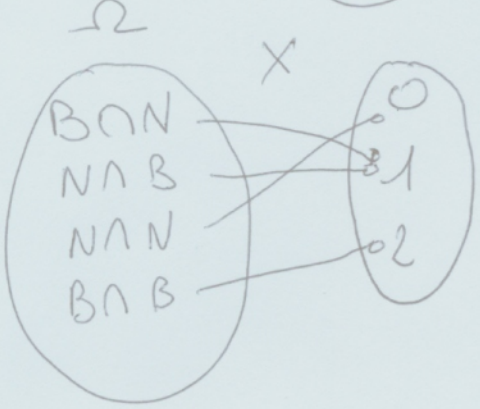
TD 5

Exercice 5 :

3 urnes (I, II, III) contenant des boules blanches et noires



$p(I) = \frac{3}{6}$

$p(II) = \frac{2}{6}$

$p(III) = \frac{1}{6}$

$$p(B \cap N) = p((B \cap N)/I)\, p(I) + p((B \cap N)/II)\, p(II) + p((B \cap N)/III)\, p(III)$$
$$= p(B) \times p(N) \times p(I) + p(B) \times p(N) \times p(II) + p(B) \times p(N) \times p(III)$$
$$= \frac{3}{5} \times \frac{2}{5} \times \frac{3}{6} + \frac{2}{3} \times \frac{1}{3} \times \frac{2}{6} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{6}$$
$$= \frac{18}{150} + \frac{4}{54} + \frac{1}{24}$$
$$\simeq 0,24$$

$$p(B \cap B) = p((B \cap B)/I)\, p(I) + p((B \cap B)/II)\, p(II) + p((B \cap B)/III)\, p(III)$$
$$= p^2(B/I)\, p(I) + p^2(B/II)\, p(II) + p^2(B/III)\, p(III)$$
$$= \left(\frac{3}{5}\right)^2 \times \frac{3}{6} + \left(\frac{2}{3}\right)^2 \times \frac{2}{6} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 \times \frac{1}{6}$$
$$= \frac{9}{25} \times \frac{3}{6} + \frac{4}{9} \times \frac{2}{6} + \frac{1}{4} \times \frac{1}{6}$$
$$= \frac{27}{150} + \frac{8}{54} + \frac{1}{24} \simeq 0,37$$

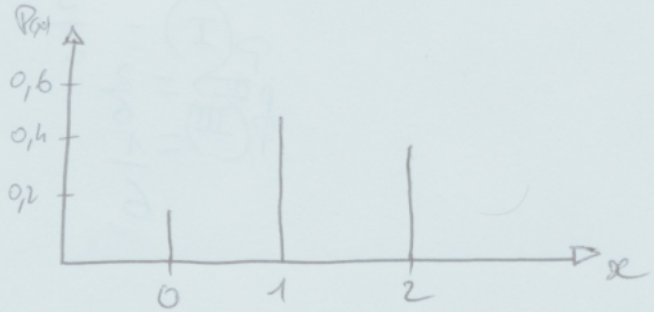
$$p(N \cap N) = \left(\frac{2}{5}\right)^2 \times \frac{3}{6} + \left(\frac{1}{3}\right)^2 \times \frac{2}{6} + \left(\frac{1}{2}\right)^2 \times \frac{1}{6}$$
$$= \frac{4}{25} \times \frac{3}{6} + \frac{1}{9} \times \frac{2}{6} + \frac{1}{4} \times \frac{1}{6} = \frac{12}{150} + \frac{2}{54} + \frac{1}{24} \simeq 0,16$$

$$p(B \cap N) + p(N \cap B) + p(B \cap B) + p(N \cap N) = 1$$

$$p(X=0) = p(N \cap N) \simeq 0{,}16$$

$$p(X=1) = p((N \cap B) \cup (B \cap N)) = 2 \times p(B \cap N) \simeq 0{,}48$$

$$p(X=2) = p(B \cap B) \simeq 0{,}37$$



$$E(X) = \sum_{x \in \mathbb{N}} x \, p_{xx} = 1 \times 0{,}48 + 2 \times 0{,}37 \simeq 1{,}22$$

$$V(X) = E(X^2) - E^2(X) = 1 \times 0{,}48 + 4 \times 0{,}37 - (1{,}22)^2 \simeq 0{,}47.$$