ANALYSE DE FOURIER

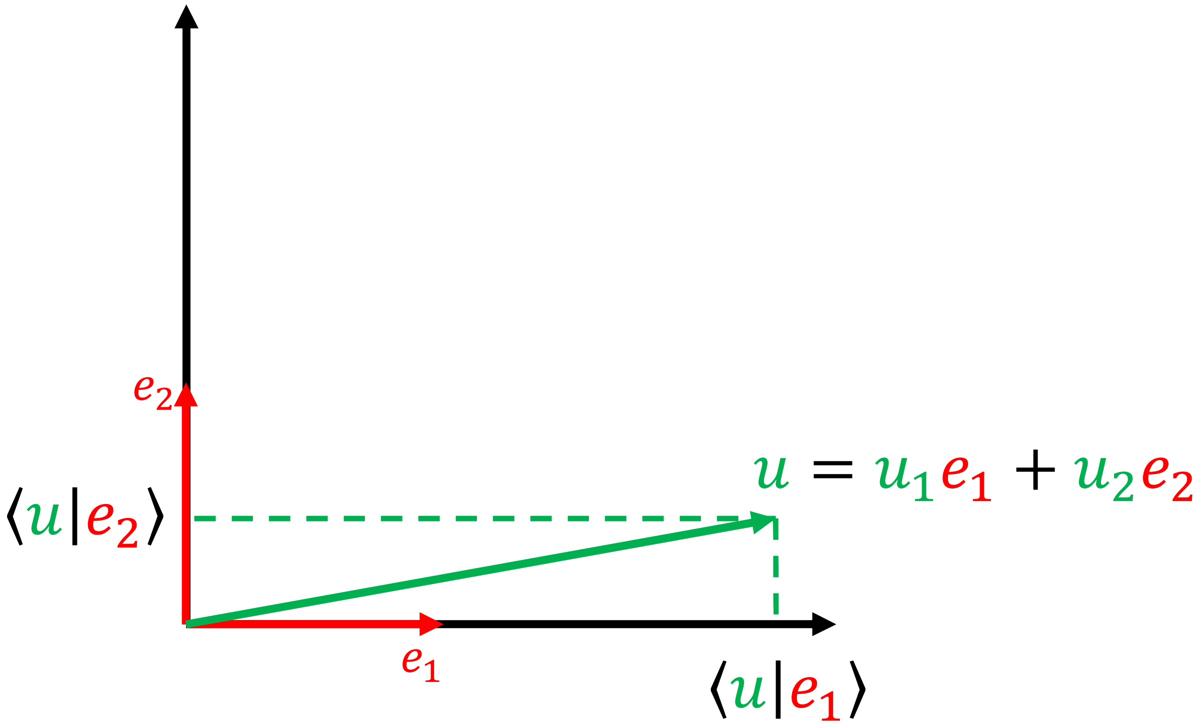
PRODUIT SCALAIRE

Objectif : disposer d'un **outil qui permet de décomposer** des objets complexes en éléments « simples »

- définir l'ensemble des objets à étudier (espace vectoriel)
- définir les éléments simples : base
- décomposer les objets de l'espace vectoriel sur la base au moyen d'un produit scalaire

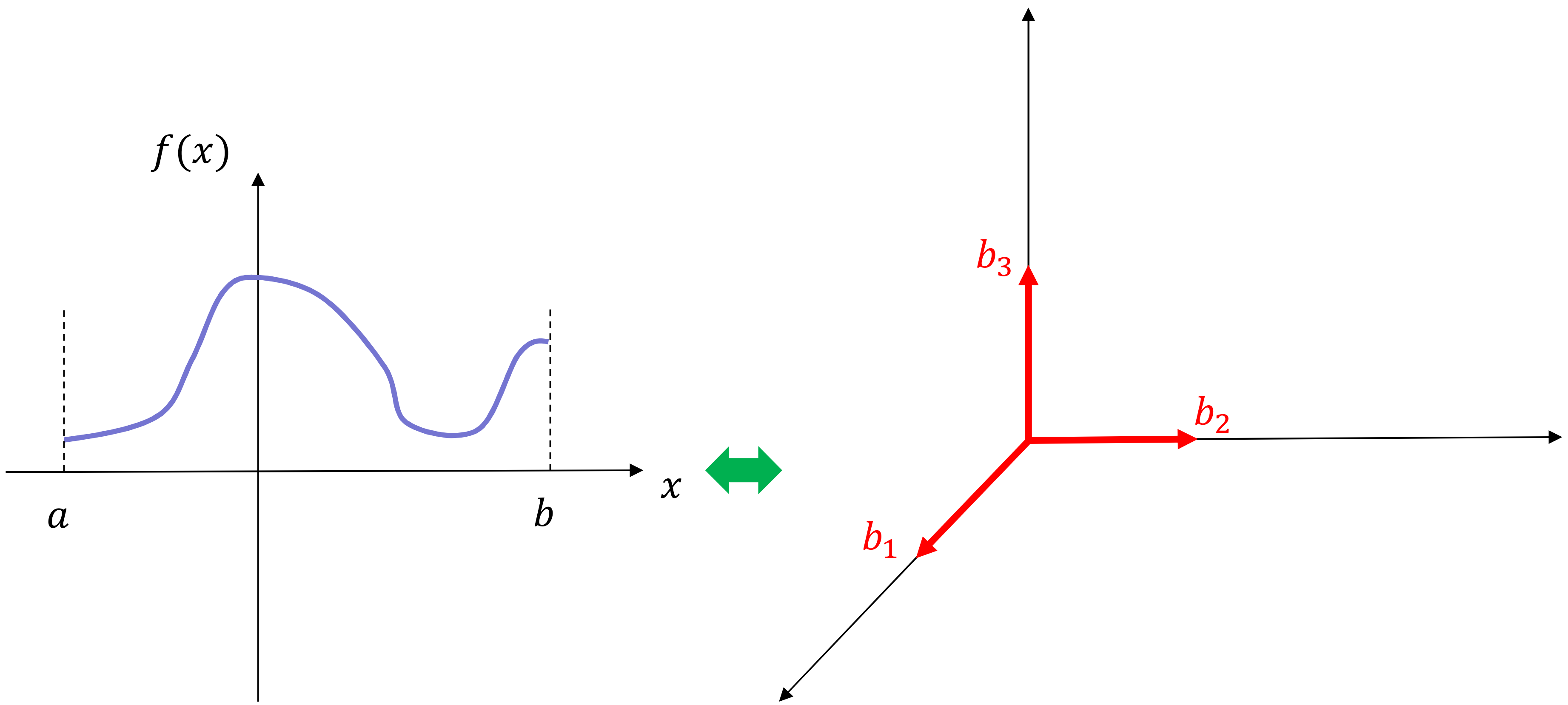
PRODUIT SCALAIRE

$$\langle u | e_i \rangle = \|u\| \|e_i\| \cos\theta_i$$

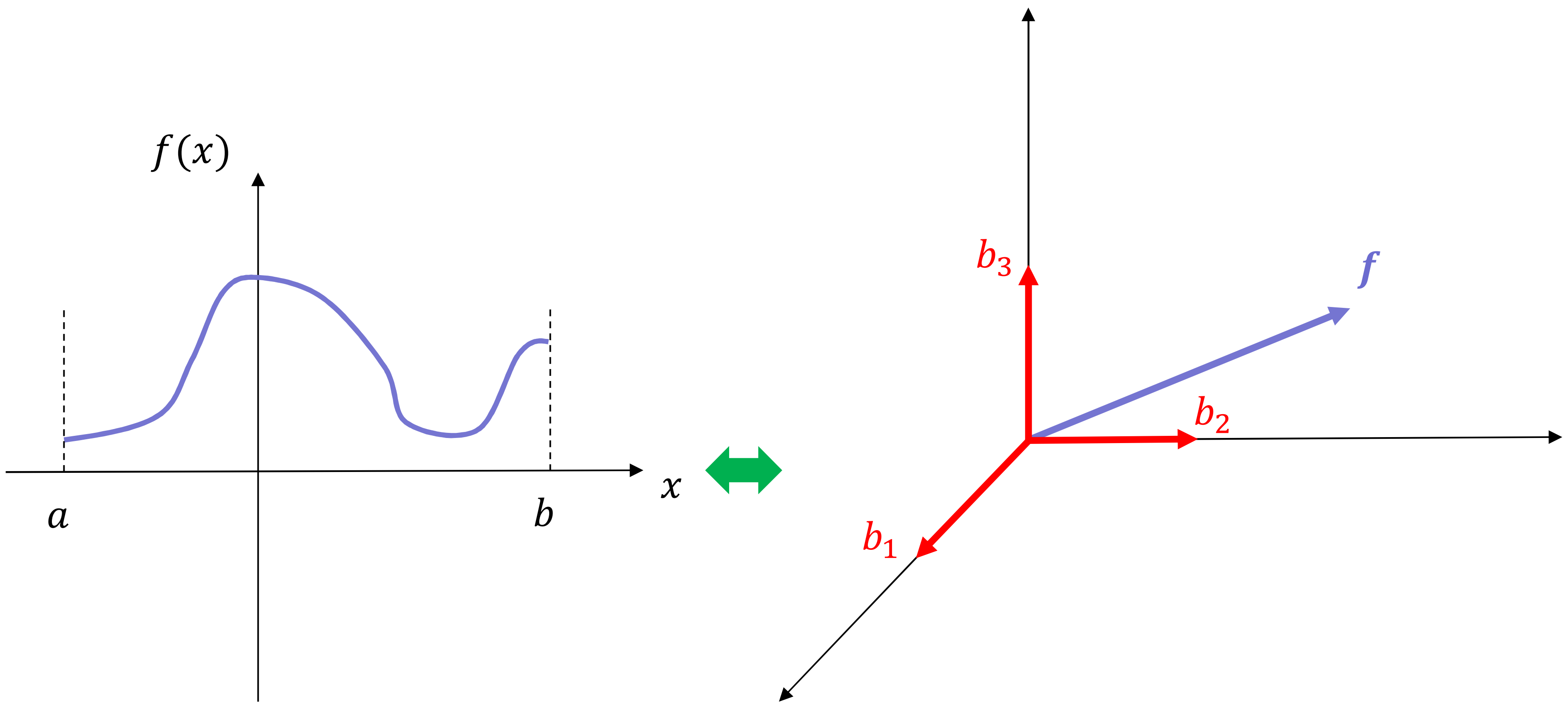


$\langle u | v \rangle = 0$ <=> u et v sont orthogonaux

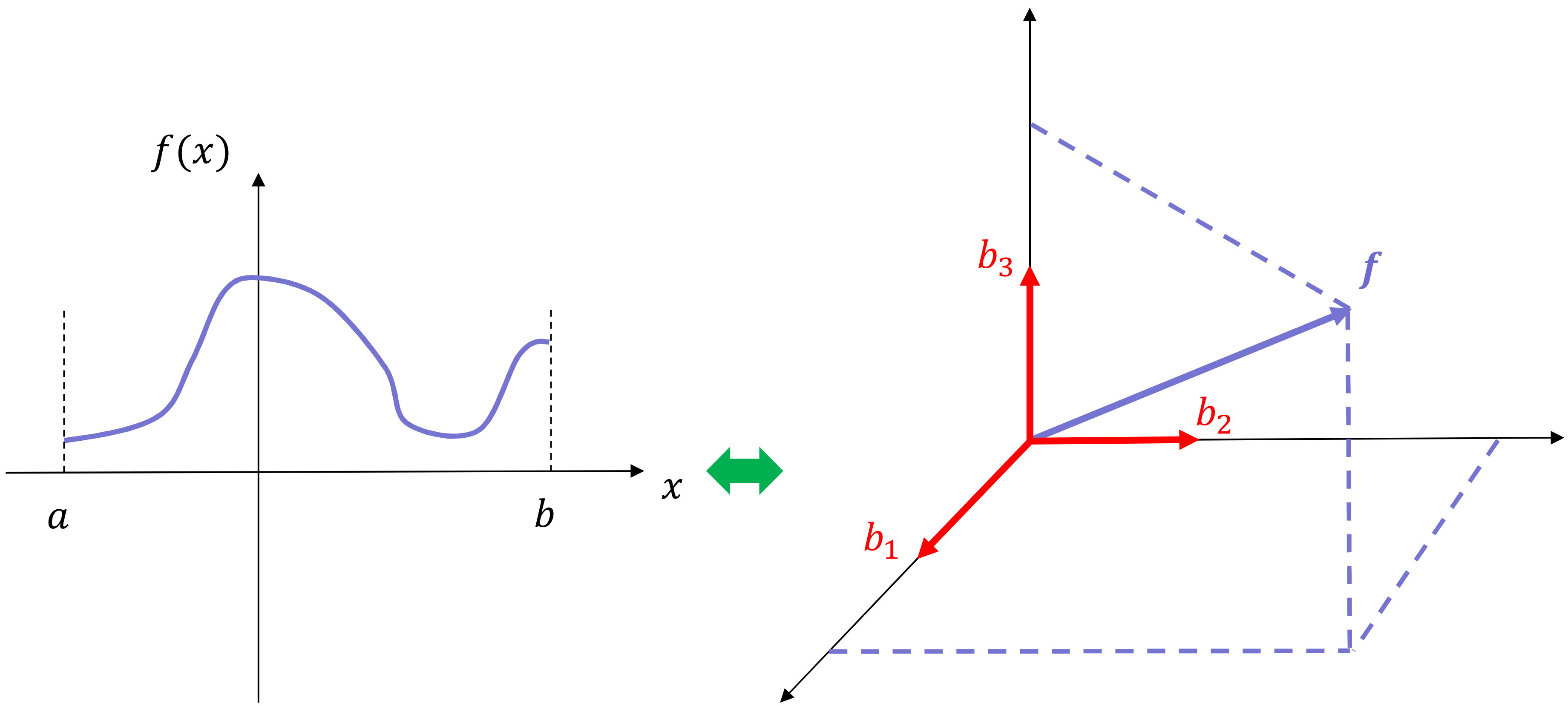
PRODUIT SCALAIRE



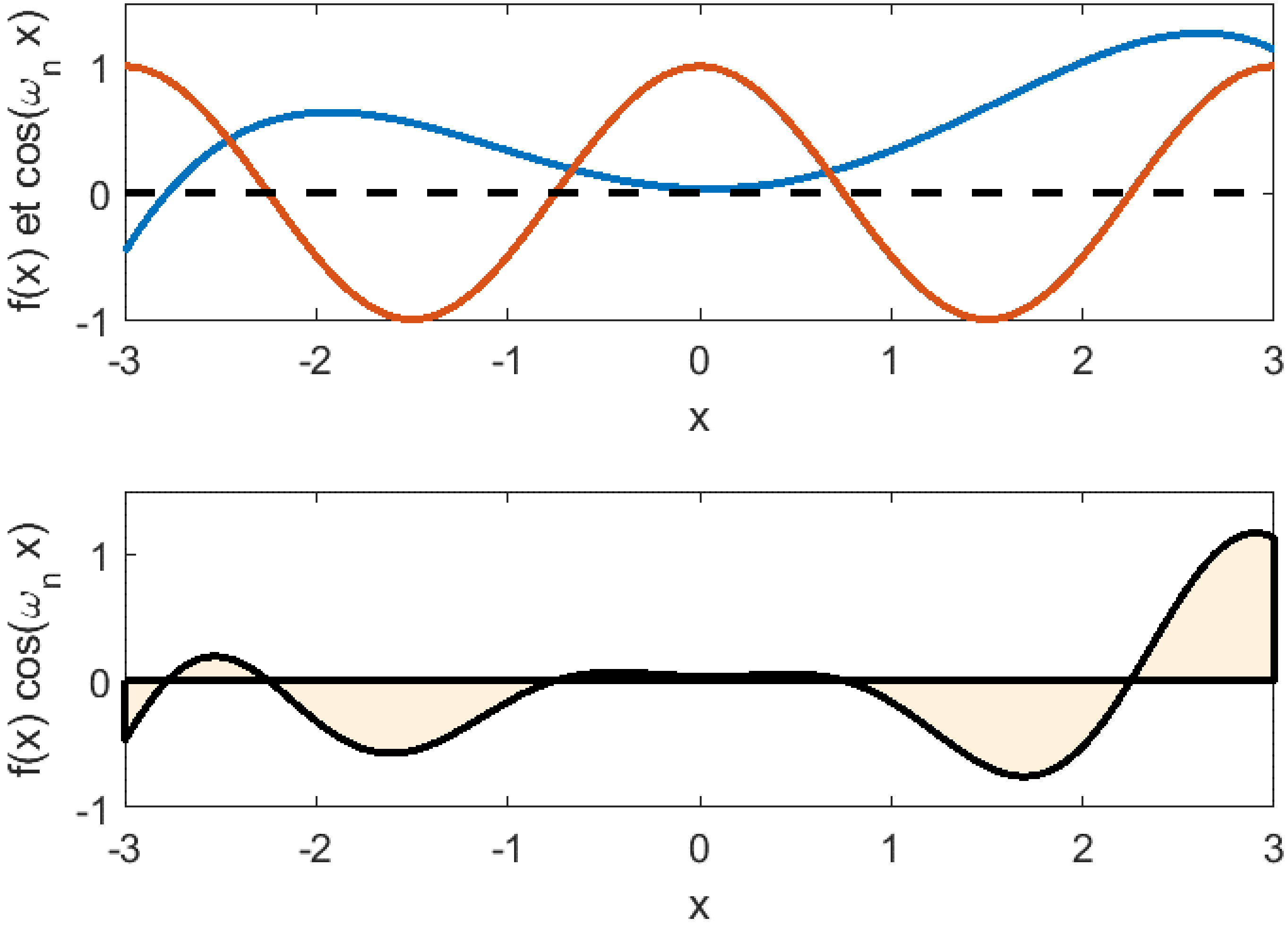
PRODUIT SCALAIRE



PRODUIT SCALAIRE



PRODUIT SCALAIRE



Joseph Fourier

(1768 – 1830)

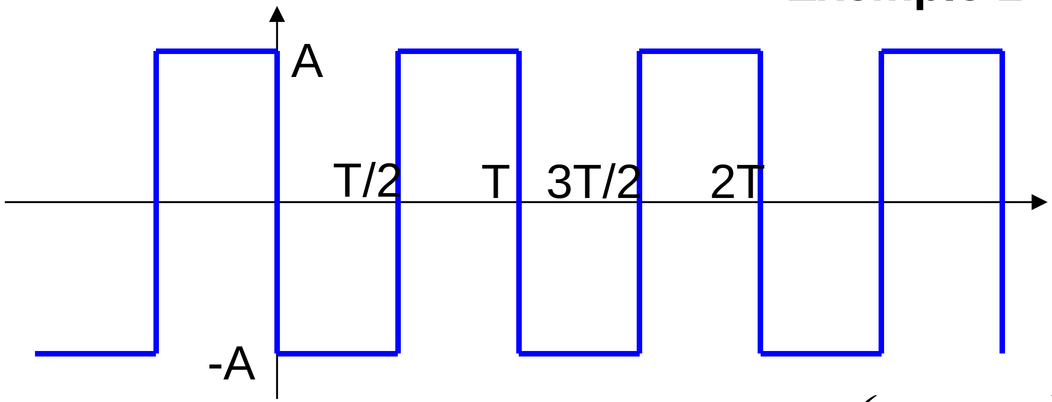
$$f(t) = a_0 + \sum_{n=1}^{+\infty} \left(a_n \cos(n\omega t) + b_n \sin(n\omega t)\right)$$

$$\omega = \frac{2\pi}{T}$$

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt \qquad a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega t)\,dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega t)\,dt$$

Exemple 1



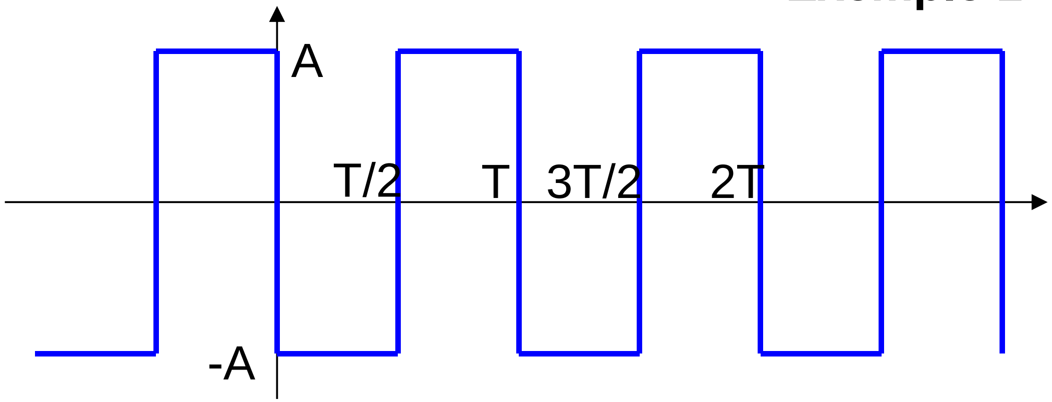
$$a_0 = 0 \qquad a_n = \frac{2}{T}\int_0^T f(t)\cos\left(n\frac{2\pi}{T}t\right)dt = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin\left(n\frac{2\pi}{T}t\right)dt$$

$$= \frac{A}{n\pi}\left(\left[-\cos\left(n\frac{2\pi}{T}t\right)\right]_0^{T/2} - \left[-\cos\left(n\frac{2\pi}{T}t\right)\right]_{T/2}^{T}\right)$$

$$= \frac{2A}{n\pi}\left(1-\cos(n\pi)\right)$$

Exemple 1

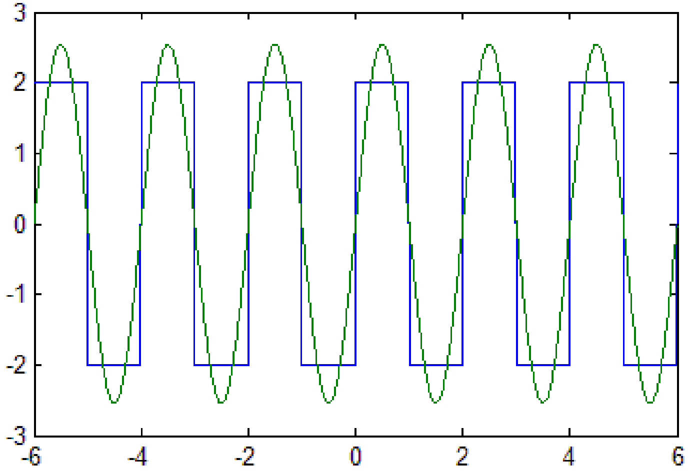


$$b_1 = \frac{4A}{\pi} \qquad b_3 = \frac{4A}{3\pi} \qquad b_5 = \frac{4A}{5\pi}$$

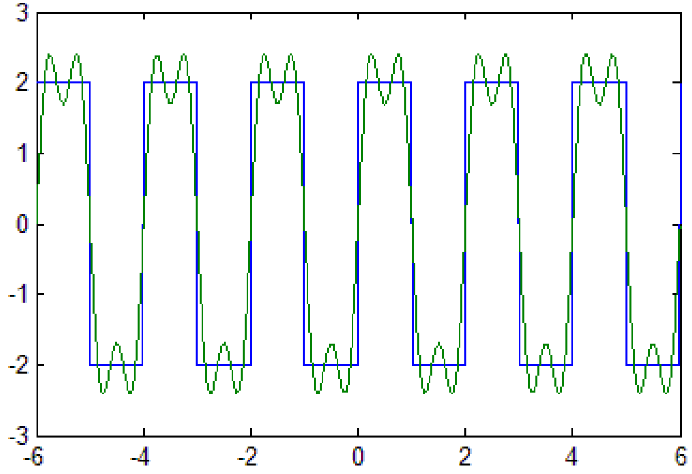
$$f(t) = \frac{4A}{\pi}\left(\sin\left(\frac{2\pi}{T}t\right) + \frac{1}{3}\sin\left(3\frac{2\pi}{T}t\right) + \frac{1}{5}\sin\left(5\frac{2\pi}{T}t\right)\right) + \ldots$$

Décomposition en série de Fourier de la fonction carrée

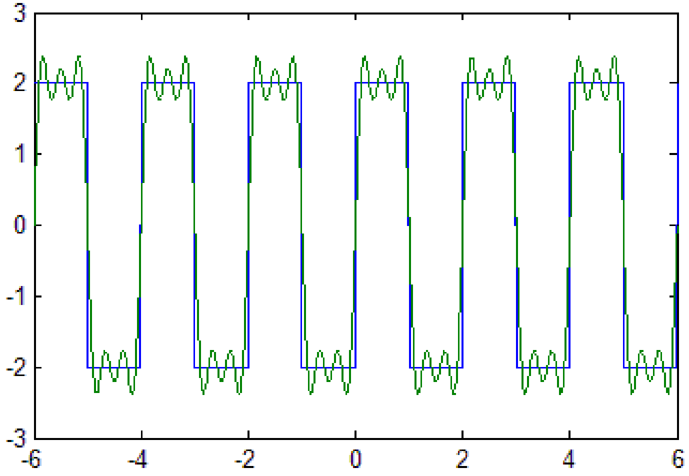
1ère harmonique



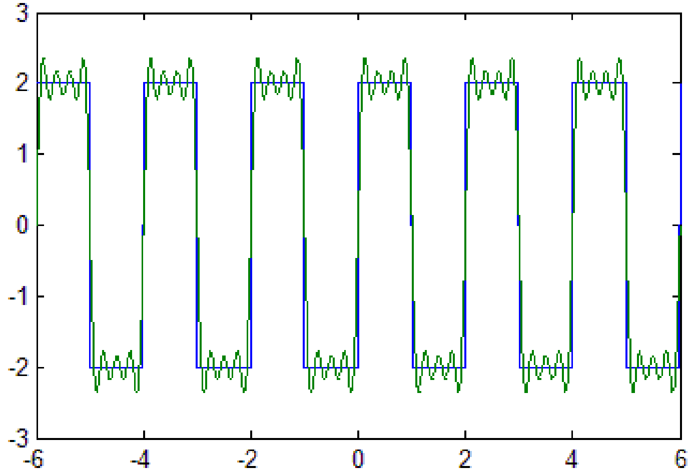
3ème harmonique



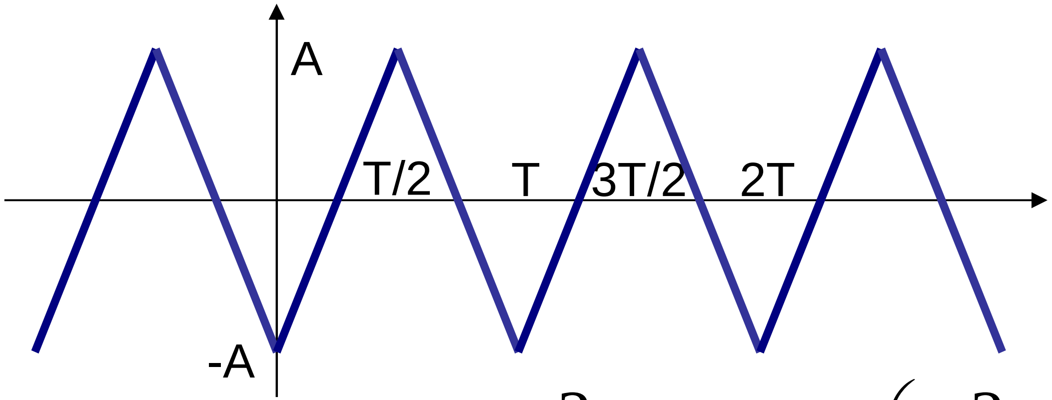
5ème harmonique



7ème harmonique



Exemple 2



$$f(t)=\frac{4A}{T}t-A \qquad t\in[0;T/2]$$

$$f(t)=-\frac{4A}{T}t+3A \quad t\in[T/2;T]$$

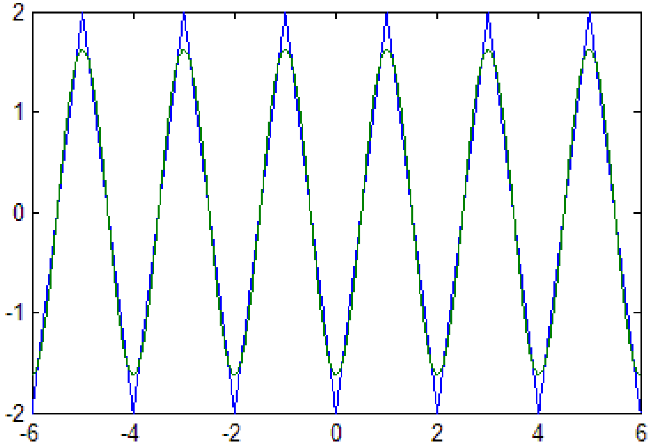
$$a_0=0 \qquad\qquad b_n=\frac{2}{T}\int_0^T f(t)\sin\left(n\frac{2\pi}{T}t\right)dt=0$$

$$a_n=-\frac{8A}{n^2\pi^2}$$ où n est impair.

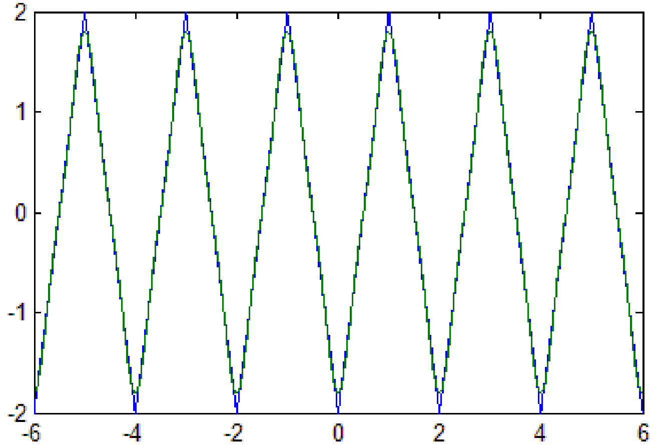
$$f(t)=-\frac{8A}{\pi^2}\left(\cos(\omega t)+\frac{1}{9}\cos(3\omega t)+\frac{1}{25}\cos(5\omega t)+\ldots\right)$$

Décomposition en série de Fourier de la fonction triangle

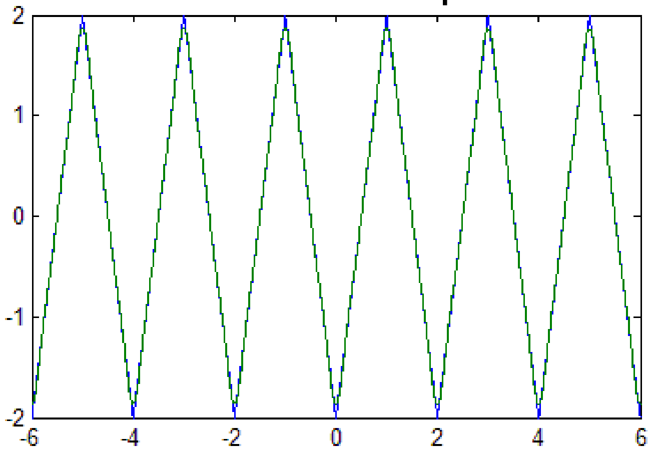
1ère harmonique



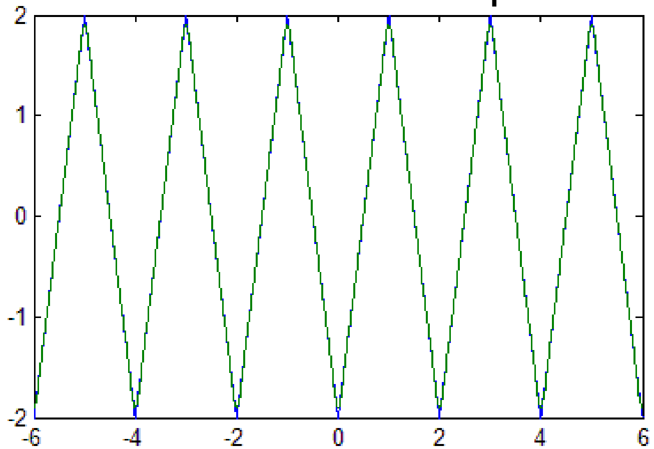
3ème harmonique

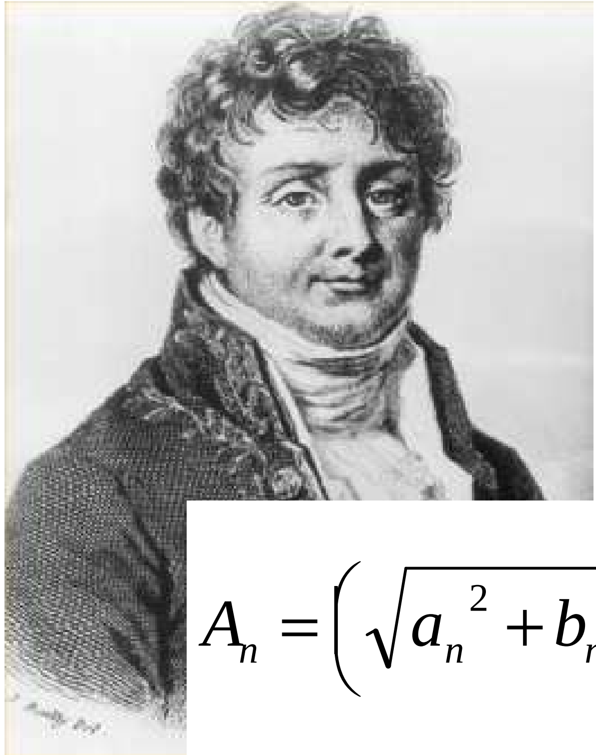


5ème harmonique



7ème harmonique





Joseph Fourier

(1768 – 1830)

On pose: $A_n = a_n \cos(n\omega t) + b_n \sin(n\omega t)$

n-ième harmonique

$$A_n = \left(\sqrt{a_n^{\,2} + b_n^{\,2}}\right)\left(\frac{a_n}{\sqrt{a_n^{\,2} + b_n^{\,2}}}\cos(n\omega t) + \frac{b_n}{\sqrt{a_n^{\,2} + b_n^{\,2}}}\sin(n\omega t)\right)$$

$$C_n = \sqrt{a_n^{\,2} + b_n^{\,2}}$$

φ_n est tel que:

$$\cos(\varphi_n) = \frac{a_n}{\sqrt{a_n^{\,2} + b_n^{\,2}}}$$

$$\sin(\varphi_n) = \frac{b_n}{\sqrt{a_n^{\,2} + b_n^{\,2}}}$$

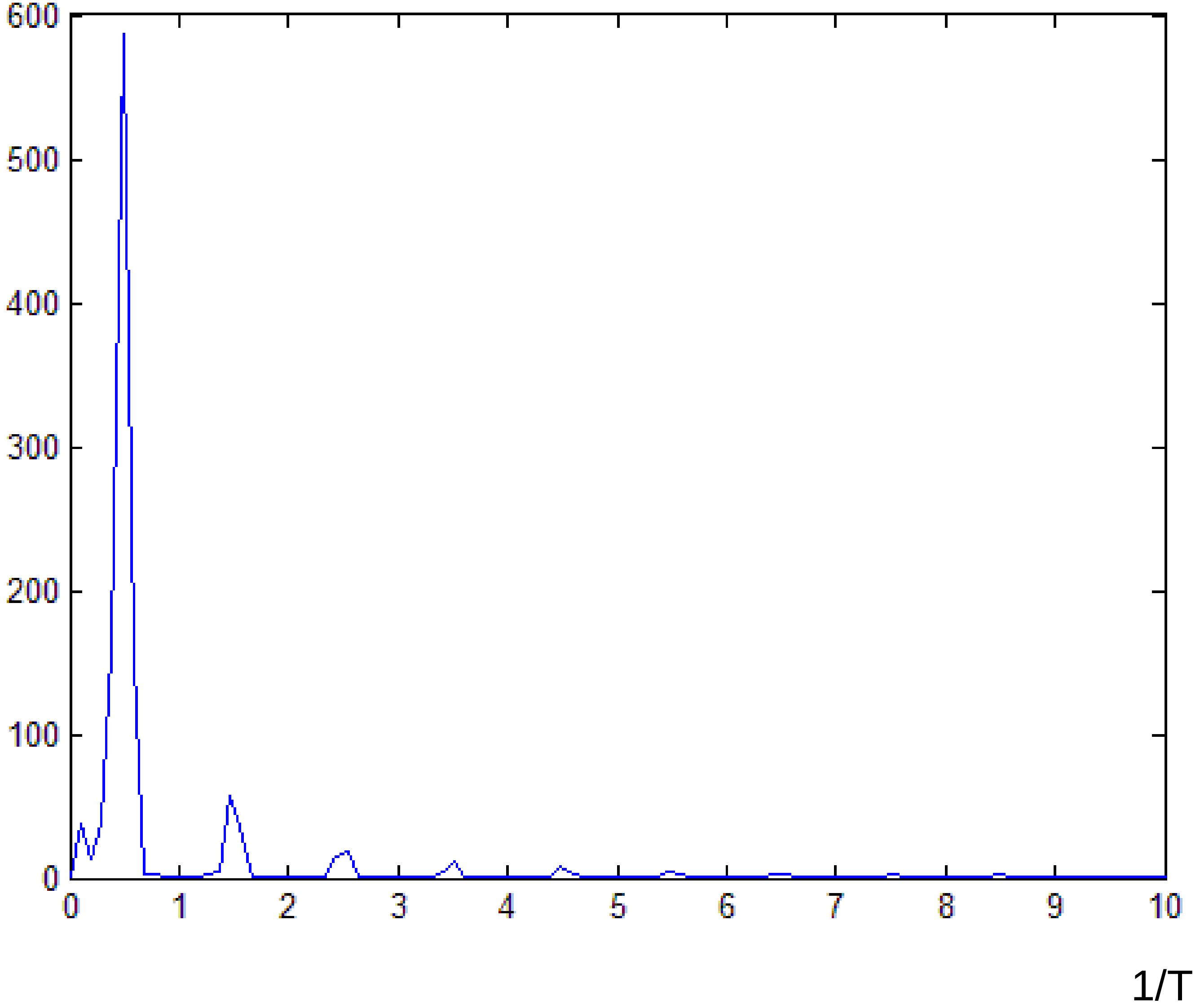
Joseph Fourier

(1768 – 1830)

$$A_n = C_n \cos\left(n\omega t - \varphi_n\right)$$

$$f\left(t\right) = a_0 + \sum_{n=1}^{+\infty} C_n \cos\left(n\omega t - \varphi_n\right)$$

Spectre de Fourier de la fonction carrée



Spectre de Fourier de la fonction triangle