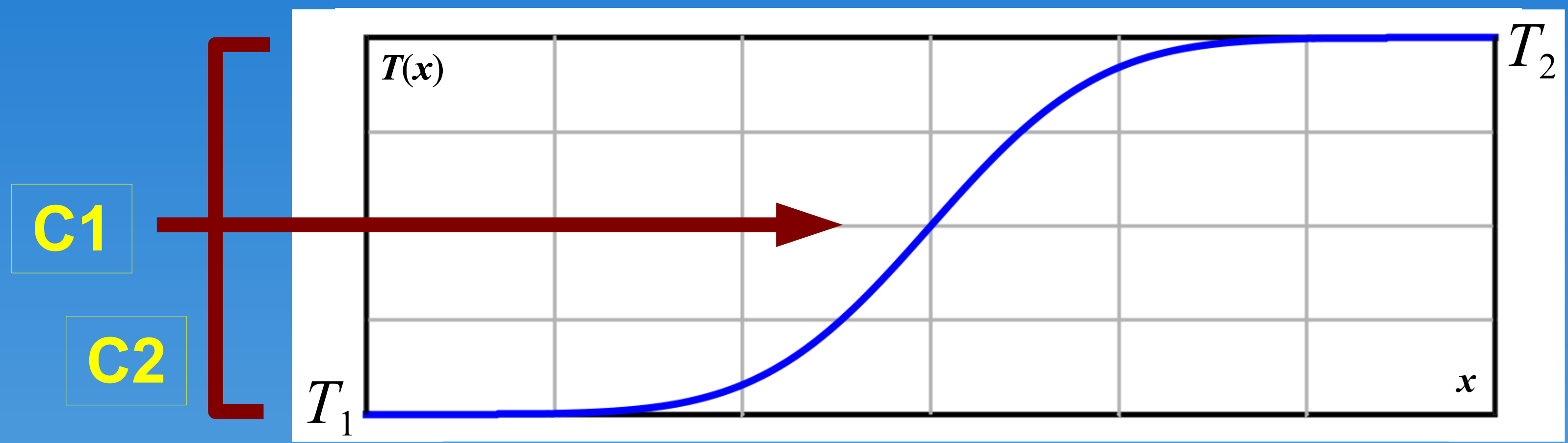
C1



$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,\mathrm{d}t$$

Analytical solution

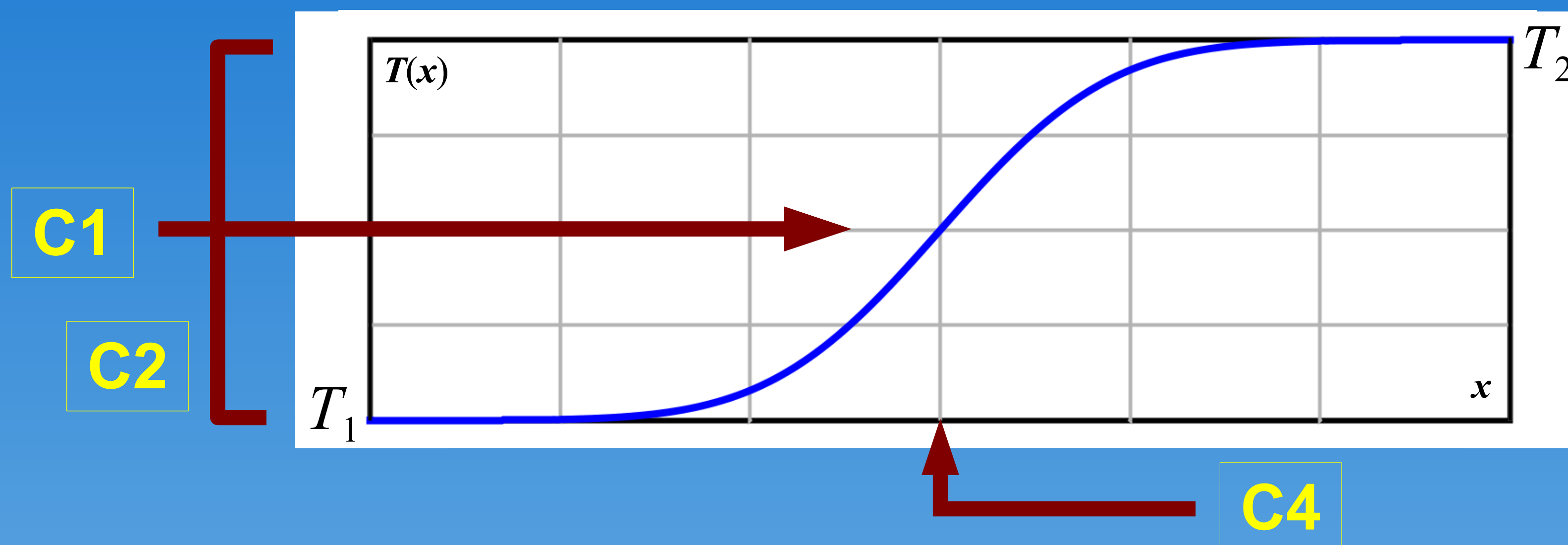
$$T(x) = \underbrace{\frac{T_2 + T_1}{2}}_{\text{C1}} + \underbrace{\frac{T_2 - T_1}{2}}_{\text{C2}} \operatorname{erf}\left(\frac{1}{\sqrt{2}} \sqrt{\frac{\gamma}{K_H}} \, (x - \mu) \right)$$



$$\boxed{\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \mathrm{d}t}$$

Analytical solution

$$T(x) = \underbrace{\frac{T_2 + T_1}{2}}_{C1} + \underbrace{\frac{T_2 - T_1}{2}}_{C2}\,\mathrm{erf}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{\gamma}{K_H}}\,(x - \underbrace{\mu}_{C4})\right)$$

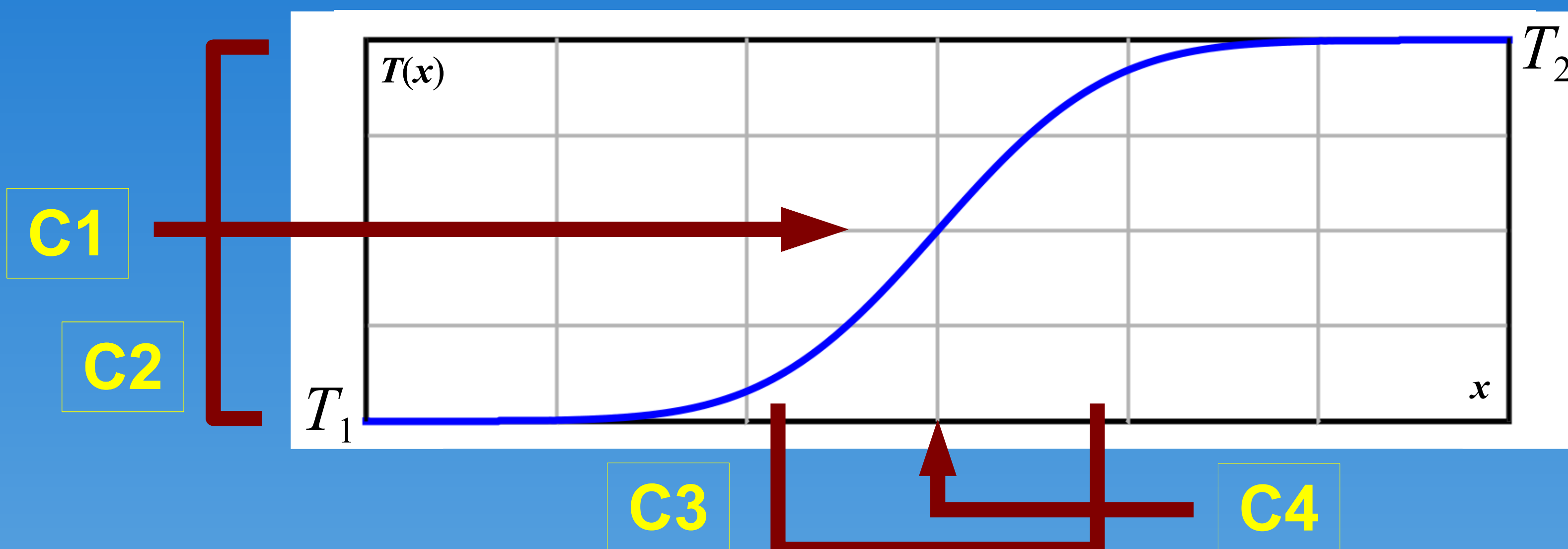


$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,\mathrm{d}t$$

3. Equation

Analytical solution

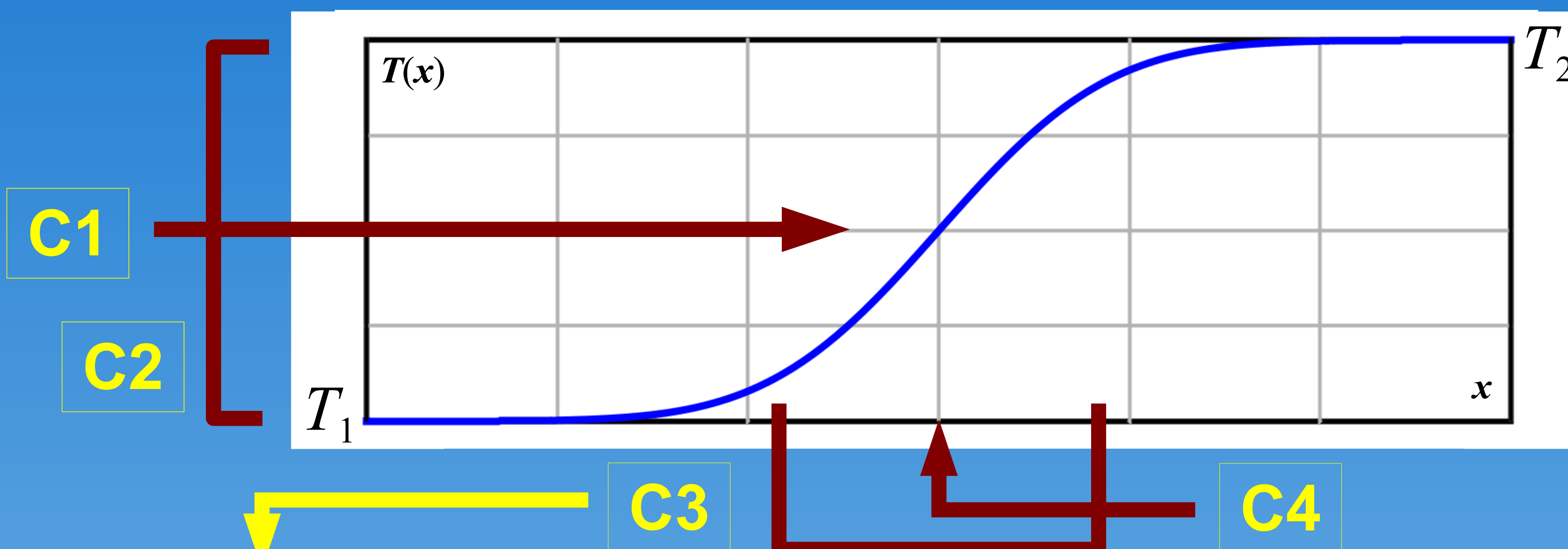
$$T(x)=\underbrace{\frac{T_2+T_1}{2}}_{C1}+\underbrace{\frac{T_2-T_1}{2}}_{C2}\,\mathrm{erf}\left(\underbrace{\frac{1}{\sqrt{2}}\sqrt{\frac{\gamma}{K_H}}}_{C3}\,(x-\underbrace{\mu}_{C4})\right)$$



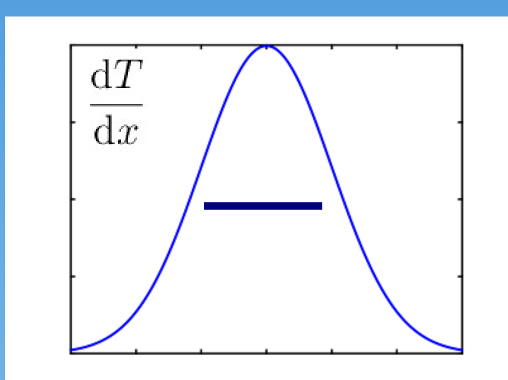
$$\mathrm{erf}(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,\mathrm{d}t$$

Analytical solution

$$T(x) = \underbrace{\frac{T_2 + T_1}{2}}_{C1} + \underbrace{\frac{T_2 - T_1}{2}}_{C2} \operatorname{erf}\left(\underbrace{\frac{1}{\sqrt{2}} \sqrt{\frac{\gamma}{K_H}}}_{C3} (x - \underbrace{\mu}_{C4}) \right)$$



$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \mathrm{d}t$$

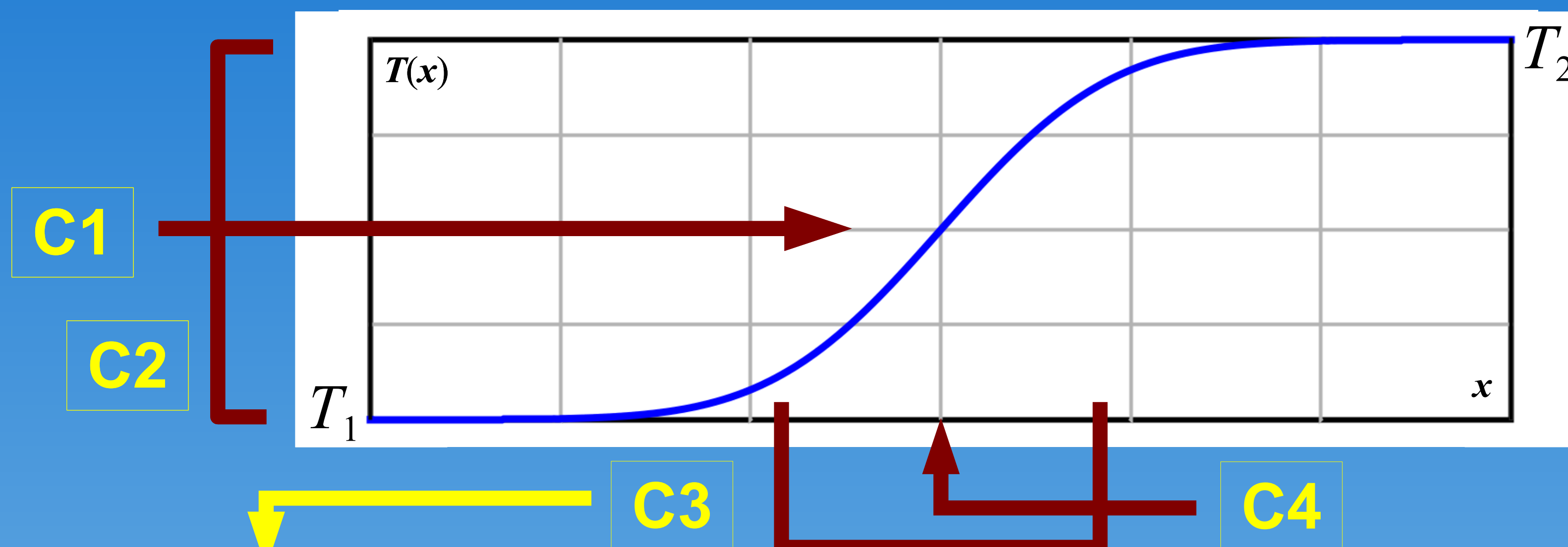


$$W_{front} = 2\sqrt{\frac{K_H}{\gamma}}$$

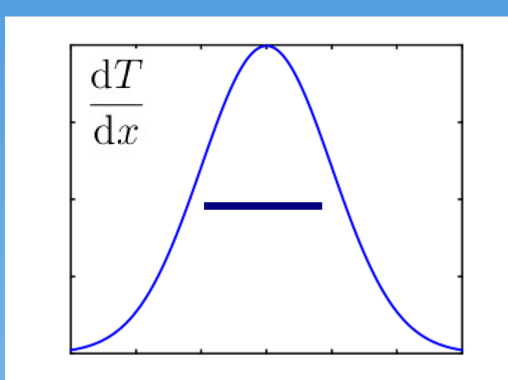
68% of T variation

3. Equation — Analytical solution

$$T(x) = \underbrace{\frac{T_2+T_1}{2}}_{C1} + \underbrace{\frac{T_2-T_1}{2}}_{C2}\,\mathrm{erf}\left(\underbrace{\frac{1}{\sqrt{2}}\sqrt{\frac{\gamma}{K_H}}}_{C3}\,(x-\underbrace{\mu}_{C4})\right)$$



$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,\mathrm{d}t$$



$$W_{front} = 2\sqrt{\frac{K_H}{\gamma}}$$

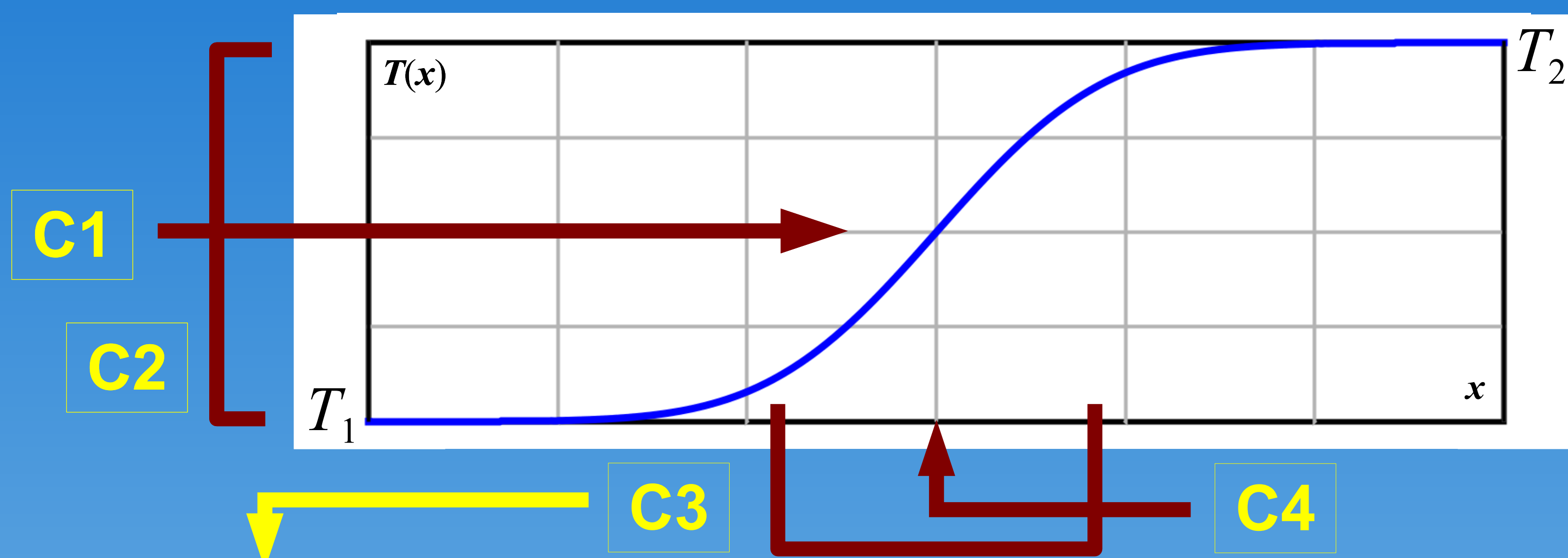
68% of T variation

- **Coefficients computed by best fitting T and S sections**

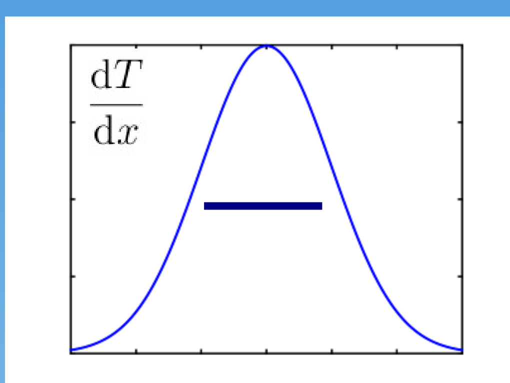
$$K_H = \frac{\gamma}{\left(2\,\mathrm{C3}^2\right)}$$

Analytical solution

$$T(x) = \underbrace{\frac{T_2 + T_1}{2}}_{\text{C1}} + \underbrace{\frac{T_2 - T_1}{2}}_{\text{C2}} \operatorname{erf}\left(\underbrace{\frac{1}{\sqrt{2}} \sqrt{\frac{\gamma}{K_H}}}_{\text{C3}} (x - \underbrace{\mu}_{\text{C4}}) \right)$$



$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \mathrm{d}t$$



$$W_{front} = 2\sqrt{\frac{K_H}{\gamma}}$$

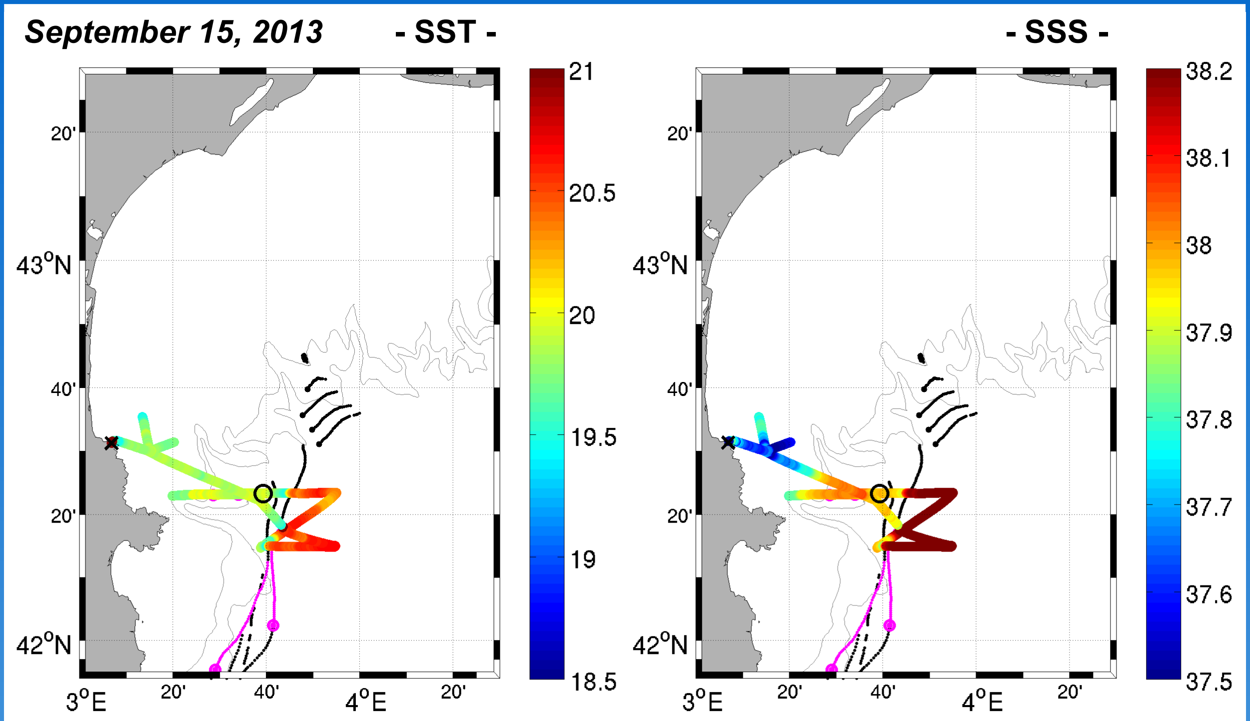
68% of T variation

- **Coefficients computed by best fitting T and S sections**

$$K_H = \frac{\gamma}{\left(2\,\mathrm{C3}^2\right)}$$

from drifter dispersion!!

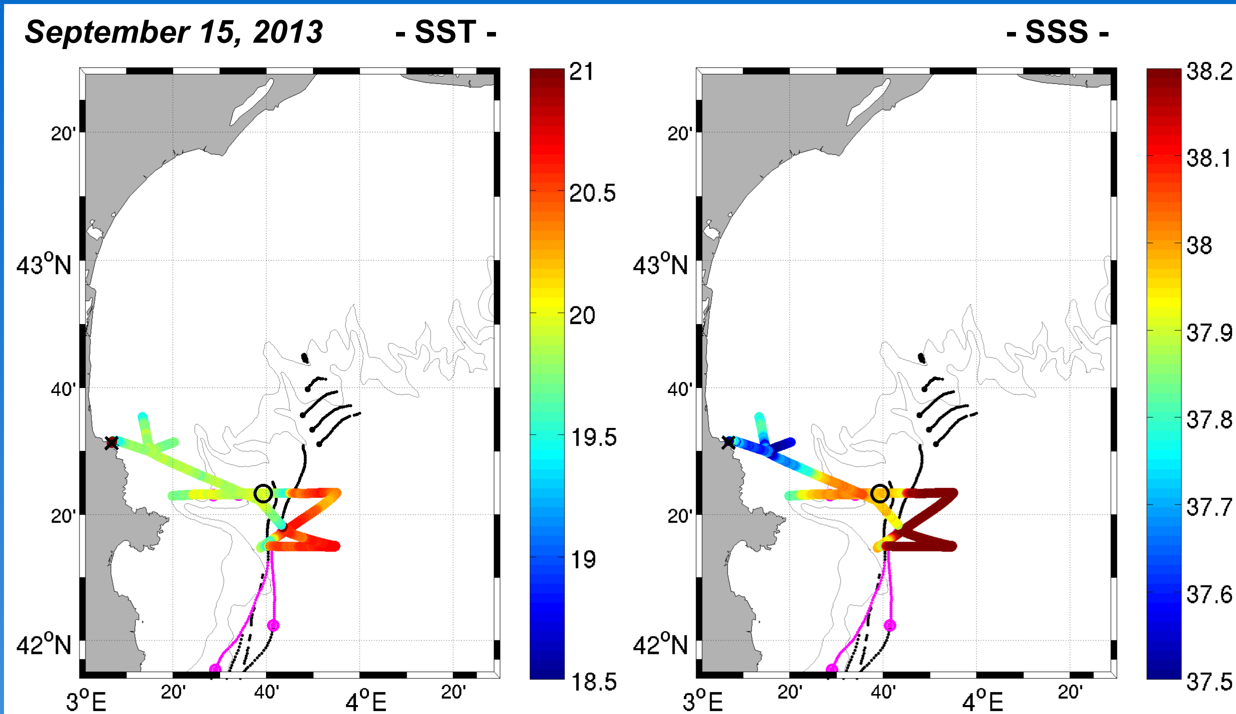
Cross-front transects

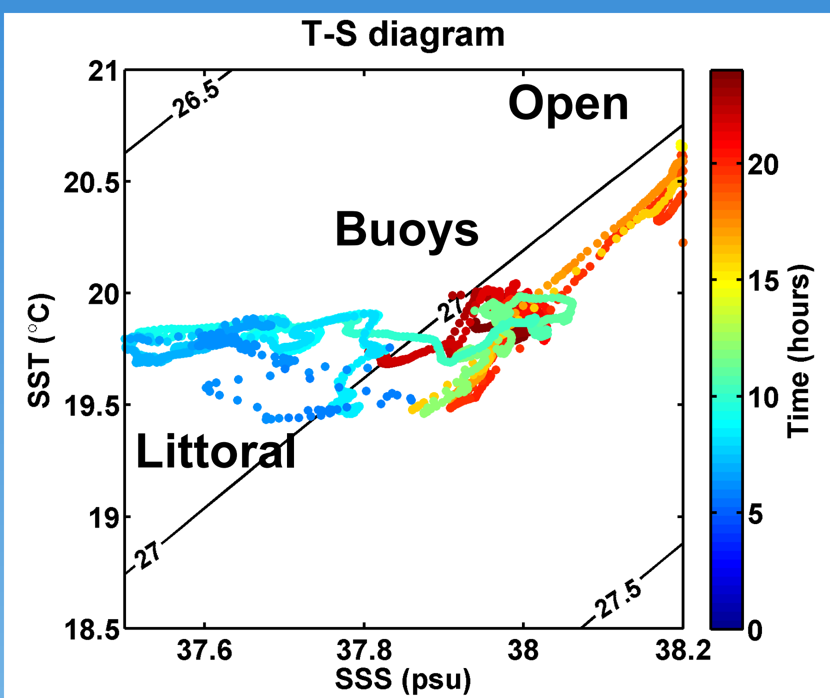


SST and SSS from ship thermosalinograph

4. Kh

Cross-front transects

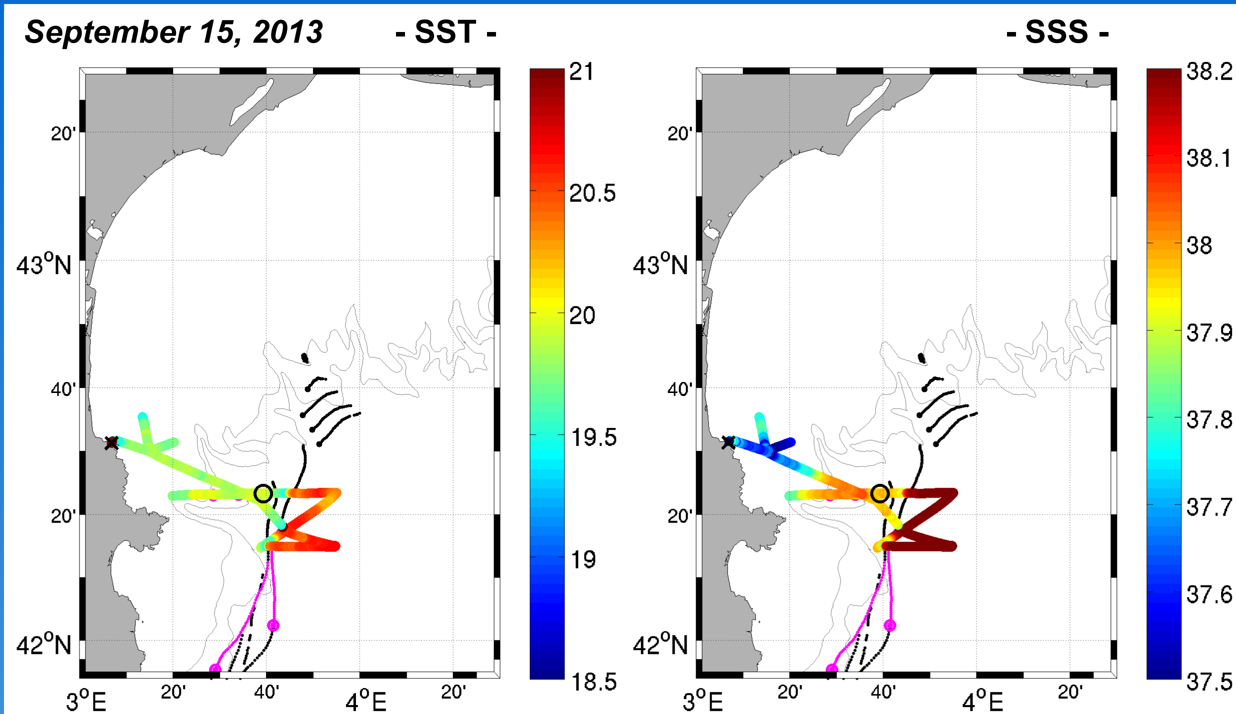


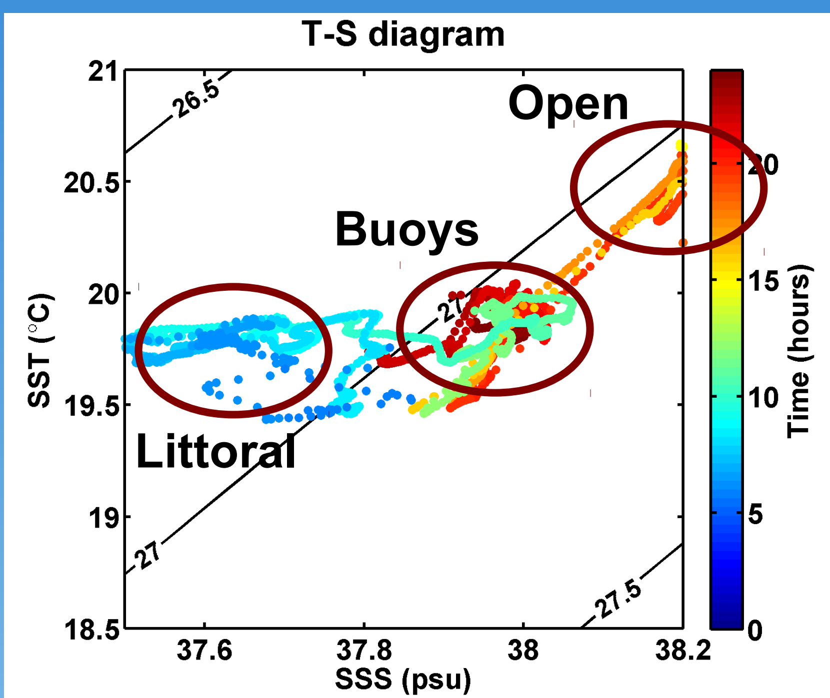


SST and SSS from ship thermosalinograph

4. Kh

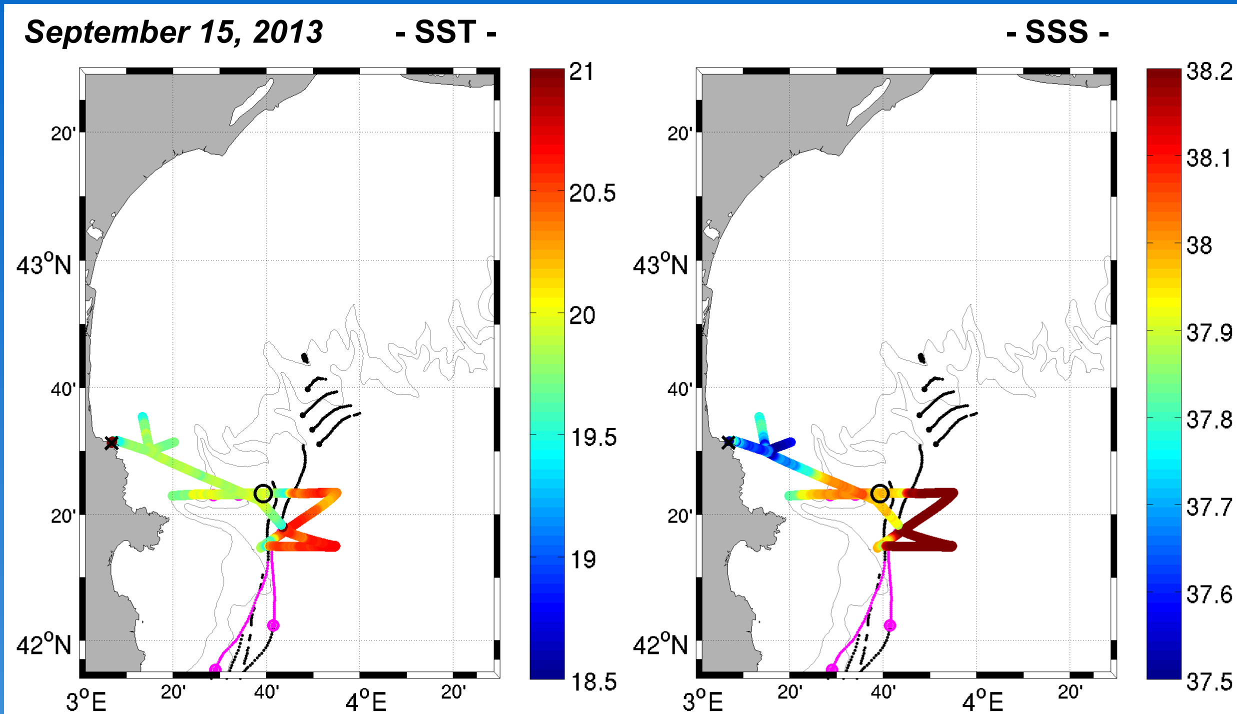
Cross-front transects



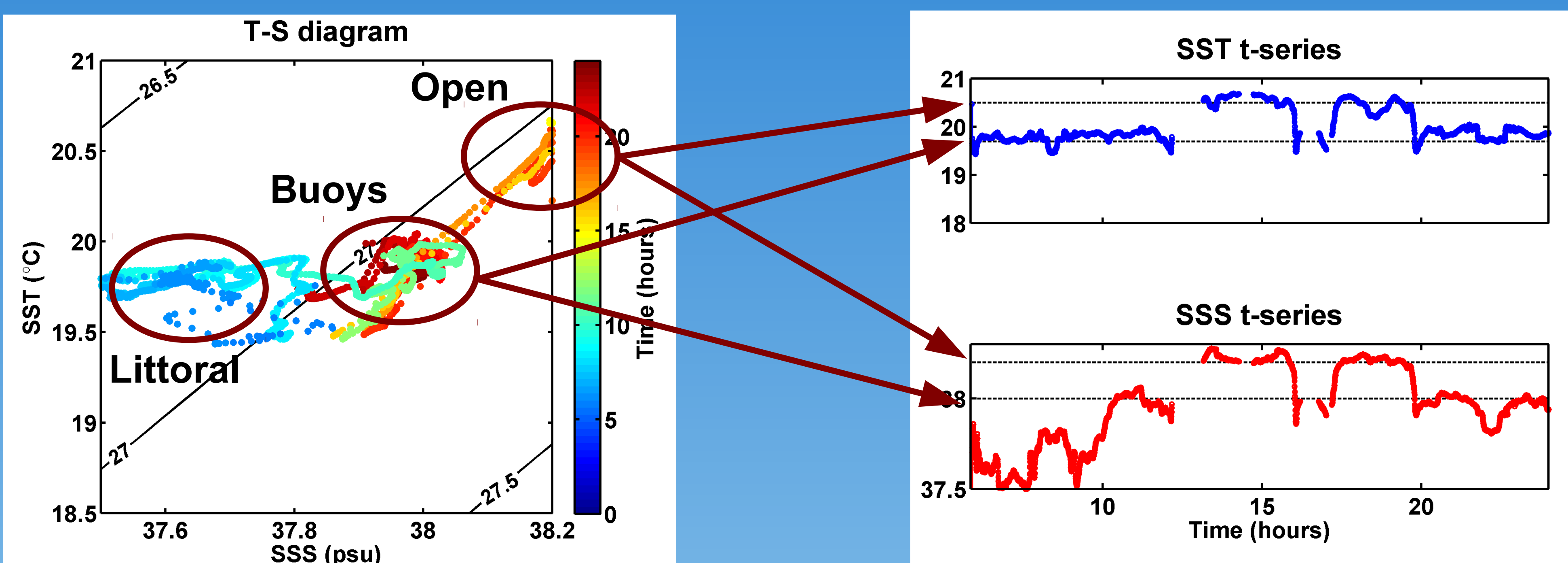


SST and SSS from ship thermosalinograph

Cross-front transects

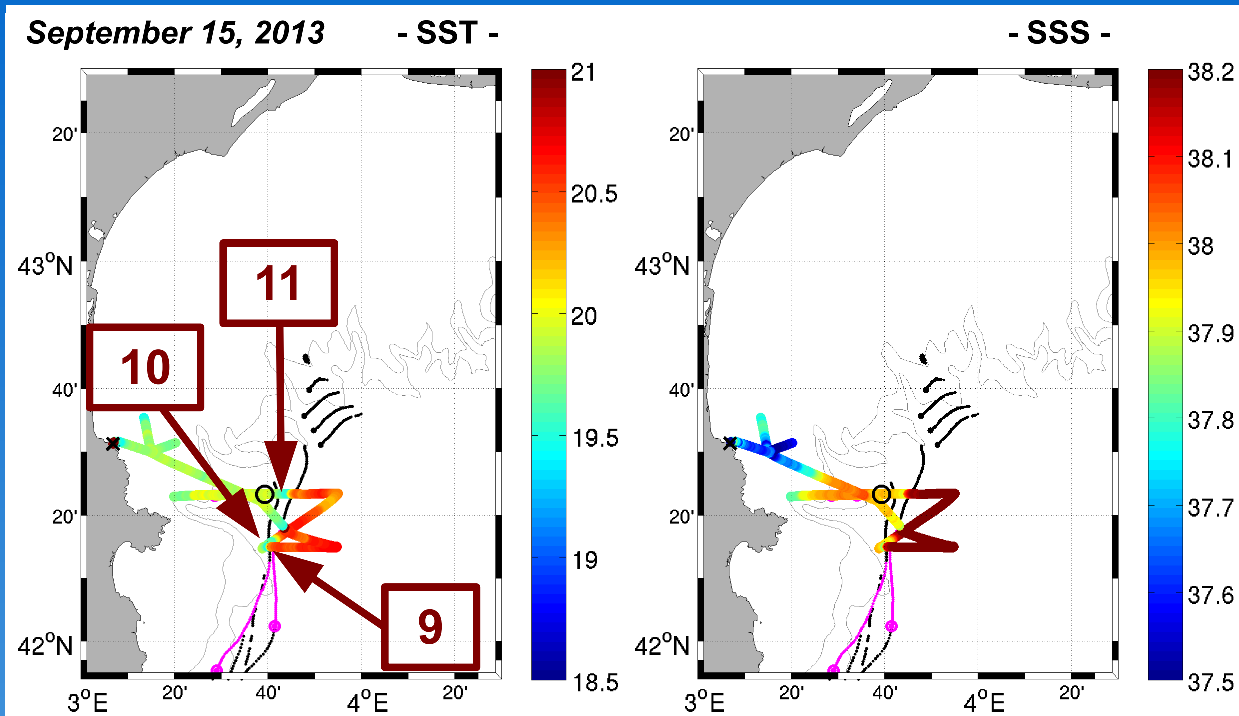


SST and SSS from ship thermosalinograph

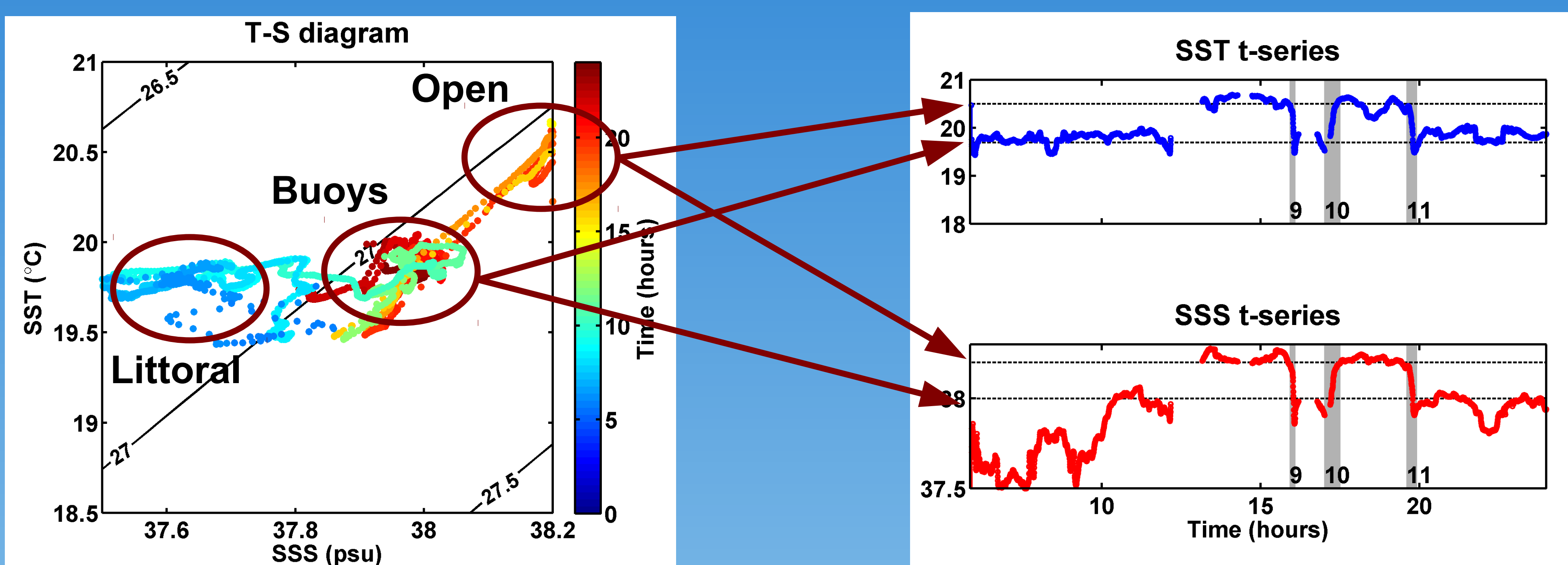


4. Kh

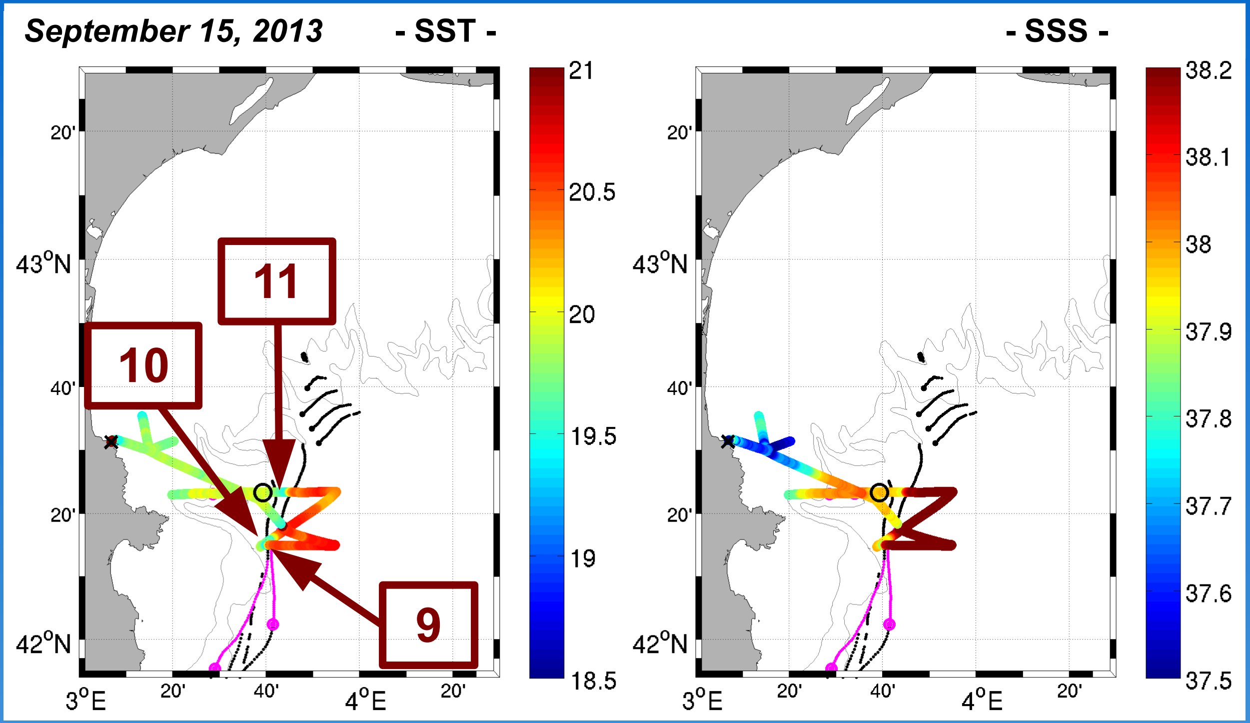
Cross-front transects



SST and SSS from ship thermosalinograph

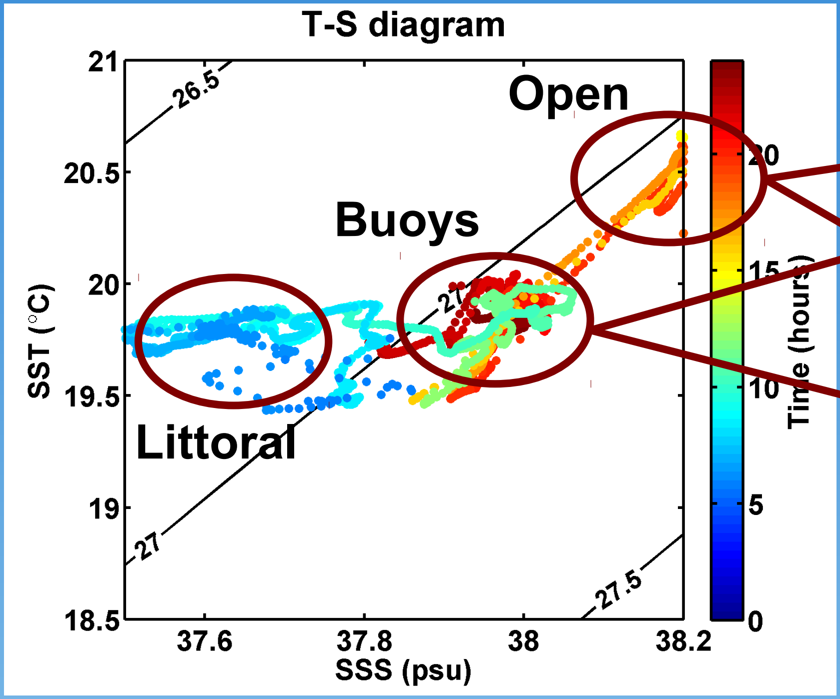


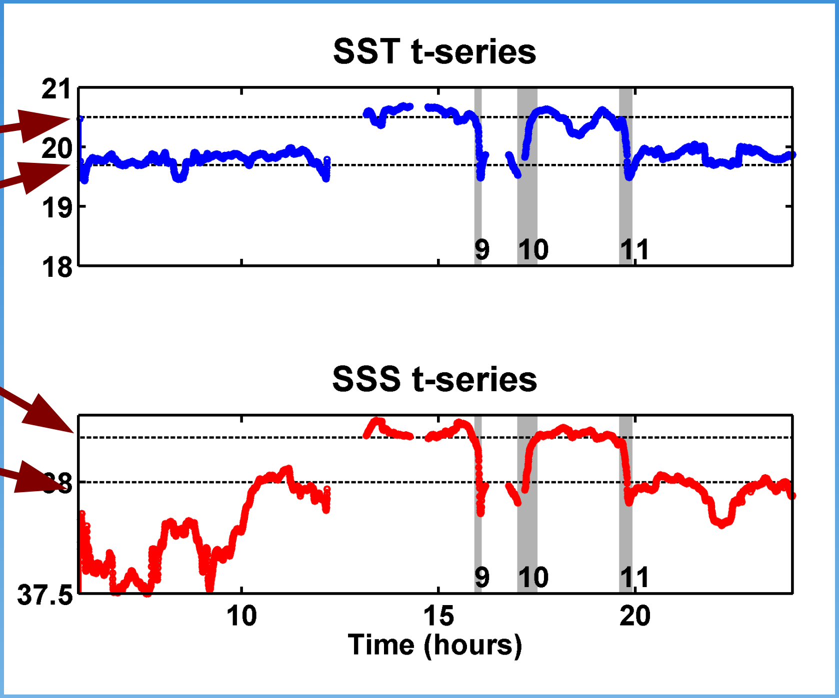
4. Kh Cross-front transects



SST and SSS from ship thermosalinograph

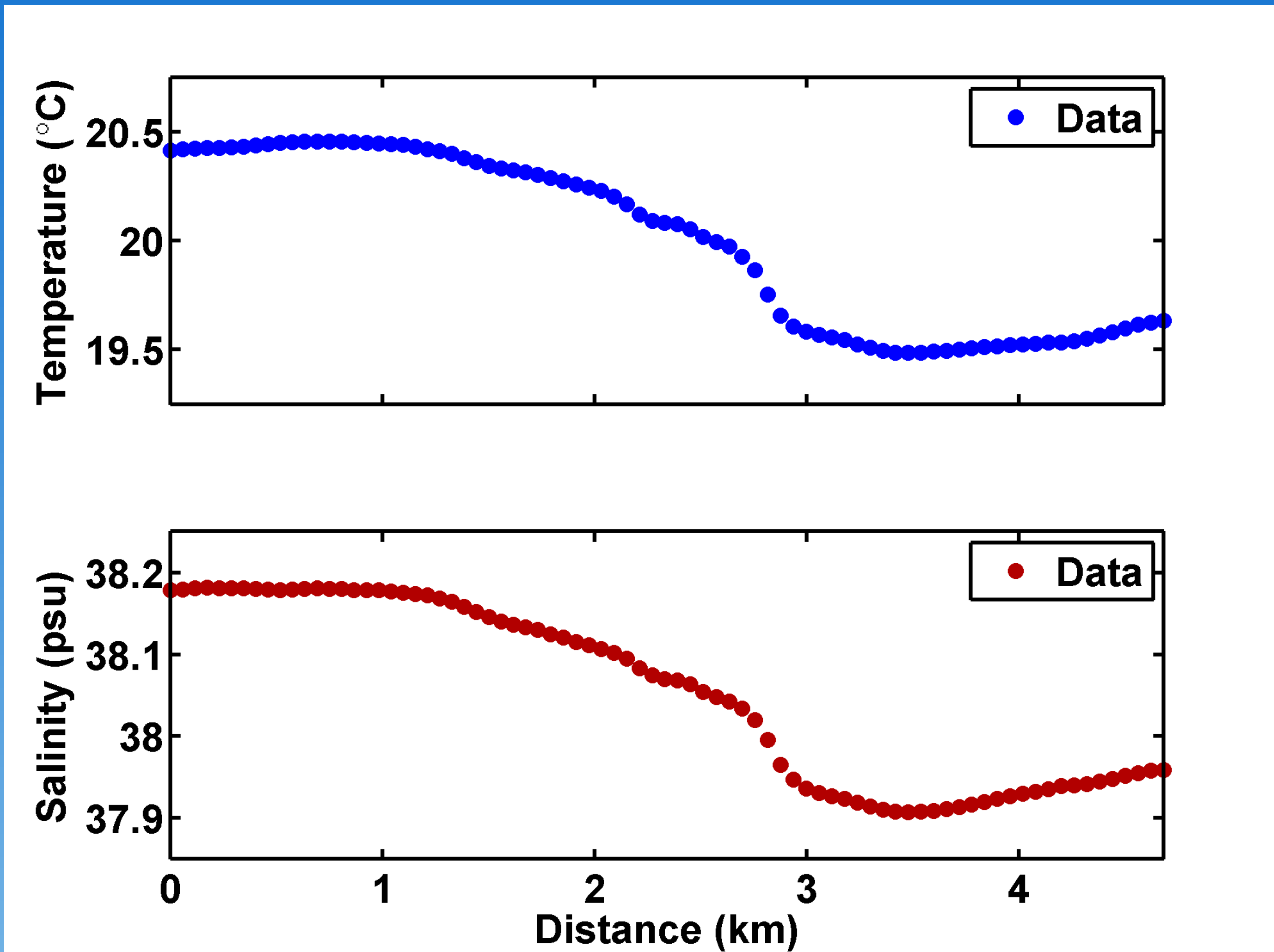
- Identified 30 cross-front transects
- Transects projected on the direction normal to the LCS axis





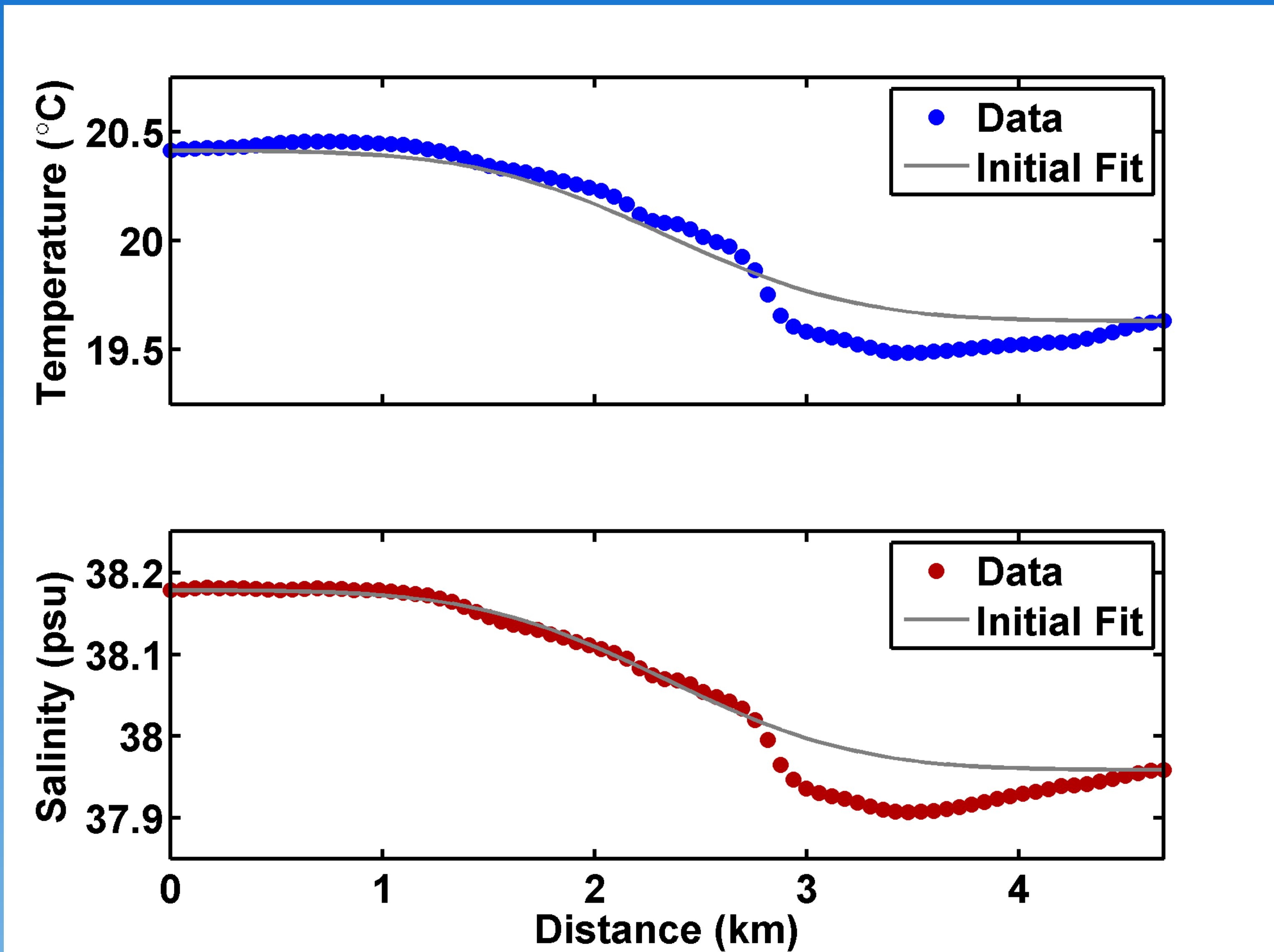
Curve fitting

Example: Transect 11



Curve fitting

Example: Transect 11



Front equation

$$T(x) = C_1 + C_2 \operatorname{erf}(C_3 \ (x - C_4))$$

Initial fit

$$C_1 = \frac{T_2 + T_1}{2}$$

$$C_2 = \frac{T_2 - T_1}{2}$$

$$C_3 = 1$$

$$C_4 = \frac{Dist}{2}$$

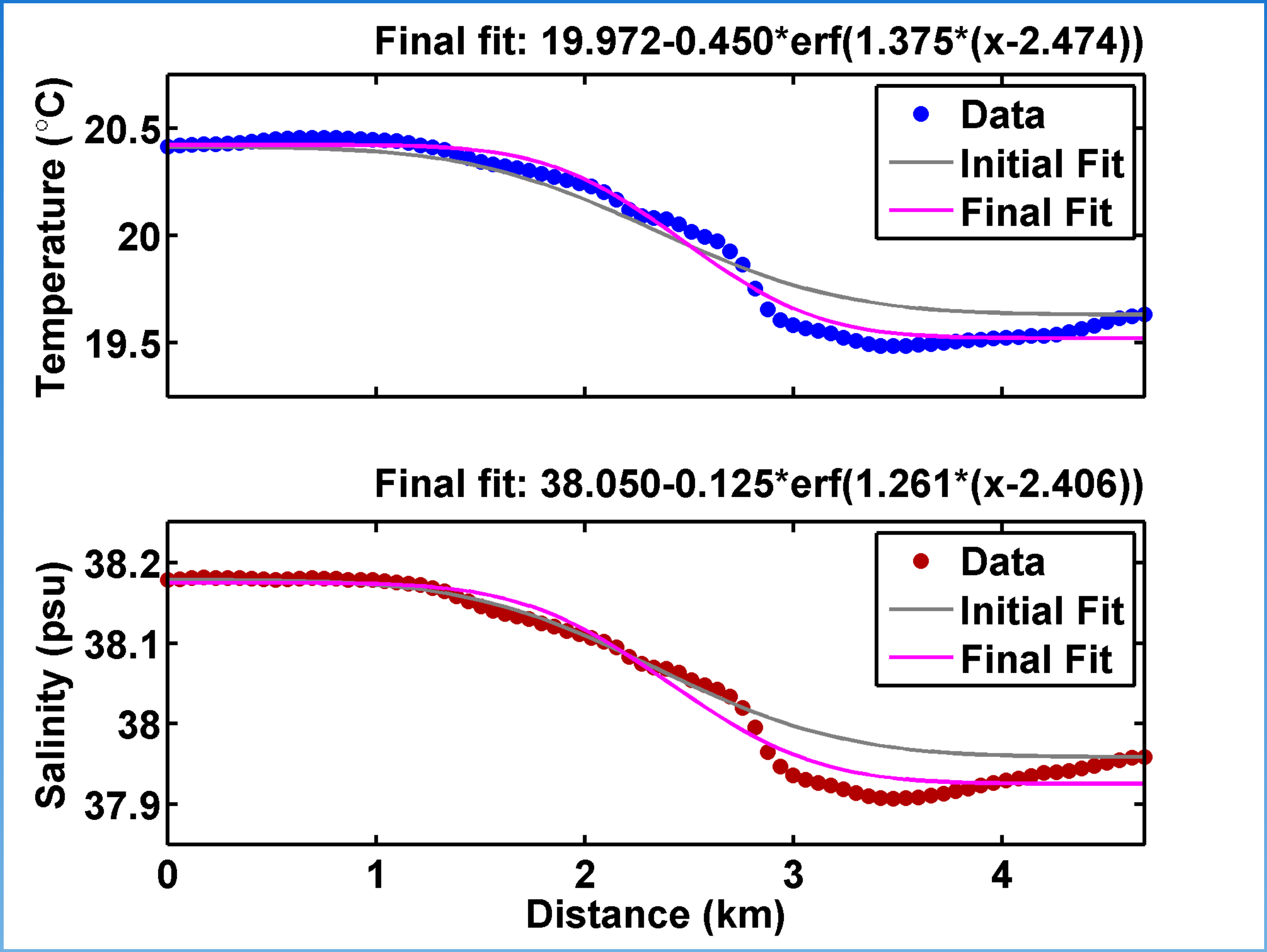
Curve fitting

Example: Transect 11

➔ Parameters from least square estimation using Nelder-Mead simplex direct search

Front equation

$$T(x) = C_1 + C_2\,\mathrm{erf}\,(C_3\ (x - C_4))$$



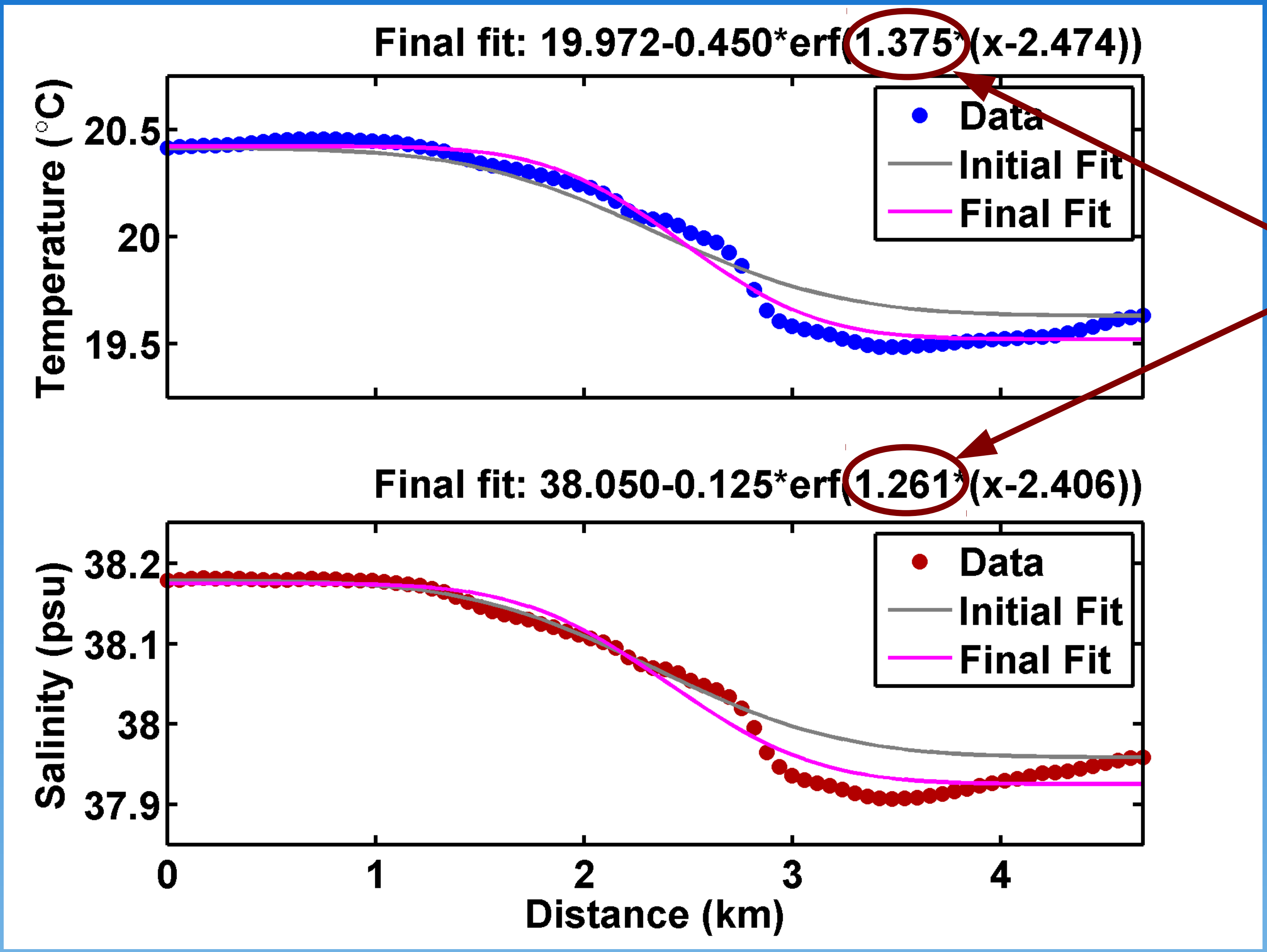
Curve fitting

Example: Transect 11

➔ Parameters from least square estimation using Nelder-Mead simplex direct search

Front equation

$$T(x) = C_1 + C_2 \operatorname{erf}(C_3 \ (x - C_4))$$

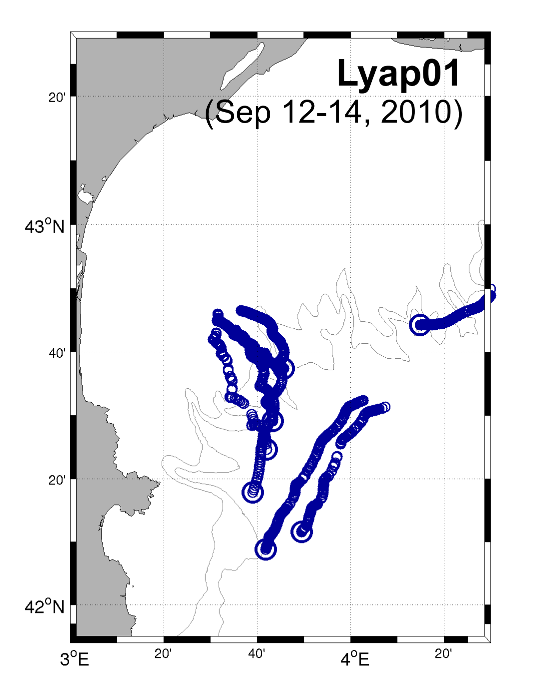


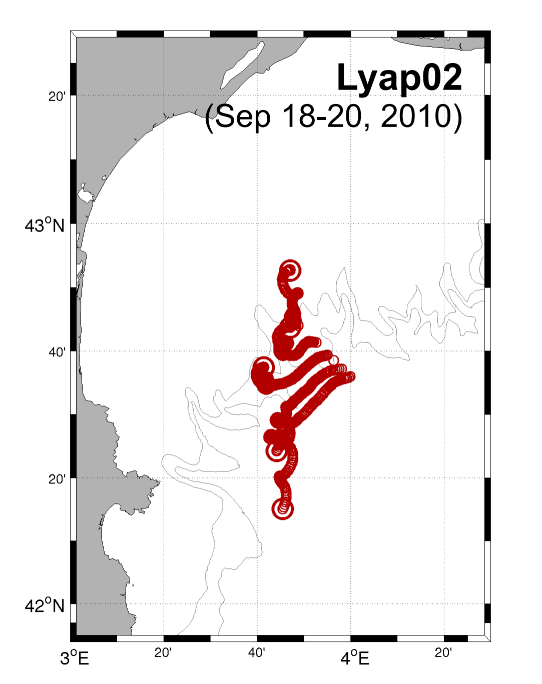
$$C_3 = \frac{1}{\sqrt{2}} \sqrt{\frac{\gamma}{K_H}}$$

- No fit for 11 out of 30 transects: limits of starting assumptions

i.e. impact of vertical dynamics

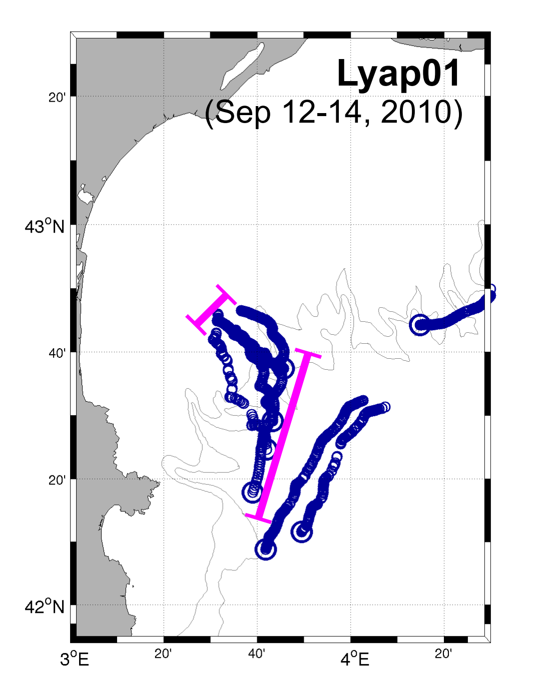
Strain rate

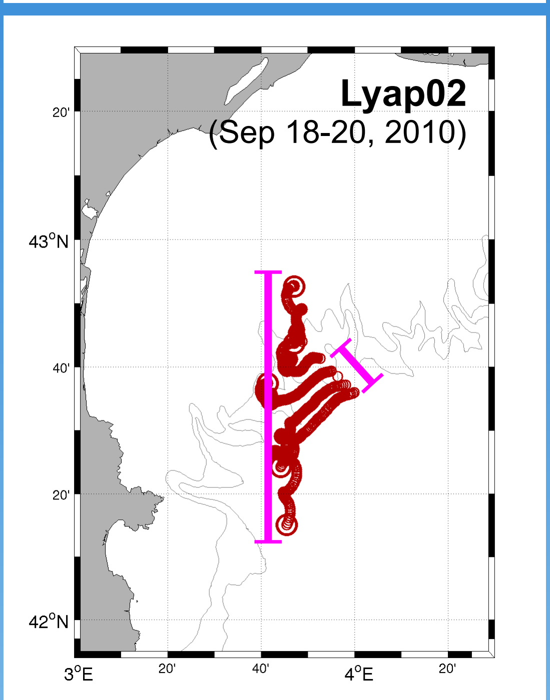




Dispersion patterns of drifter arrays

Strain rate





Dispersion patterns of drifter arrays

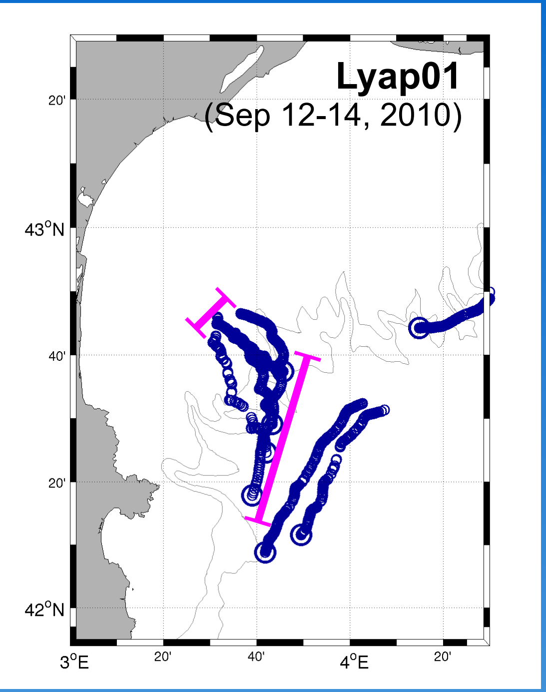
- For each deployment, computed fastest separation rate between buoy couples (analogous to Lyapunov exponent)

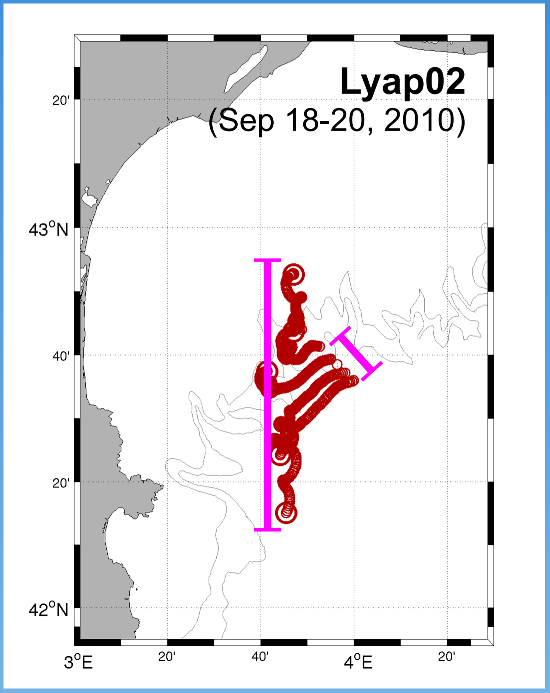
4. Kh

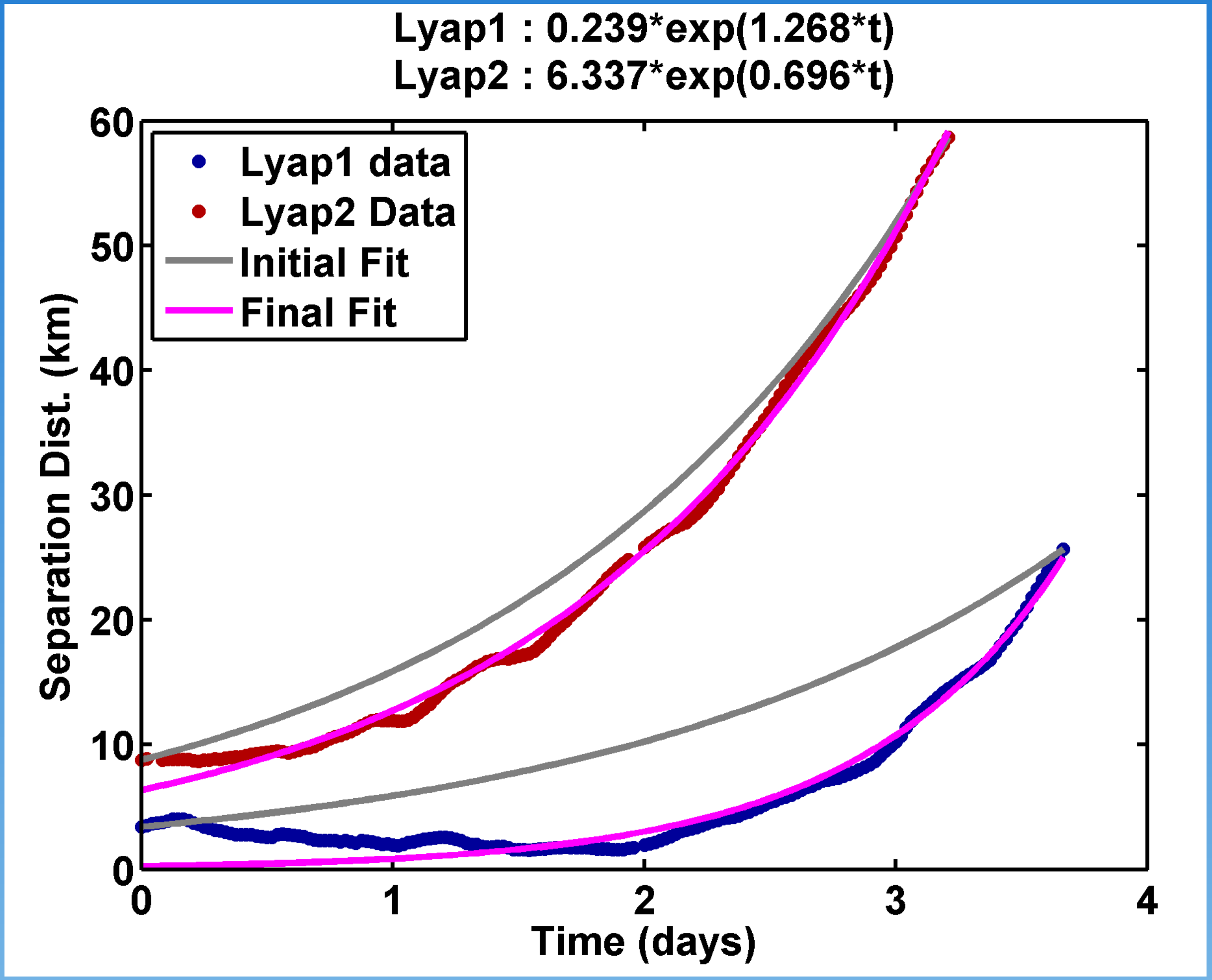
Strain rate

Dispersion patterns of drifter arrays

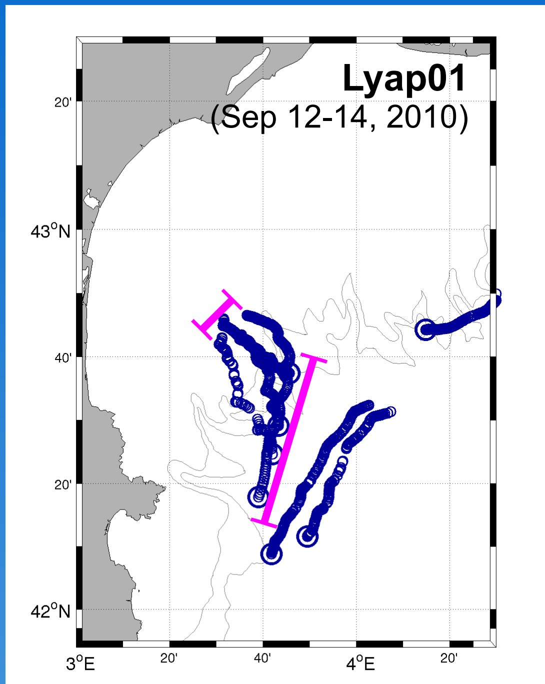
- For each deployment, computed fastest separation rate between buoy couples (analogous to Lyapunov exponent)

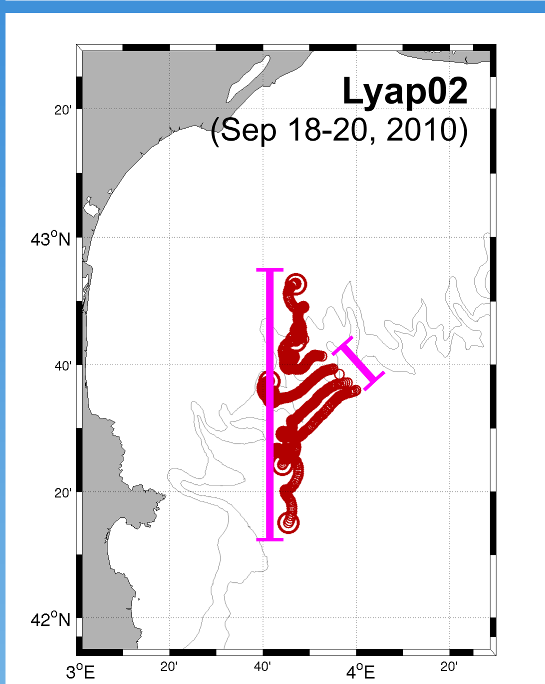






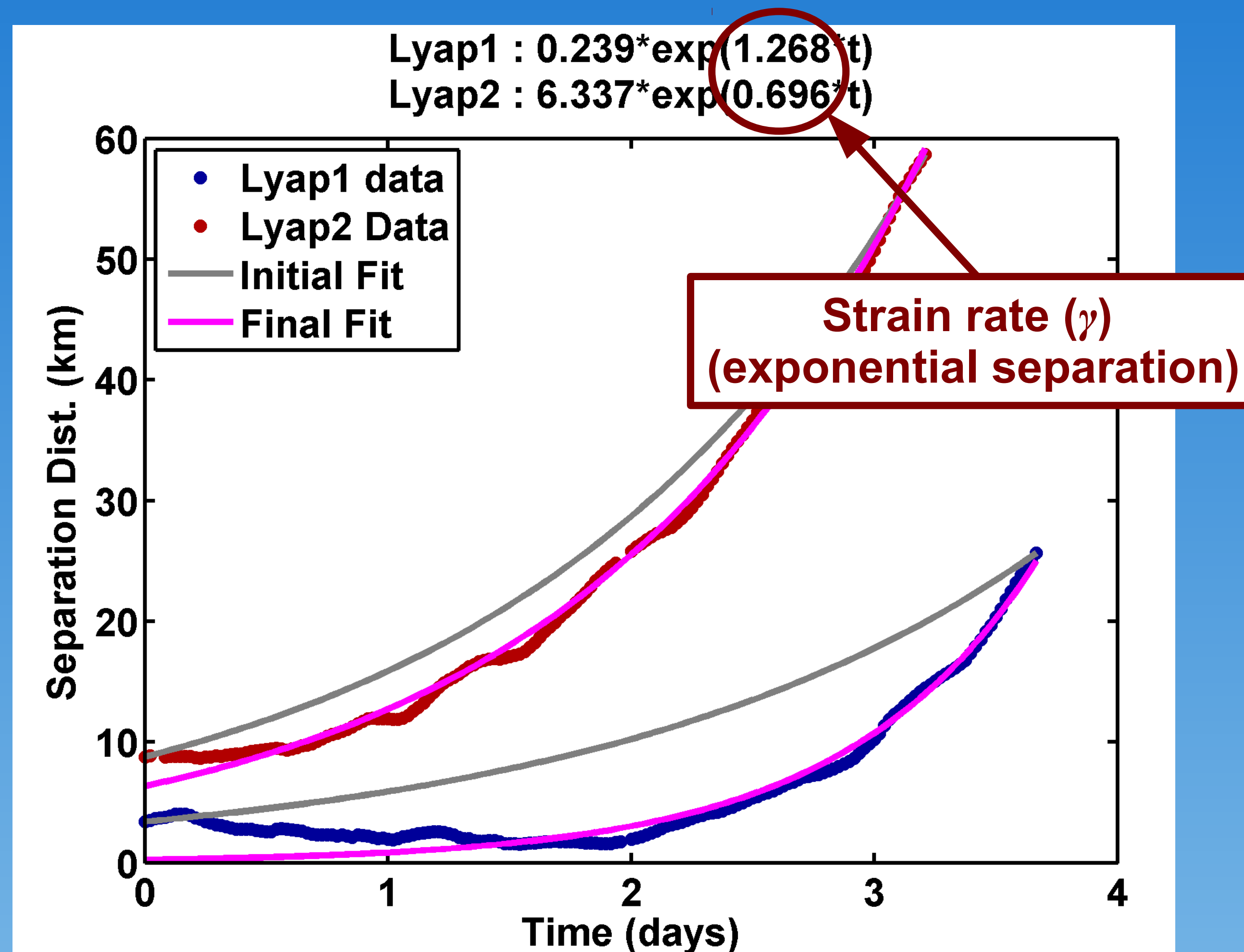
Strain rate





Dispersion patterns of drifter arrays

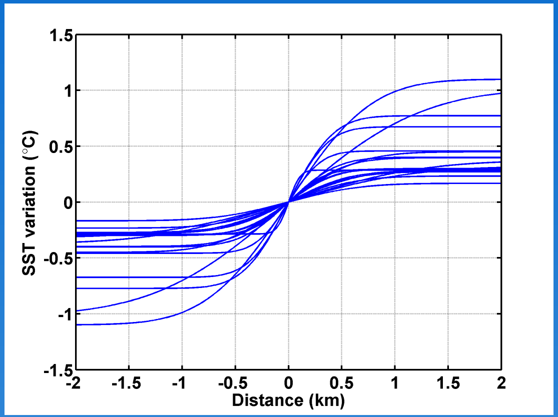
- For each deployment, computed fastest separation rate between buoy couples (analogous to Lyapunov exponent)



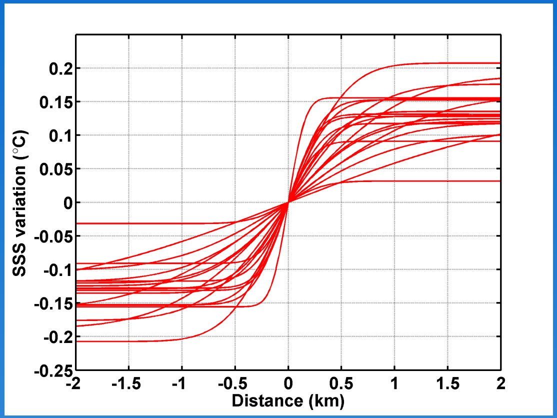
5. Results

Eddy diffusivity coefficients

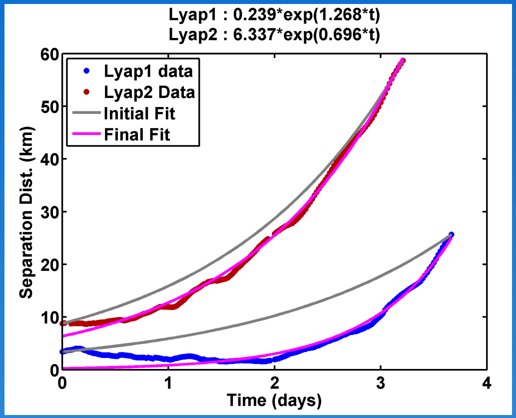
T Front



S Front



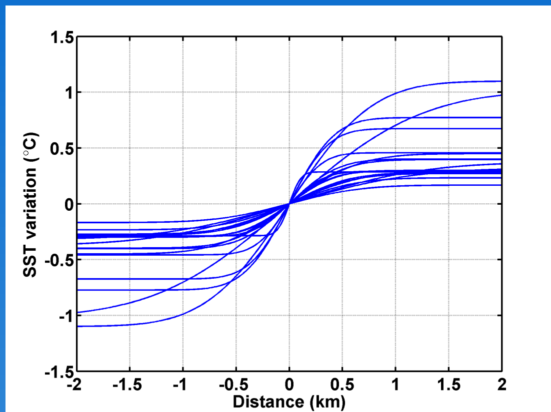
Strain rate



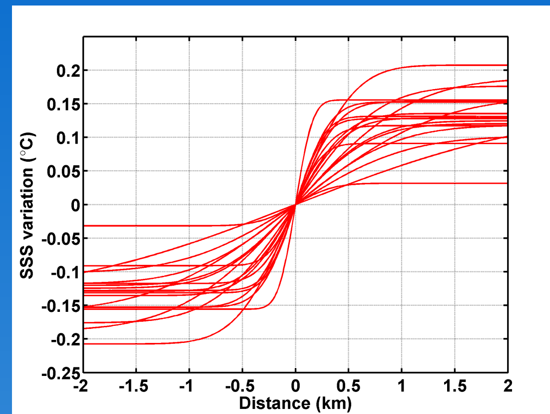
5. Results

Eddy diffusivity coefficients

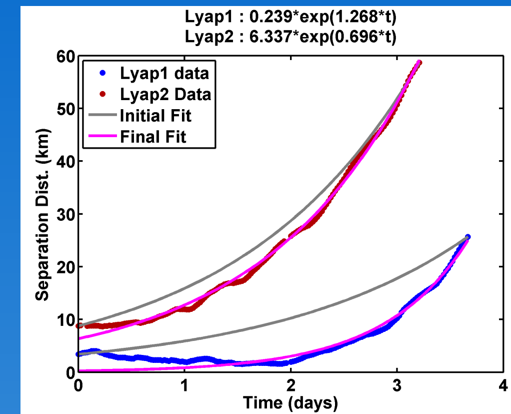
T Front



S Front

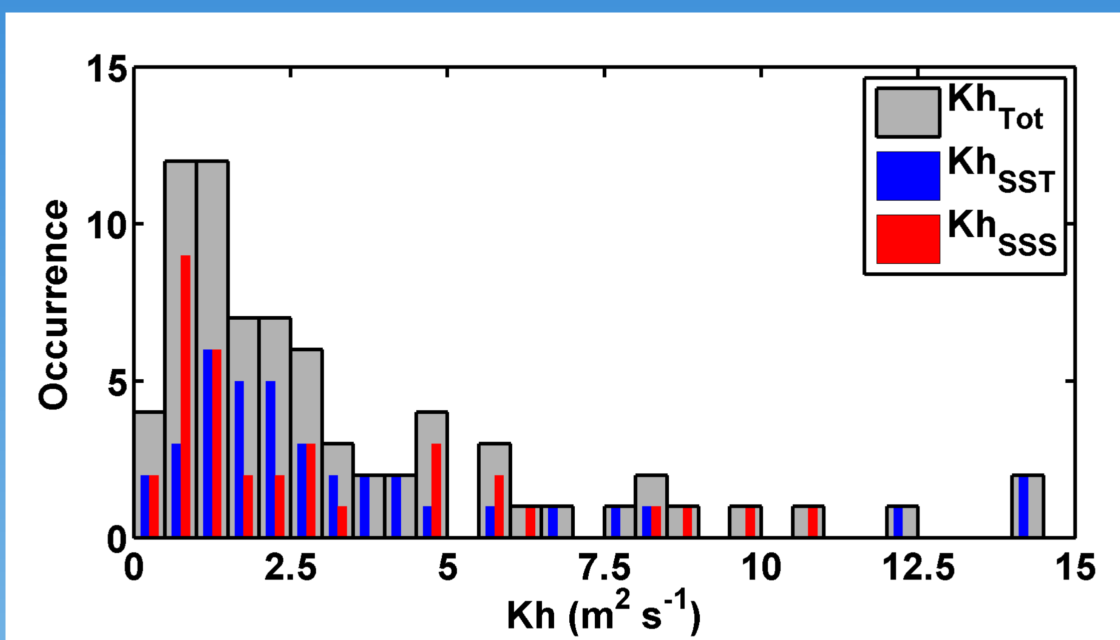


Strain rate



$$K_H = \frac{\gamma}{\left(2\,C3^2\right)}$$

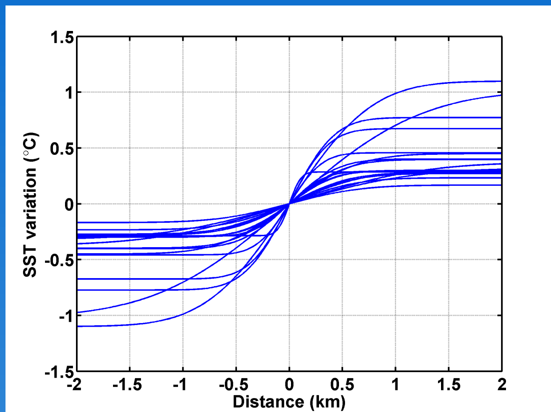
Eddy diffusivity coefficients



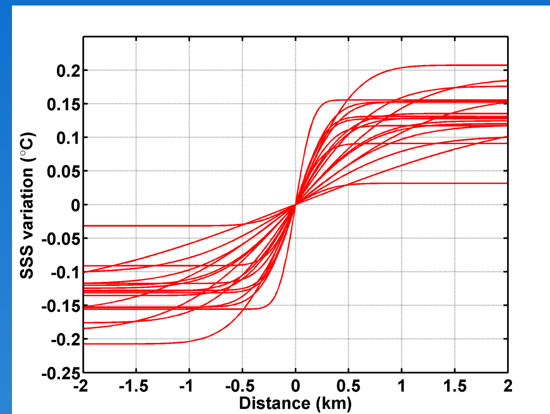
5. Results

Eddy diffusivity coefficients

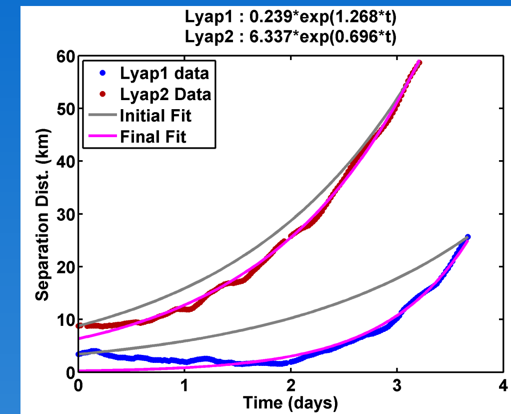
T Front



S Front

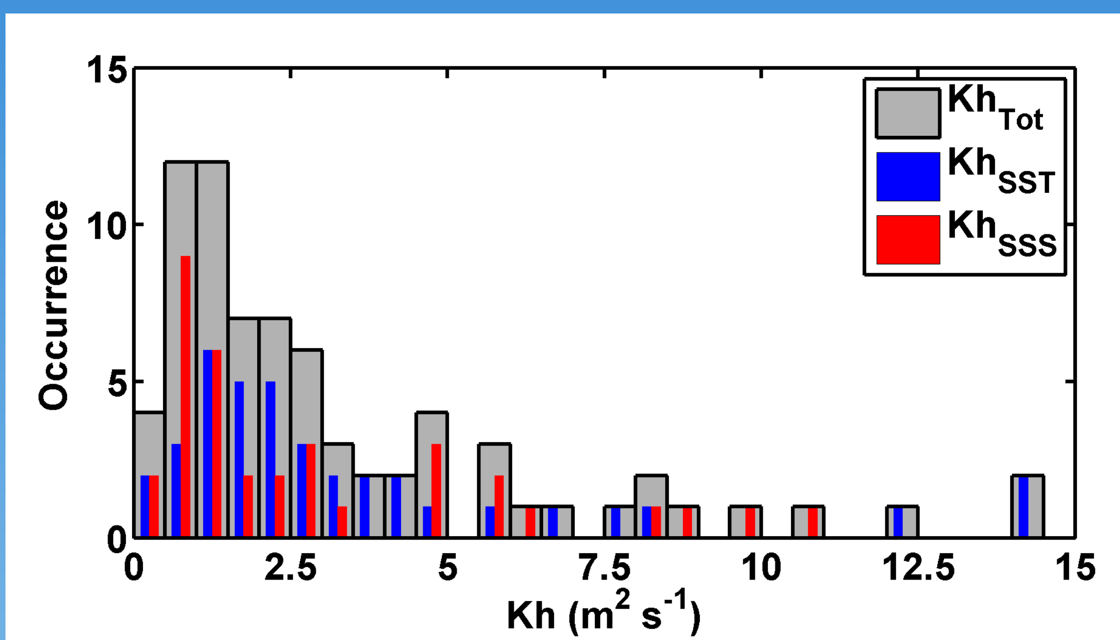


Strain rate



$$K_H = \frac{\gamma}{\left(2\,\mathrm{C}3^2\right)}$$

Eddy diffusivity coefficients



- Kh_{SST} similar to Kh_{SSS}
- 75% of estimates between 0.5 - 5 m s^{-2}
- Front widths range from 250 m to 2km
- Long tail

Conclusions

➔ **New approach relatively simple and cheap**
(i.e. compared to passive tracer release experiments)

➔ **In-situ estimates of Kh at the submesoscale in line with values used in high-resolution numerical models**

➔ **Tail of high values affects Kh statistics (mean and standard deviation) => check starting assumptions:**

- **Steady state**
- **Uniform strain rate**
- **Vertical motions**
- **...**

Perspectives

- ➔ Further dedicated in-situ experiments
- ➔ Test approach from high-resolution models
- ➔ Extend analysis of Kh over wider regions/the global ocean using remote sensed datasets

Perspectives

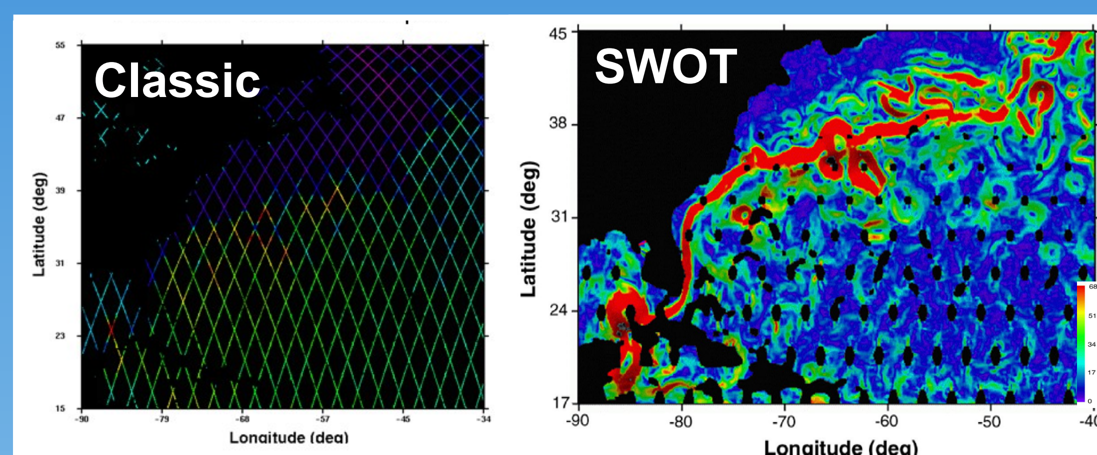
- ➔ **Further dedicated in-situ experiments**
- ➔ **Test approach from high-resolution models**
- ➔ **Extend analysis of Kh over wider regions/the global ocean using remote sensed datasets**

Airborne SWOT sensor tested over key ocean regions (2013-2020)

Surface Water and Ocean Topography

NASA – CNES mission

- **New generation, high-resolution (1Km) altimeter**
- **Launch: Fall 2020**



References

This work has been developed within the project:

Lyapunov Analysis in the CoaSTal Environment (LACOSTE)

Marie Curie Intra-European Fellowship
Call: ***FP7 - PEOPLE - 2011 - IEF***
Leading PI: ***F. Nencioli***

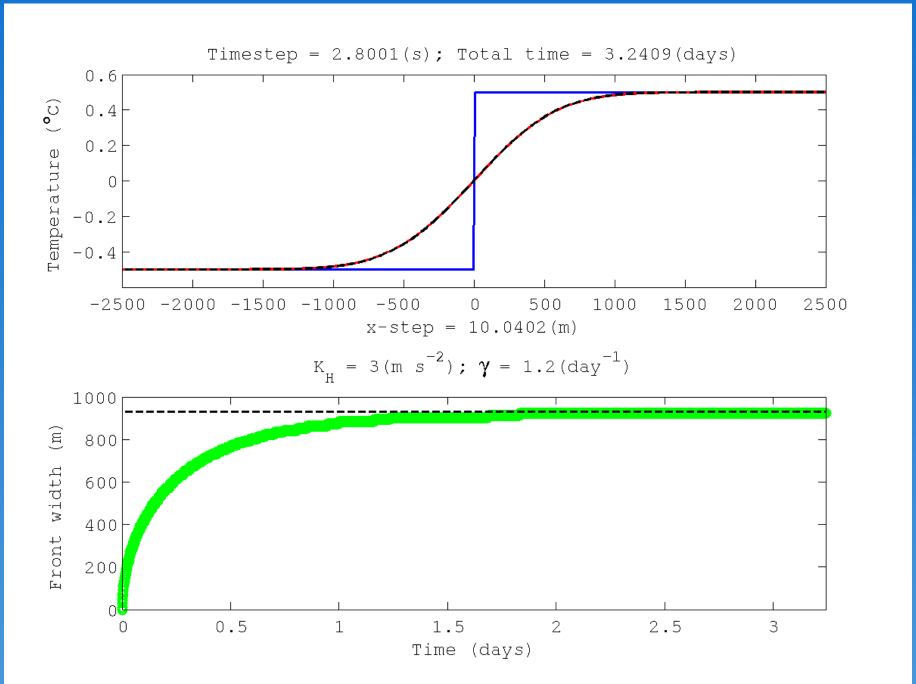
F. Nencioli, F. d'Ovidio, A. Doglioli, A. Petrenko
Surface coastal circulation patterns by in-situ detection of Lagrangian Coherent Structures.
Geophysical Research Letters, 38, L17604, doi:10.1029/2011GL048815

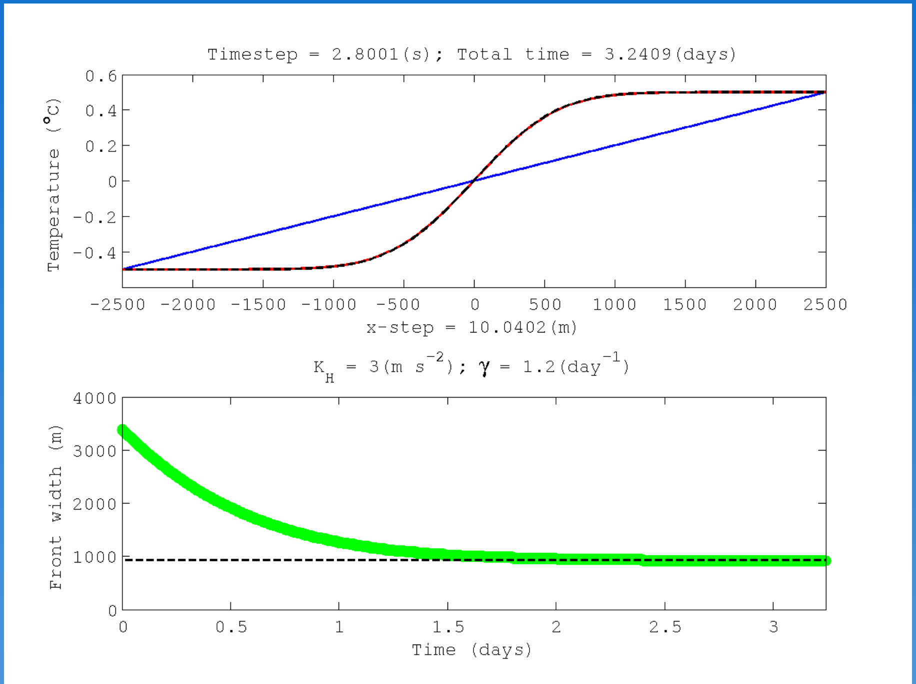
LATEX website: *www.com.univ-mrs.fr/LOPB/LATEX*

EXTRA SLIDES

Numerical solution

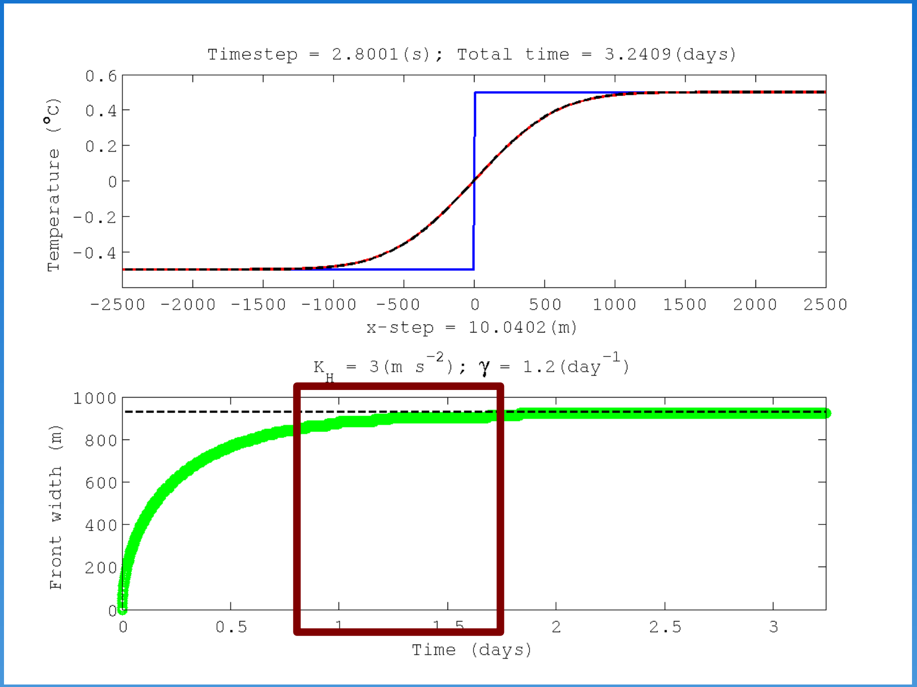
First order upwind scheme

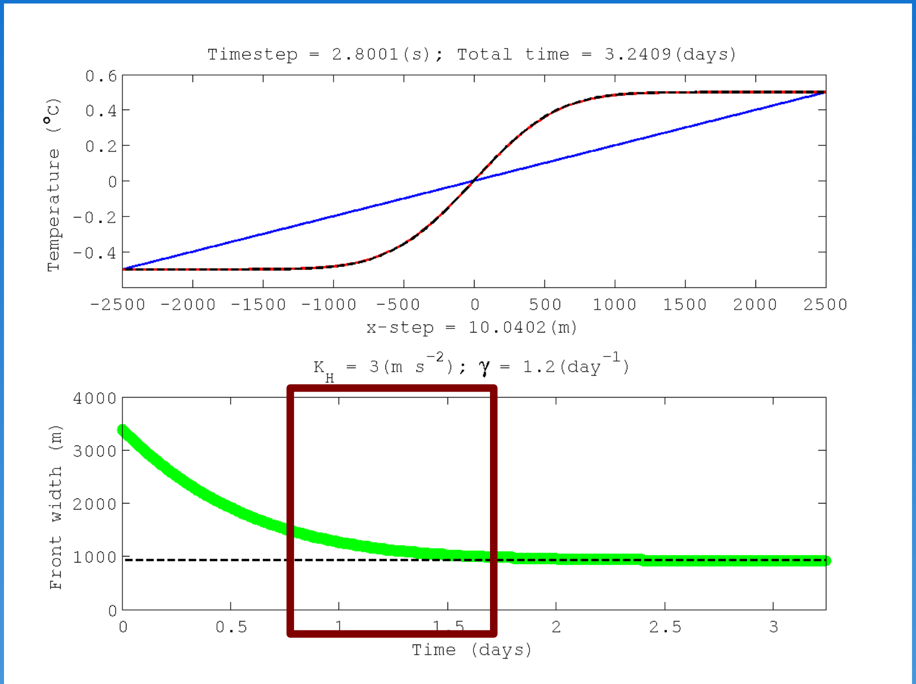




Numerical solution

First order upwind scheme





- Fast adjustment to equilibrium (within 1 day)
- However Kh proportional to square of width
- Even small errors in width could affect estimate of Kh