### Lagrangian model of zooplankton dispersion: numerical schemes comparisons and parameter sensitivity tests

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**Abstract** This paper presents two comparisons or tests for a Lagrangian model of zooplankton dispersion: numerical schemes and time steps. Firstly, we compared three numerical schemes using idealized circulations. Results showed that the precisions of the advanced Adams-Bashfold-Moulton (ABM) method and the Runge-Kutta (RK) method were in the same order and both were much higher than that of the Euler method. Furthermore, the advanced ABM method is more efficient than the RK method in computation - al memory requirements and time consumption. We therefore chose the advanced ABM method as the Lagrangian particle-tracking algorithm. Secondly, we performed a sensitivity test for time steps, using outputs of the hydrodynamic model, Symphonie. Results showed that the time step choices depend on the fluid response time that is related to the spatial resolution of velocity fields. The method introduced by Oliveira et al. (2002) is suitable for choosing time steps of Lagrangian particle-tracking models, at least when only considering advection.

Keywords: Lagrangian; Numerical scheme; Time steps; Zooplankton

### **1 INTRODUCTION**

Zooplankton is an important part of marine ecosystems. Investigating the relationships between zooplankton and marine ecosystems is crucial for understanding the mechanisms guiding biological production (Batchelder et al., 2002). Owing to their natural complexity, zooplankton cannot be fully studied using only in-situ observations and laboratory experiments. In such a context, numerical modelling is a particularly useful tool since it permits a synthesis of information from different sources: laboratory experiments, in-situ parameters and measurements. Furthermore, it allows the construction and investigation of scenarios with a large number of complex and interactive physical and biological processes that act simultaneously and at different scales.

Two different approaches exist in zooplankton numerical modelling: Eulerian and Lagrangian. Eulerian models have been mainly developed for biogeochemical purposes, with usually a simple representation of zooplankton entities (Fasham et al., 1990; Carlotti et al., 2000). In such models zooplankton is treated as a global chemical component (carbon stock), even if several functional groups are considered. Lagrangian models have been mainly developed for zooplankton dispersal, to follow the behaviour of the zooplankton as individuals. Recent researches have highlighted the potential of Lagrangian models for zooplankton dispersal simulation, to examine the role played by various physical processes, to study transport processes over an entire basin and to simulate complex and interactive processes acting at different scales (Miller et al., 1998; Guizien et al., 2006; Speirs et al., 2006; Lett et al., 2007).

It is still a challenge to build a Lagrangian zooplankton dispersal model. How zooplankton organisms exchange material, find prey, avoid predation and reproduce is greatly dependent on their physical environment. Behaviours such as diel vertical migration or ontogenetic vertical migration may be influenced by both extrinsic (light, food concentration) and intrinsic (individual's size, or nutritional status) factors. Therefore, the model parameter space is multidimensional and includes the time integration scheme, the maximum acceptable time step, the flow field temporal averaging, the inclusion of convective events and motility, the treatment of random fluctuations and trajectories near model boundaries and so on. It is not feasible to discuss all these factors in one paper. Here we focus on the choices of time integration schemes and time steps. We will perform further sensitivity tests of more parameters and schemes in later work.

The time integration scheme is critical to a Lagrangian zooplankton dispersal model because of its great impacts on the accuracy, efficiency, and memory requirements. The three widely used schemes are the Euler method (Parada et al., 2003; Guizien et al., 2006; Lett et al., 2007), the Runge-Kutta (hereafter RK) method (Batchelder et al., 2002; Oliveira et al., 2002; Tittensor et al., 2003) and the Adams-Bashfold-Moulton (ABM in the following) method (Peliz et al., 2007; Carr et al., 2008). Darmofal et al. (1996) found that multi-stage schemes required at least three times more internal data storage than multi-step schemes of equal order. They also observed that multistage schemes were generally more accurate within the stability bounds. Garcia et al. (1999) have also compared the Euler method and the RK method by using simple numerical experiments. However, to our knowledge, there is no study comparing the three schemes for zooplankton dispersal simulations.

The work is a contribution to the project LAPLACE (CNRS Programme EC2CO) and the Fund for Creative Research Groups by NSFC (No. 40821004) and the project by NSFC (No. 40706059).

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Time step is an important parameter in a Lagrangian particle-tracking model, which is related to the accuracy and efficiency of the simulations. Darmofal et al. (1996) showed that the time step should be at most in the order of the physical timescale of the flow. Oliveira et al. (2002) suggested that the time step should depend on the ratio of the particle to fluid response times, namely the Stokes number. Guizien et al. (2006) also pointed out that the integration time step is constrained by the particle-eddy interaction time, which is close to the barotropic model time step of the 3D current model. Furthermore, different time steps have been used when considering turbulent diffusion (Visser, 1997; Ross et al., 2004; Peliz et al., 2007) and biological factors (Miller et al., 1998; Batchelder et al., 2002).

In the next section, we compare the three different numerical schemes using idealized circulation conditions. Then, in section 3, we perform sensitivity tests by choosing different time steps. In the following text, we will use the term "particles" to describe zooplankton individuals.

### 2 NUMERICAL SCHEMES COMPARISON

For one particle, the Lagrangian particle-tracking algorithm is based on the following vector equation (e.g. Garcia et al., 1999):

$$\frac{dX}{dt} = \vec{V_a}(X,t) + \vec{V_d}(X,t)$$
(1)

where *X* is the particle location

 $V_a$  is the advective velocity at the particle location and

 $V_d$  is a stochastic fluctuation, which is related to the turbulence and small scale eddies.

If we set  $\vec{V} = \vec{V}_a + \vec{V}_d$  then integrate Eq. (1), we can attain:

$$X(t^{n+1}) = X(t^n) + \int \vec{V}(X,t) dt$$
 (2)

Here *dt* is the time step with  $dt=t^{n+1}-t^n$  and *n* is the time index such that  $t^n = ndt$ .

Different mathematical methods used to integrate Eq. (2) numerically give rise to various particle-tracking schemes. In this paper, we compare the three methods which have been widely used to solve Eq. (2).

The first scheme is the Euler method, which is very simple to program (Parada et al., 2003; Guizien et al., 2006; Lett et al., 2007). Generally, it is implemented as:

$$X^{n+1} = X^n + \vec{V}_p(X^n, t^n) dt$$
 (3)

An advantage of this scheme is that only one velocity field  $\vec{V}_f(X,t^{n+1})$  is needed to calculate the particle velocity  $\vec{V}_p(X,t^{n+1})$  at the new particle position  $(X^{n+1})$ . The current particle position  $(X^n)$  and particle velocity  $\vec{V}_p(X,t^n)$  are also needed, but they are only point data for each particle (*i.e.*, not field data). In a three-dimensional flow field, this adds 6 additional words of storage for each particle per particle path (Darmofal et al., 1996). The storage of the data can be referred to as the internal memory usage (IMU hereafter) because it is generally done in the computer's internal memory for each step of the integration. Moreover, IMU can also measure the amount of I/O between disk and the CPU required per time step when the data are stored on disk. For the Euler scheme, the total IMU is bounded by  $IMU \le N_f + 6N_p$ , where  $N_f$  is the number of words necessary to store the velocity field and  $N_p$  is the number of particles.

The main drawback of this method is that it is only first-order accurate. The time step must be very small, otherwise particle trajectories may diverge from the real ones (Bennett et al., 1987).

The second scheme is the classic 4-stage RK algorithm, which is widely used in physical or biological Lagrangian particle-tracking modelling (Batchelder et al., 2002; Oliveira et al., 2002). Its application to Eq. (2) may be written as:

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$$X^{n+1} = X^n + (a+2b+2c+d)dt/6$$
 (4)

where

$$a = \vec{V}(X^{n}, t^{n})$$
  

$$b = \vec{V}\left(X^{n} + \frac{dt}{2}a, t^{\frac{n+1}{2}}\right)$$
  

$$c = \vec{V}\left(X^{n} + \frac{dt}{2}b, t^{\frac{n+1}{2}}\right)$$
  

$$d = \vec{V}\left(X^{n} + c\,dt, t^{n+1}\right)$$

This method requires velocities at intermediate times and therefore the intermediate values must be interpolated. Darmofal et al. (1996) expressed a fourth-order interpolation for the velocity, to maintain accuracy:

$$\vec{V}\left(X,t^{\frac{n+1}{2}}\right) = \frac{5}{16}\vec{V}\left(X,t^{n+1}\right) + \frac{15}{16}\vec{V}\left(X,t^{n}\right) - \frac{5}{16}\vec{V}\left(X,t^{n-1}\right) + \frac{1}{16}\vec{V}\left(X,t^{n-2}\right)$$
(5)

The IMU of the RK scheme can be significant because 4 planes of velocity field data ( $N_i$ ) are needed to construct the interpolation. In a 3D flow field, 6 additional words of storage ( $N_p$ ) are needed for each particle per particle path (as has been introduced in the previous sections). The total IMU for the RK scheme is as  $IMU \le 4N_i + 6N_p$ .

The third scheme is an advanced ABM method, which has been already used in zooplankton modelling (Peliz et al., 2007; Carr et al., 2008). The ABM method is probably the most common implementation of the multi-step methods to solve Eq. (2), which is to integrate over a finite step using a cubic interpolating polynomial approximation for  $\vec{V}$ . The advanced ABM method is a predictor-corrector method, combining the Adams-Bashford method (the predictor step) and the advanced Adams-Moulton method (the corrector step).

$$\begin{aligned} \widetilde{X}^{n+1} &= X^n + \\ \frac{\mathrm{d}t}{24} \begin{pmatrix} 55 \vec{V}(X^n, t^n) - 59 \vec{V}(X^{n-1}, t^{n-1}) \\ + 37 \vec{V}(X^{n-2}, t^{n-2}) - 9 \vec{V}(X^{n-3}, t^{n-3}) \end{pmatrix} \end{aligned}$$
(6)

$$X^{n+1} = \frac{19}{270} \widetilde{X}^{n+1} + \frac{251}{270} \left[ X^n + \frac{dt}{24} \begin{pmatrix} 9\vec{V} \left( \widetilde{X}^{n+1}, t^{n+1} \right) + 19\vec{V} \left( X^n, t^n \right) \\ -5\vec{V} \left( X^{n-1}, t^{n-1} \right) + \vec{V} \left( X^{n-2}, t^{n-2} \right) \end{pmatrix} \right]$$
(7)

It should be noted that this scheme is not self-starting, therefore, the RK method is needed to initiate the scheme (*i.e.* the locations in the first four time steps are computed using the RK method).

To calculate the new particle position,  $(X^{n+1})$ , and the correlating particle velocity,  $\vec{V}_p(X^{n+1}, t^{n+1})$ , we need one velocity field,  $\vec{V}_f(X, t^{n+1})$ , the current particle position,  $(X^n)$ , and four particle velocities,  $\vec{V}_p(X^n, t^n)$ ,  $\vec{V}_p(X^{n-1}, t^{n-1})$ ,  $\vec{V}_p(X^{n-2}, t^{n-2})$  and  $\vec{V}_p(X^{n-3}, t^{n-3})$ . Here the velocity field is stored in  $N_f$  words. In a 3D flow field, the current particle position and four particle velocities are stored in 15  $N_p$  words. The total IMU for the advanced ABM scheme is as  $IMU \leq N_f + 15N_p$ .

In general, the storage for the field information is much greater than that for particles. Therefore, the Euler scheme and the advanced ABM scheme require much less memory than the RK scheme. In addition, the RK scheme involves four velocity evaluations, while the advanced ABM scheme performs only one velocity evaluation during one time step particle integration.

We simulated the particle trajectories using the three schemes then compared the accuracy. Following Garcia et al. (1999), we considered a two-dimensional flow field:

The work is a contribution to the project LAPLACE (CNRS Programme EC2CO) and the Fund for Creative Research Groups by NSFC (No. 40821004) and the project by NSFC (No. 40706059).

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 $V_x = ax - by$ 

$$T_y = ay + bx$$
 (8)

where x, y are the Cartesian coordinates and a, b are constants. The analytical solution of particle trajectories is given by

 $x_{\varepsilon}(t) = \exp(at)[x_0 \cos bt - y_0 \sin bt]$  $y_{\varepsilon}(t) = \exp(at)[x_0 \sin bt + y_0 \cos bt]$ (9)

where  $x_0$  and  $y_0$  are the initial particle locations and  $x_{\varepsilon}(t)$  and  $y_{\varepsilon}(t)$  are the exact coordinates at time *t*. We compared model results (*x*,*y*) with the analytical solution ( $x_{\varepsilon}, y_{\varepsilon}$ ) using the following error measure:

$$Err = \sqrt{\left(\frac{Var(x) - Var(x_e)}{Var(x_e)}\right)^2 + \left(\frac{Var(y) - Var(y_e)}{Var(y_e)}\right)^2}$$
(10)

In this experiment, we set a=0,b=0.001, which gives a circular rotating flow field (Fig. 1). The 1000 particles are released randomly in a 10m diameter circular cluster centered at the point (1500, 0). At that point, the velocity is 1.5ms<sup>-1</sup> and therefore a particle takes about 1.75h to complete a revolution.

Fig. 1 presents the velocity field and the trajectories of the 1000 particles, computed using different methods. The trajectory with dt=60s simulated by the advanced ABM scheme is shown as black points. It is indistinguishable from the analytical solution and the trajectory simulated by the RK scheme (data not shown). Instead, the trajectory simulated using the Euler method with dt=60s causes an outward drift (red points in Fig. 1). At about 5 hours after release, the cluster area of the particles simulated using the Euler method was approximately three times larger than the analytical result (data not shown). To obtain the same trajectories of particles as calculated using the advanced ABM scheme with dt=60s, we have to choose the time step dt=1s for the Euler method (data not shown).



Fig 1. Circular flow field given by Eq. (8) for (a=0,b=0.001) and the trajectories of the 1000 particles starting in a 10m diameter circular cluster centered at the point (1500, 0). The simulations with dt=60s of the Euler method and the advanced Adams-Bashfold-Moulton scheme are marked in red and black, respectively.

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Fig. 2 illustrates the error evolution for the three schemes with dt=60s, which grows continuously with time. The error growing speed of the RK scheme is close to the advanced ABM scheme. Both are much lower than that of the Euler scheme.

We performed a convergence test where all 1000 particles were tracked for 5h using the three schemes, with different time steps. The errors were computed using Eq. (10) and are presented in Fig. 3. The RK scheme and the advanced ABM scheme can attain very high precision. For example the errors were 3.6E-06 and 5.0E-05 with dt=100s, respectively. Note that to reach an error of 0.025, the Euler method requires dt=1s while the RK scheme needs dt=520s and the advanced ABM scheme needs dt=400s. This implies that the RK and the advanced ABM scheme are much faster than the Euler method when compared on an equal error basis.

From the idealized circulation experiment, we could observe that the advanced ABM method can attain almost the same precision as the RK method. Both are much higher than the Euler method at the same time step. Thus, the RK scheme and the advanced ABM scheme are much better than the Euler method for numerical computation accuracy.

In conclusion, considering a combination of computation accuracy, efficiency and memory requirements, we chose the advanced ABM scheme as the best Lagrangian particle-tracking algorithm. We used this scheme in the following tests.



Fig 2. Error evaluations for the Euler method, the Runge-Kutta scheme and the advanced Admas-Bashfold-Moulton scheme simulated with the time step *dt*=60s.



Fig 3. Convergence tests of different time steps for the Euler method, the Runge-Kutta scheme and the advanced Admas-Bashfold-Moulton scheme.

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#### **3 MODEL SENSITIVITY OF TIME STEP**

To test the model sensitivity of time step settings, we simulated the particle tracking in a so-called "offline" way, which is opposite to "online". Outputs of the 3D circulation model Symphonie,(described in detail by Marsaleix et al., (2008) and references therein) provided the velocity,  $\vec{V}$  in Eq. (2). The velocity field was daily-averaged and provided every 24 hours. The horizontal mesh was of 3 km and the 41 vertical levels were set in the hybrid coordinate system.

Oliveira et al. (2002) found that the time step depends on the value of the Stokes number,  $S_t$ . If  $S_t \ll 1$ , the particle essentially follows the fluid flow and the time step is set as  $dt=0.1\tau$ . If  $S_t \approx 1$  or  $S_t \gg 1$  the time step is set as  $dt=0.1\tau^P$ . The Stokes number is the ratio of the particle to fluid response time:  $S_t=\tau^P/\tau$ , where  $\tau^P$  and  $\tau$  are the particle and fluid velocity response times, respectively. The fluid response time can be estimated as  $\tau=\delta/u$ , where  $\delta$  is a characteristic dimension of the grid. The particle response time,  $\tau^P$ , can be computed using the formula provided by Oliveira et al. (2002).

In this section, we set  $\delta$  as the horizontal mesh size, *i.e.*  $\delta$ =3000*m*. Therefore, the fluid response time  $\tau$  must be larger than 3000s because the maximum velocity is less than 1ms<sup>-1</sup>. Here we set  $\tau$ =3000s. Using equations. (7), (8) and (9) in the paper of Oliveira et al. (2002) we calculated the particle response time and then attained the value of the Stoke number  $S_t \ll 1$ . Therefore, we set the integration time step as dt=0.1 $\tau$ , *i.e.* dt=300s. To test the model sensitivity of time step choice, we also selected two other time steps: dt=60s and dt=900s. Simulations with the time step dt=60s were considered as definitely having acceptable precision.

We released 21 particles at 20m depth in the line from the point (6°E, 42.5°N) to (6°E, 43°N) (Fig. 4). All particles were tracked passively for 15 days from March 1, 2001.

Fig. 4 and Fig. 5 show the trajectories and the depth variations of particles, respectively. Simulations with the time steps dt=60s, dt=300s and dt=900s are shown in black, green and red, respectively.

Simulations with the time steps 300s and 60s only show small differences in both the trajectories and the depth variations. In Fig.4 the trajectories of particles simulated with the time step of 300s are similar to those simulated with the time step of 60s. Fig. 5 shows similar results: the depth variations of the two different simulations almost overlap each other, especially in simulations of the first 10 days. Compared with simulations with the time step of 300s, simulations with the time step of 900s showed larger discrepancy to those with the time step of 60s. At the end of 15 days tracking we found that the final locations of particles simulated with 900s do not match those with the time step of 60s. Obvious differences between simulations with the time steps 900s and 60s, can be seen in the trajectories (Fig.4) and the depth variations (Fig.5).

Fig. 2 shows that the integrated errors grow as the integrated time increases. This led us to think that if we can keep tracking particles over relatively short integrating periods, the errors might be restrained to a low level. To test this possibility, we introduced an error-control scheme, which separated a long-term simulation into several short time particle-tracking processes.

As shown in Fig. 4 and 5 the discrepancies were very small in the first 5 days for simulations with the two different time steps dt=60s and dt=300s. Therefore, we took 5 days as the short period in the error-control scheme. We tracked particles for 5 days several times instead of simulating for a long time directly. For example, if we track particles for 60 days with error-control scheme, we actually simulate 12 times with only 5 days each time. When particles have been tracked for 5 days, we save all the information and set it as the initial conditions for the following 5-day simulations. We continue to track particles for another 5 days and repeat these steps until the proper simulation period is met.

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Fig 4. Trajectories of the 21 particles tracked 15 days with three different time steps. The initial locations are marked in black circulations and lines in black, green and red represent the trajectories of particles simulated with time steps *dt*=60*s*, *dt*=300*s* and *dt*=900*s*, respectively.



# Fig 5. Depth variations of the 5 particles (of total 21) simulated with three different time steps. The lines in black, green and red represent the depth variations of the particles simulated with time steps dt=60s, dt=300s and dt=900s, respectively.

Fig. 6 shows the depth variations of 5 particles simulated with the time step dt=300s. The results with and without the error-control scheme are presented in green and red, respectively. We also plotted, in black, the simulations with the time step dt=60s without the error-control scheme. We can see that the discrepancy of depth variations has been slightly reduced by using the error-control scheme. Because the error-control scheme does not increase the computation time, it is appropriate to introduce it into our Lagrangian particle-tracking model.

In our model we chose the time step as 300s with the error-control scheme. Note that this time step only fits for the velocity fields with the horizontal precision of 3km. In other simulations with different spatial resolutions, the time step should be chosen according to the method introduced by Oliveira et al. (2002).

In this paper we are only considering the factors of the horizontal advection to restrain the time steps. In fact, the vertical velocity field, fluctuating velocities, sub-grid diffusion, and biological factors (such as zooplankton swimming behaviours) are also important for the time step choices. With these factors, new constraints should be

added for choosing the integration time step. We will introduce the correlated analysis and discussion in another paper.



## Fig 6. Depth variations of the 5 particles (of total 21) simulated as *dt*=300*s* with and without the error-control scheme in green and red respectively, also the simulations with *dt*=60*s* are marked in black.

### **4 CONCLUSIONS**

This article summarizes the fundamentals and testing of a Lagrangian model to simulate zooplankton dispersion. After comparing three numerical schemes, we chose the advanced ABM scheme as the best Lagrangian particle-tracking algorithm. Subsequent model sensitivity tests of time step showed that the time step choices depend on the fluid response time that is related to the spatial resolution of the velocity fields. The method introduced by Oliveira et al. (2002) is suitable for choosing time steps for Lagrangian particle-tracking models, at least when only considering advection. Furthermore, the error-control scheme improves the precision of the simulations to a certain degree.

### Acknowledgments

The authors thank Patrick MARSALEIX and Ziyuan HU for kindly providing the Symphonie output data.

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- The work is a contribution to the project LAPLACE (CNRS Programme EC2CO) and the Fund for Creative Research Groups by NSFC (No. 40821004) and the project by NSFC (No. 40706059).

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