

Aix-Marseille University  
Master 2, OPB  
2023 - 2024

**Aude Joël**  
June 12, 2024

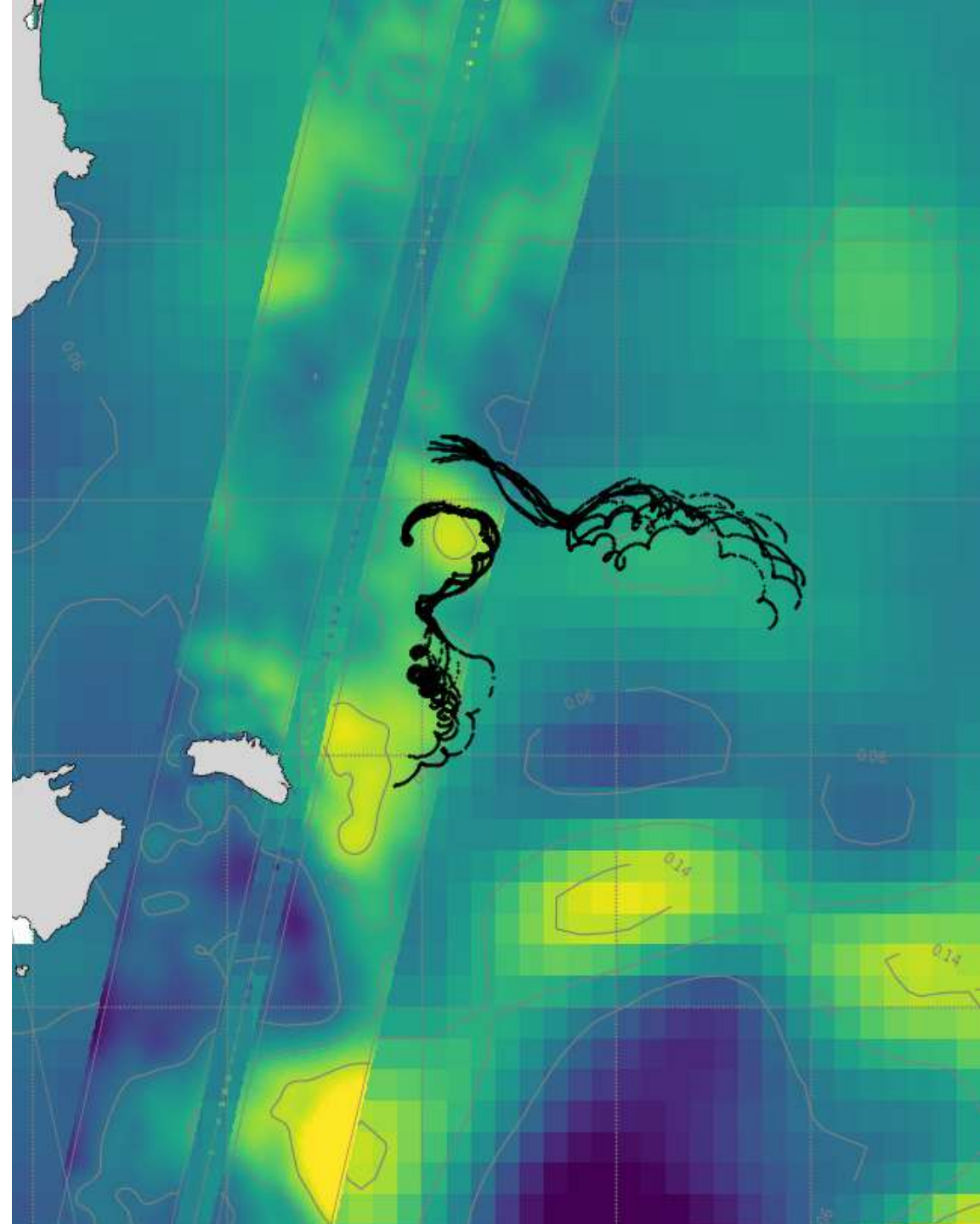
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# Lagrangian analysis of fine-scale dynamics during BioSWOT-MED

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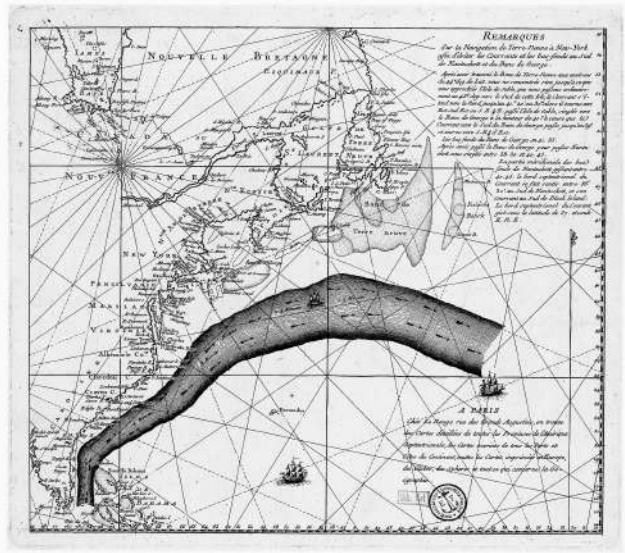
Tutors:

Andrea Doglioli (MIO),  
Maristella Berta (CNR-ISMAR)  
Louise Roussellet (LOCEAN)



# 1. Oceanic circulation & fine-scale dynamics

## Large-scale



sources: Franklin, 1769

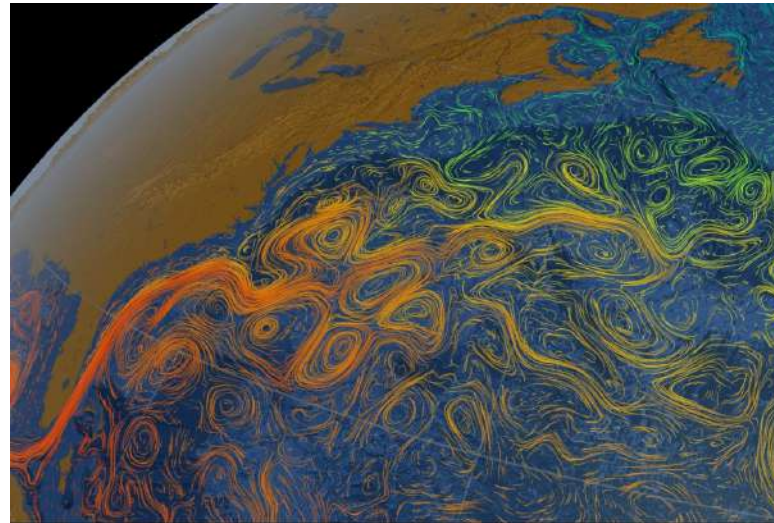
1000 - 10000 km

1 to > 1000 years

**Thermo-haline circu.**

**Decadal oscillation**

## Fine-scale



NASA

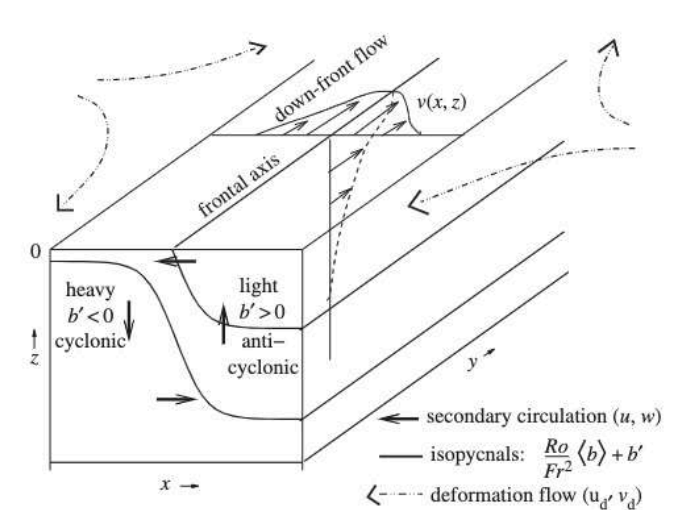
1 - 100 km

1 day up to a year

**Eddies & meander features (mesoscale)**

**Front & filaments features (sub-mesoscale)**

**→ Impact on biogeochemical processes due to 3-D circulation**



McWilliams, 2016

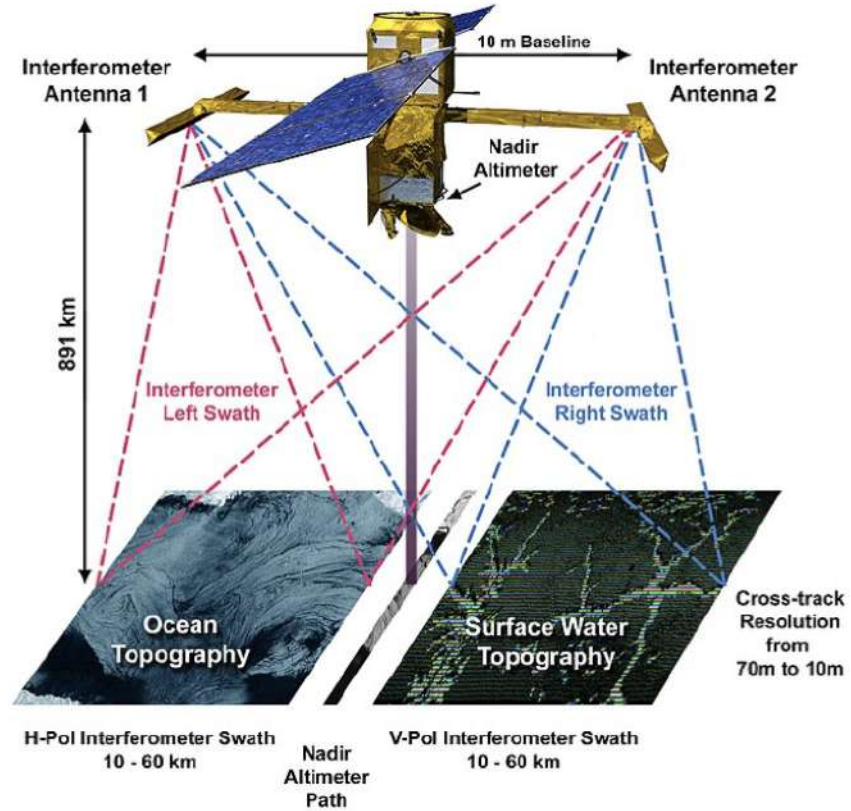
# 1. The new SWOT-satellite

## SWOT

### Surface Water and Ocean Topography

4-months CalVal phase – SWOT-AdAC

(d'Ovidio et al., 2019)

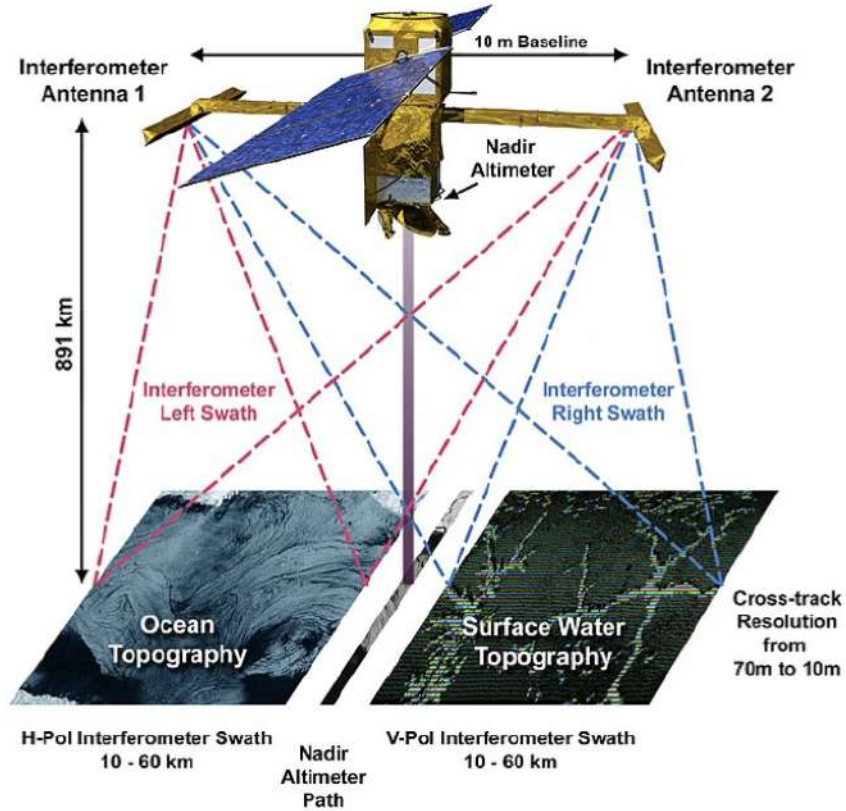


Source: NASA

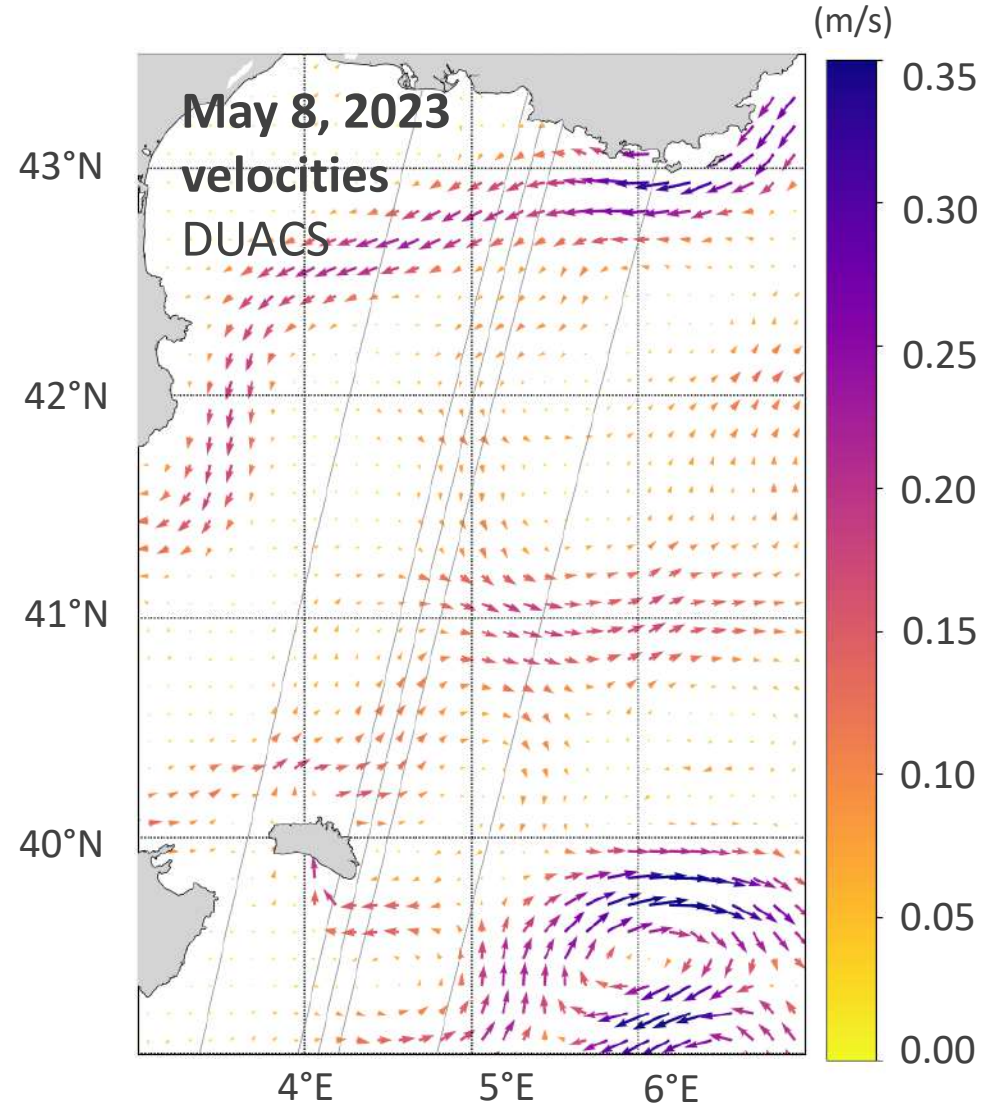
# 1. The new SWOT-satellite

## SWOT

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4-months CalVal phase – SWOT-AdAC  
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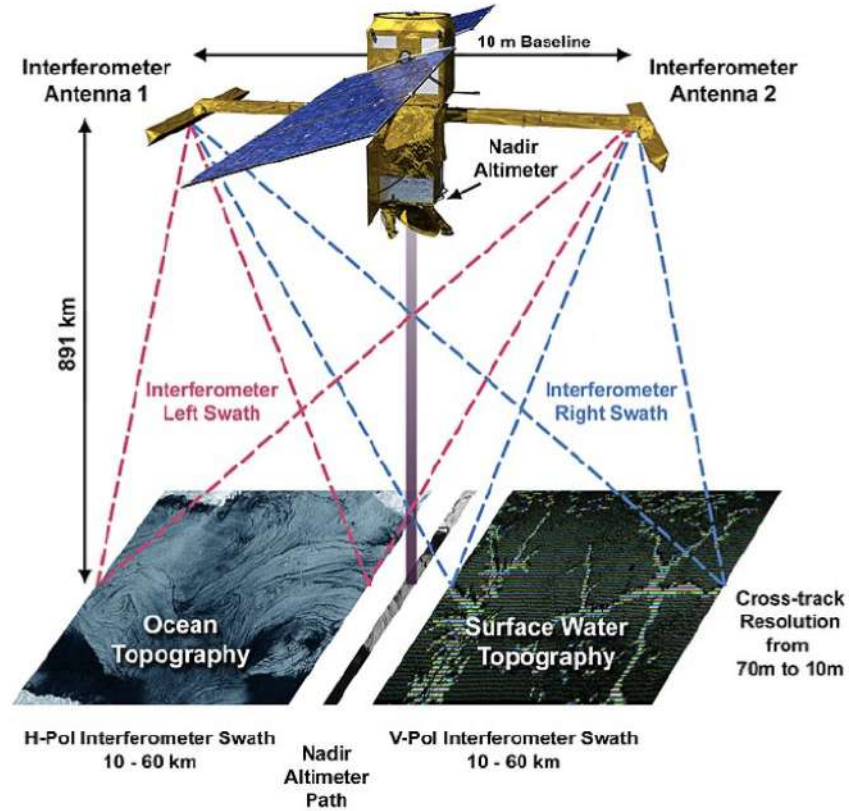
Source: NASA



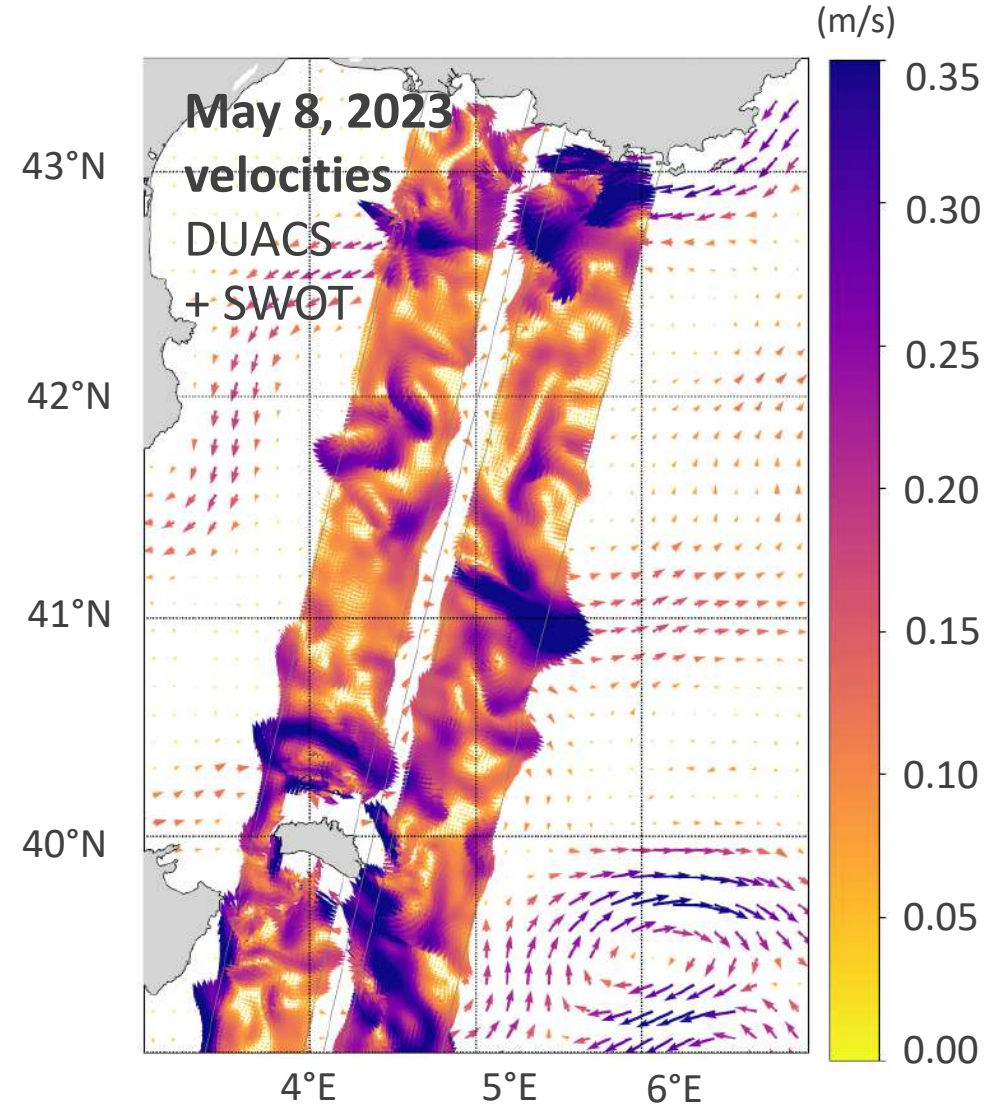
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## SWOT

Surface Water and Ocean Topography  
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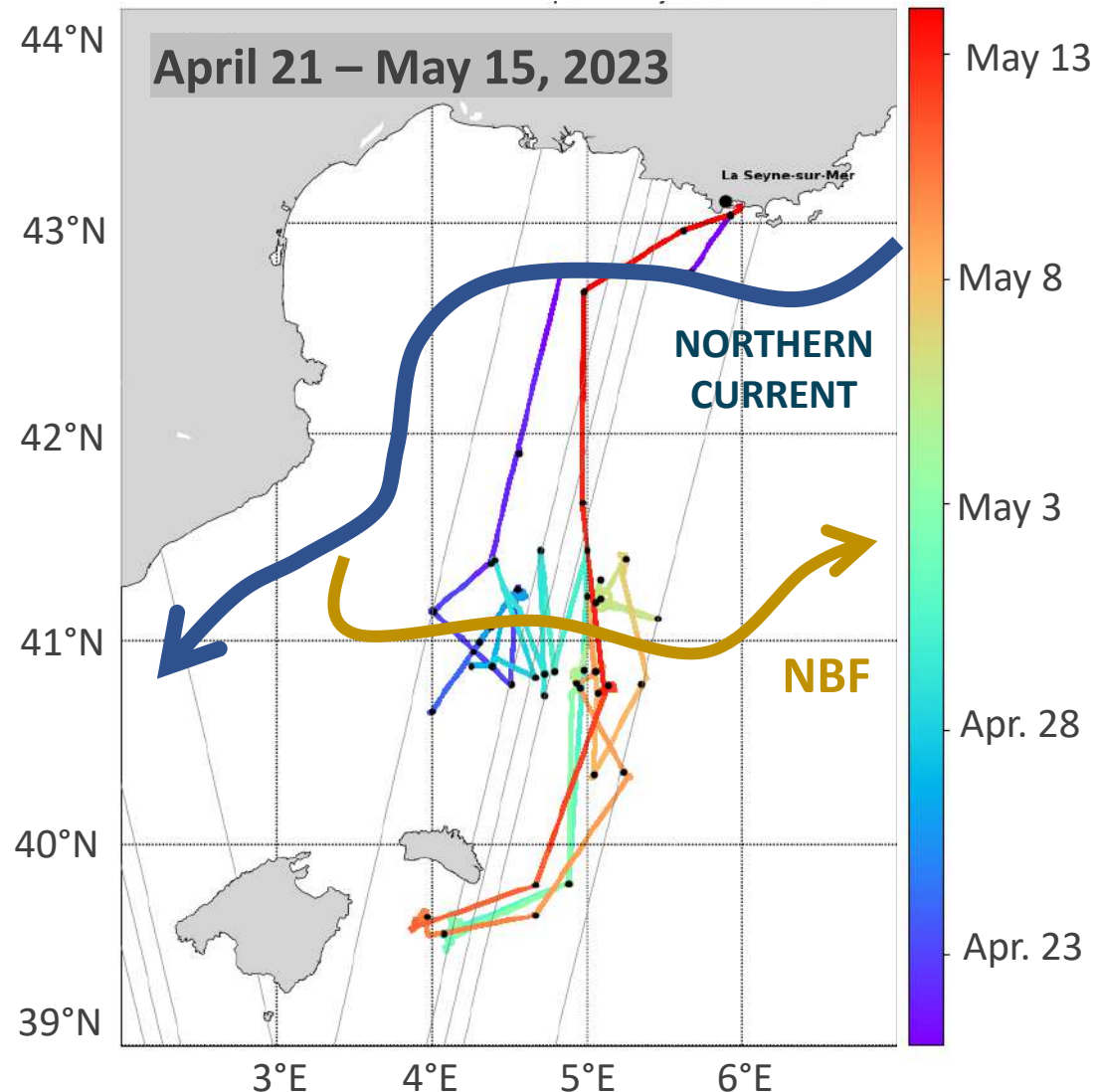
Source: NASA



# 1. The BioSWOT-Med campaign

<https://doi.org/10.17600/18002392>

3

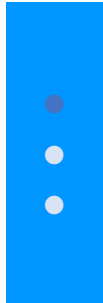


- Area: **North Balearic Front (NBF)**
- Study of **fine-scale dynamics & interactions with biogeochemical processes**
- **Adaptative** (to target oceanic features) and **Lagrangian** (to follow them) **sampling strategy** (Doglioli et al, 2024 cruise report)
- In particular, Lagrangian instruments: drifters and floats

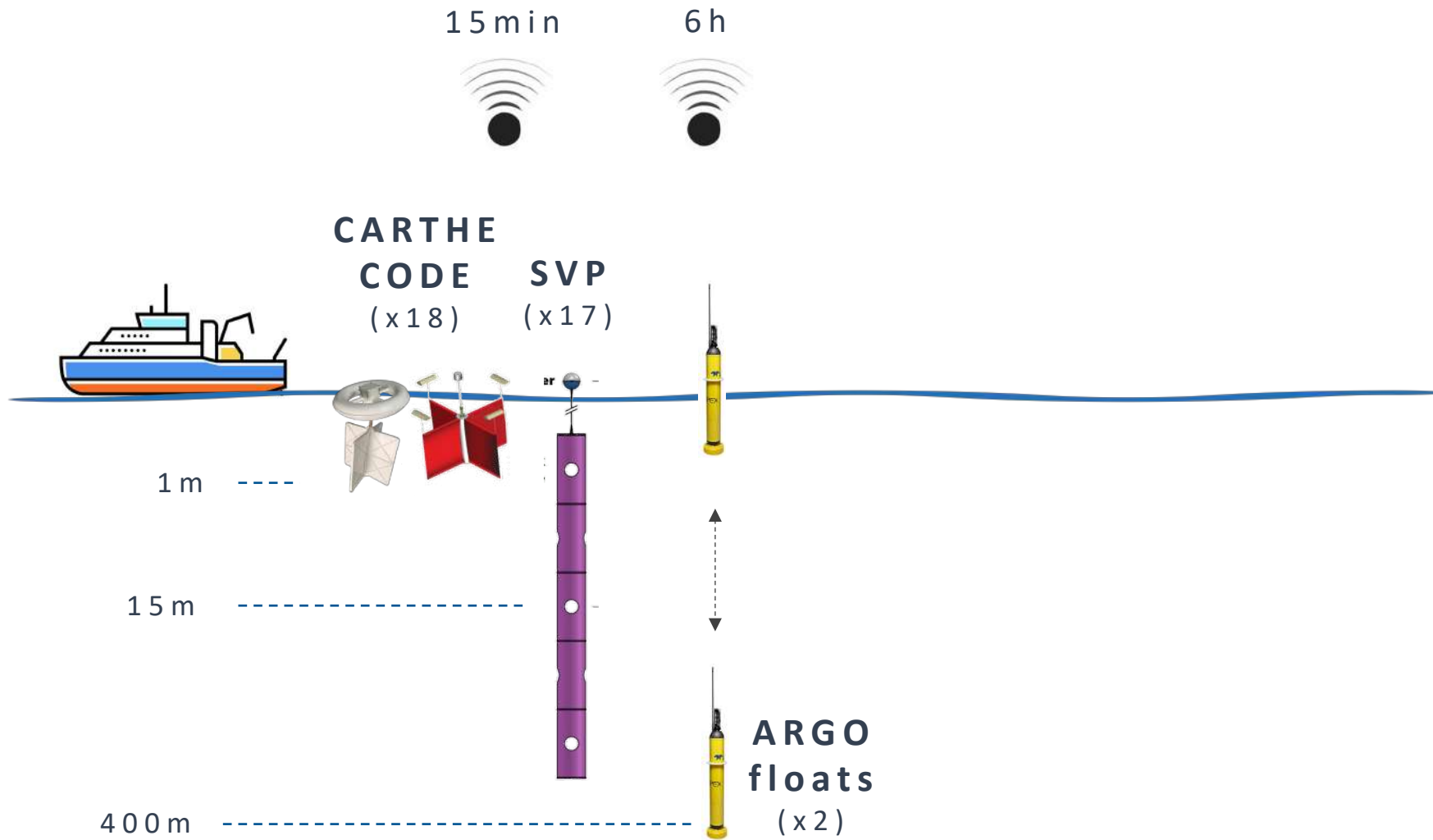
# 1. Scientific objectives

- Characterize **local surface ocean dynamics** thanks to **Lagrangian instruments** and **SWOT data**
  - Contribute to the understanding of **fine-scale structures in moderate-energetic marine environments**

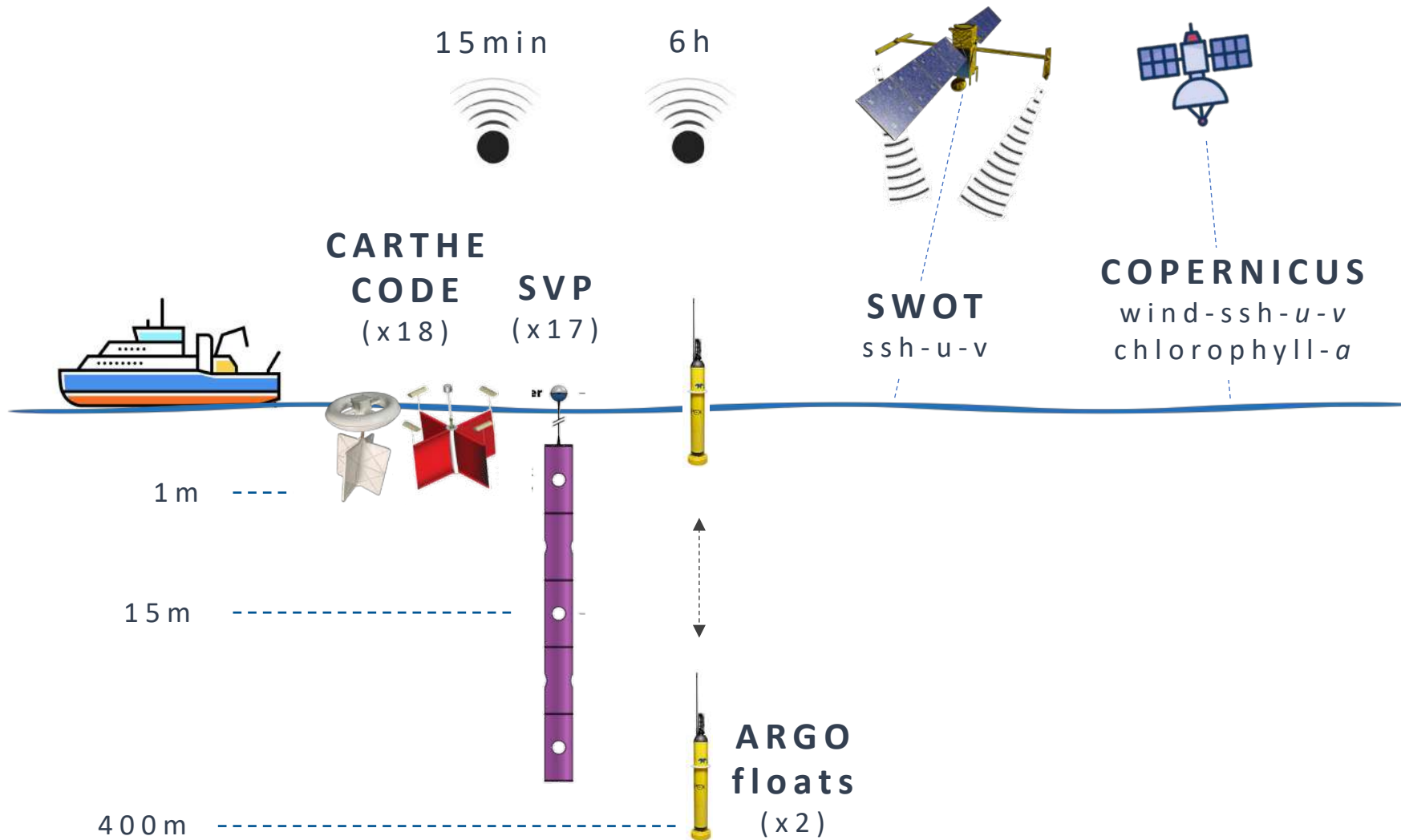
## 2. Data collection



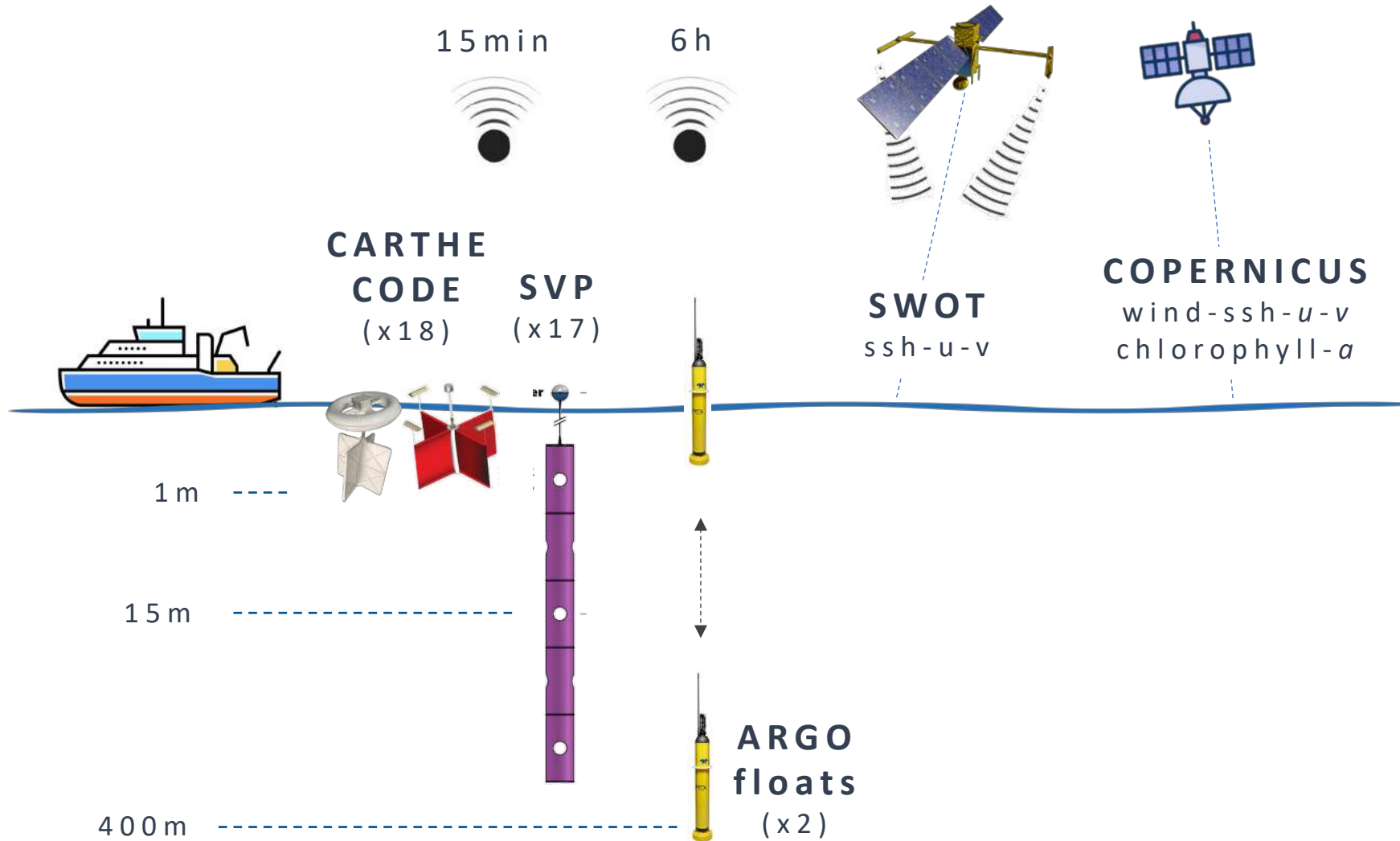
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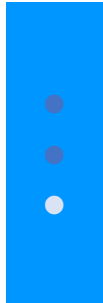
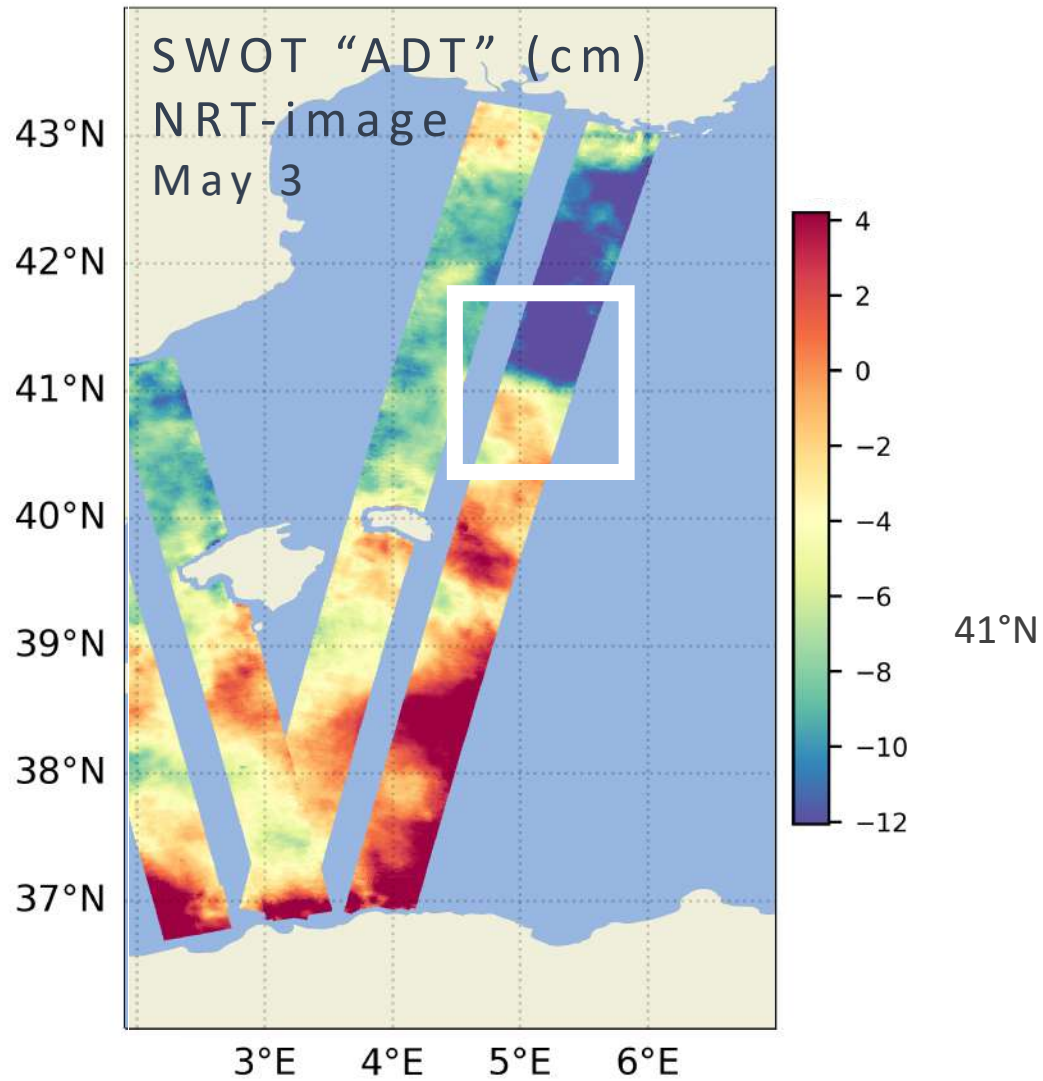
# 2. Data collection



**LAMTA**  
Num. simulation  
advection

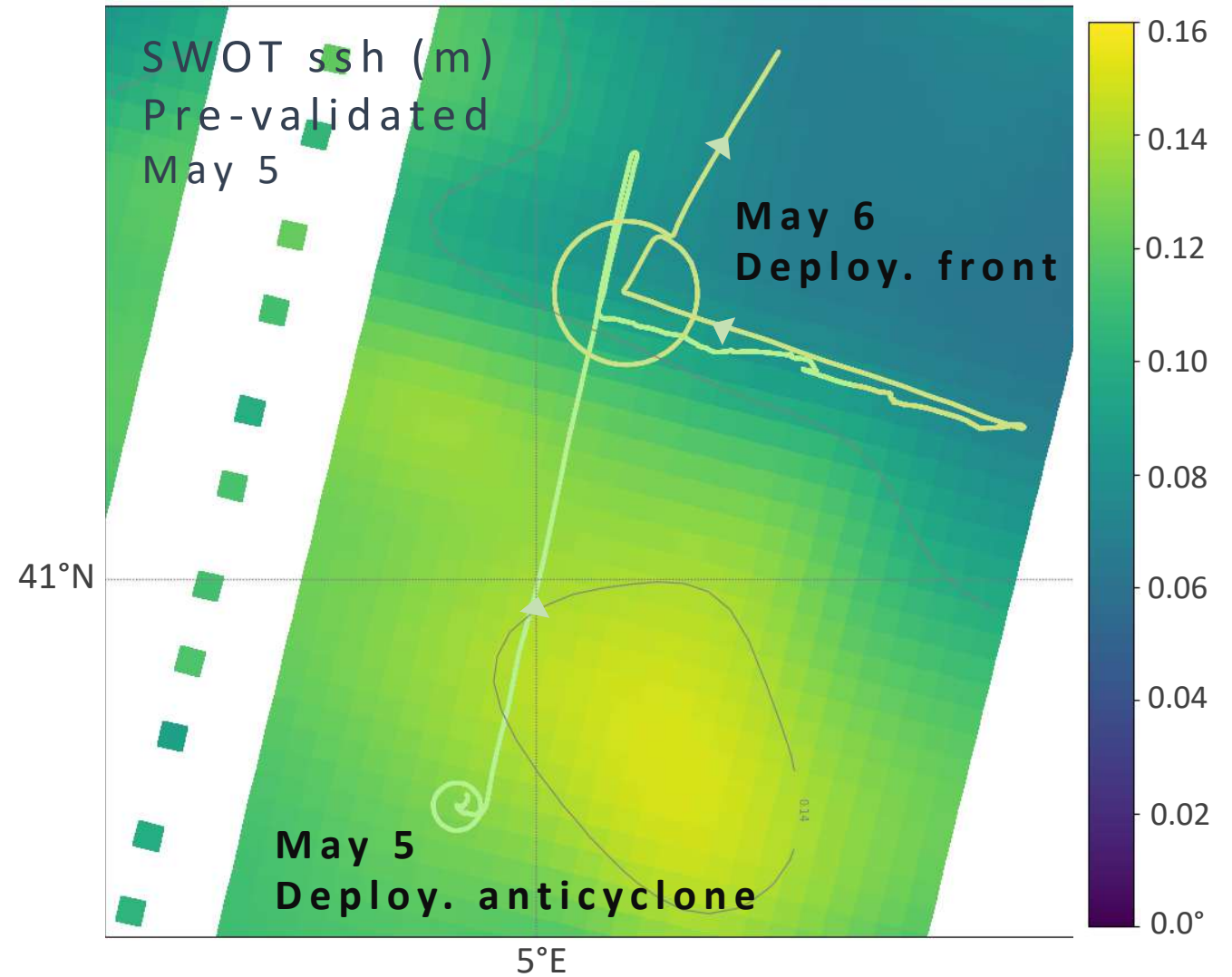
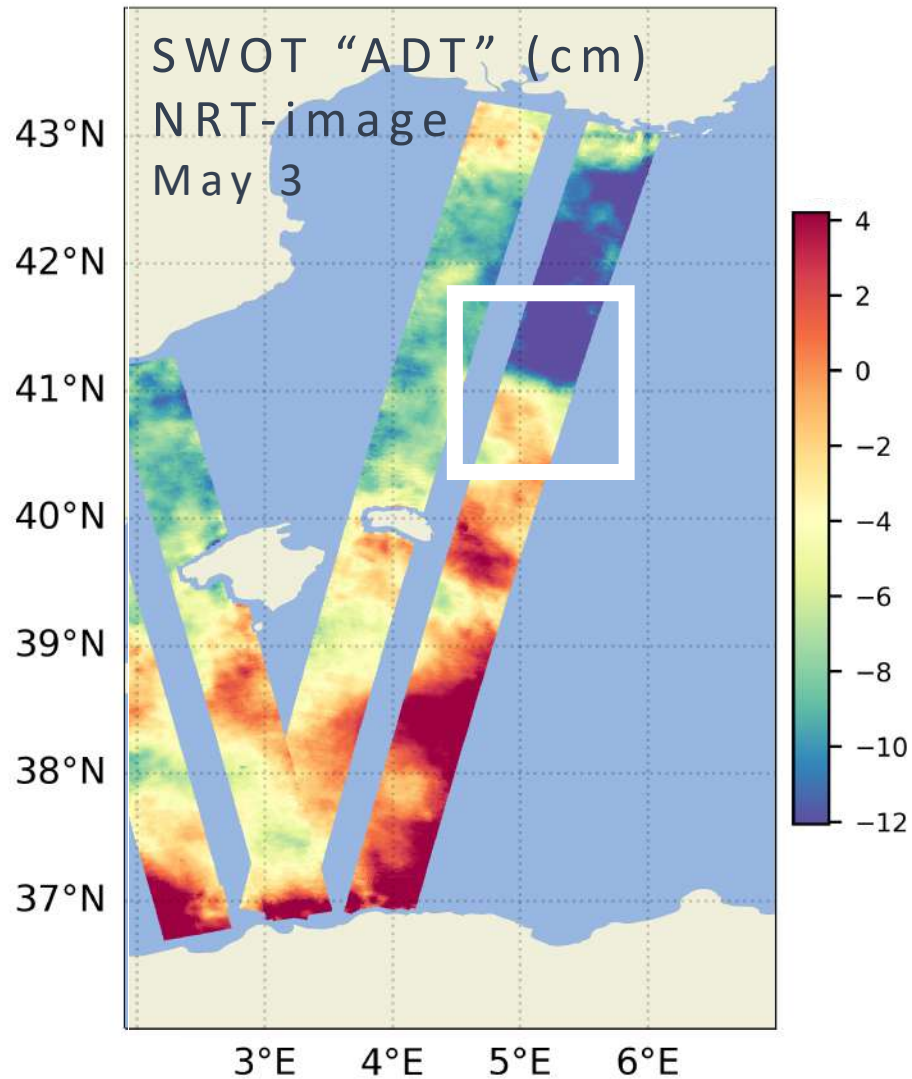


## 2. Fine-scale features targeted: May 5 & 6



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6



## 2. Drifter deployment strategy & KP

7

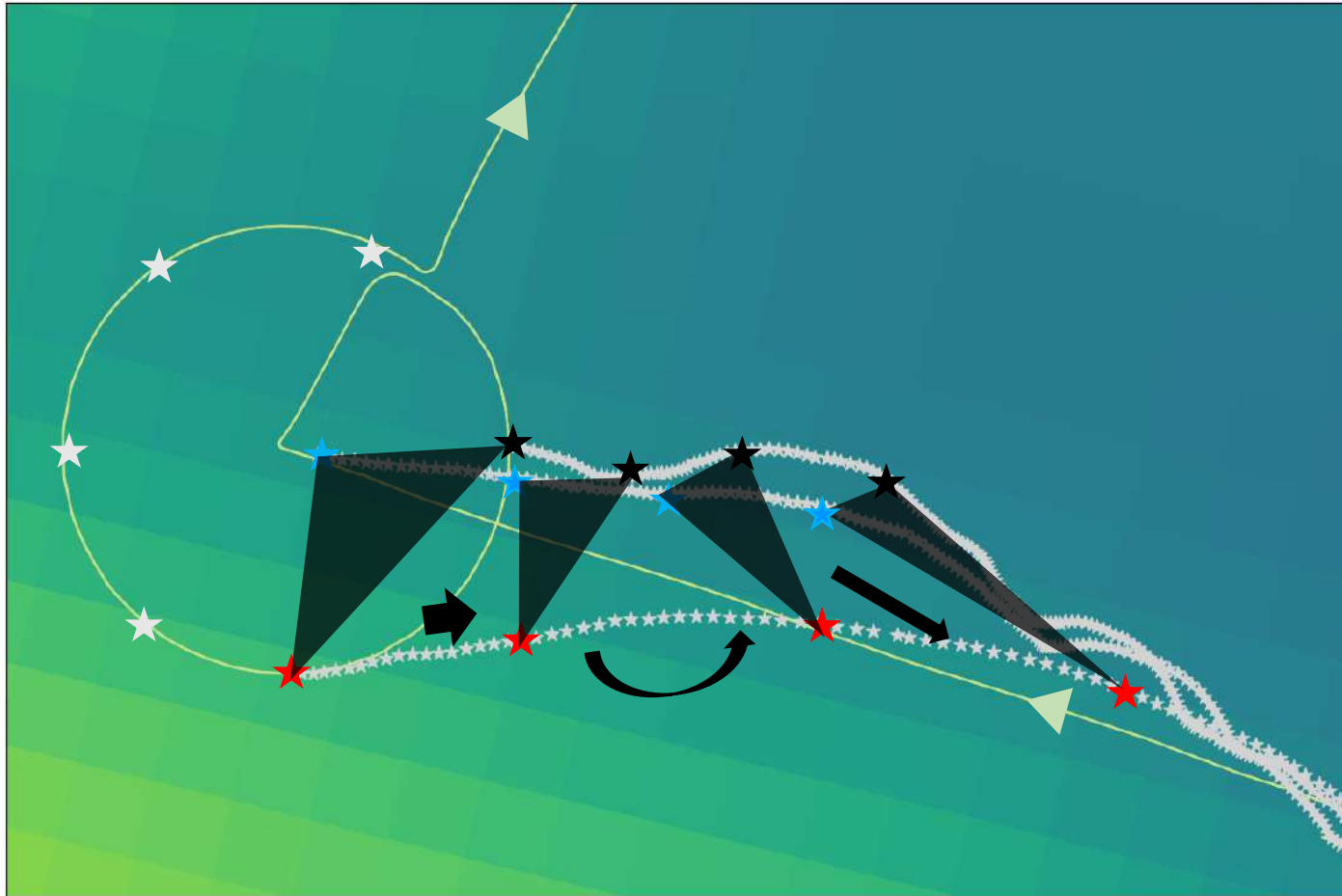


**AREA RATE OF CHANGE  
METHOD**  
(Molinari & Kirwan, 1975):



## 2. Drifter deployment strategy & KP

7



### AREA RATE OF CHANGE METHOD

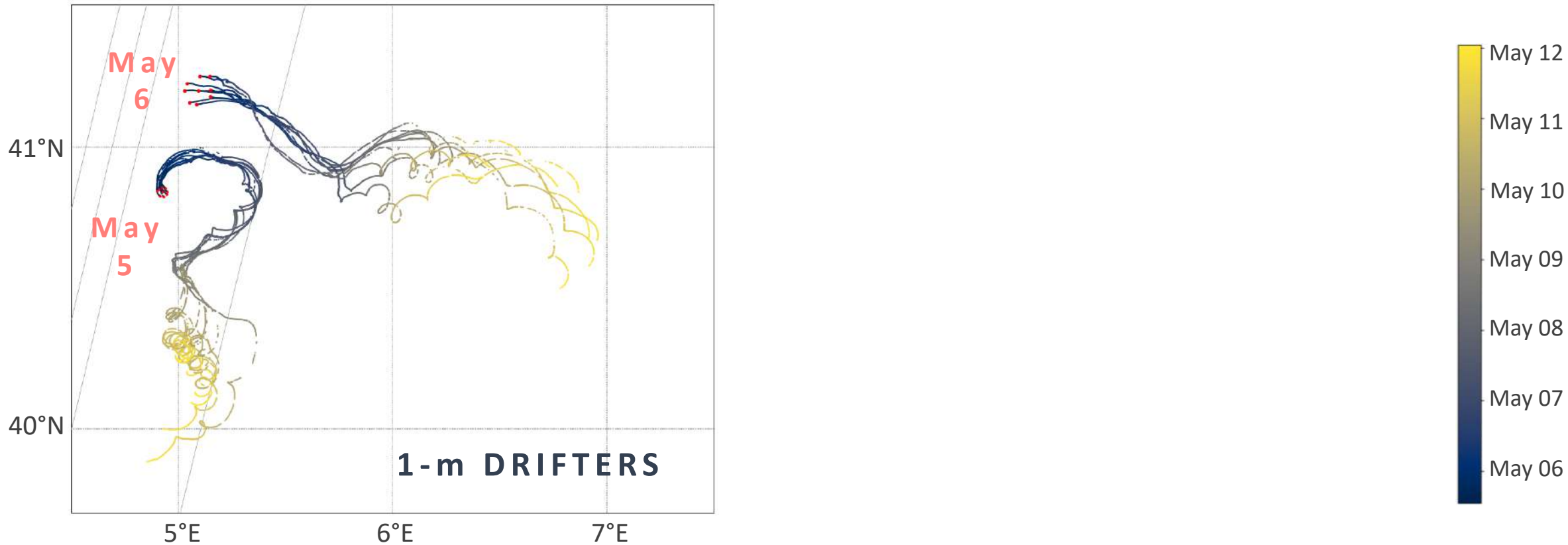
(Molinari & Kirwan, 1975):

- size: **DIVERGENCE**

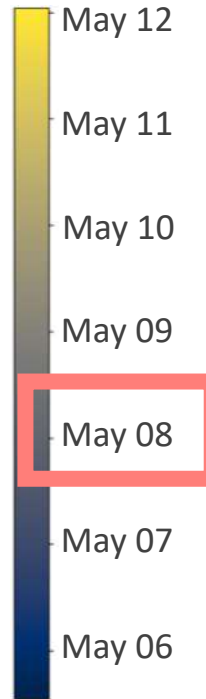
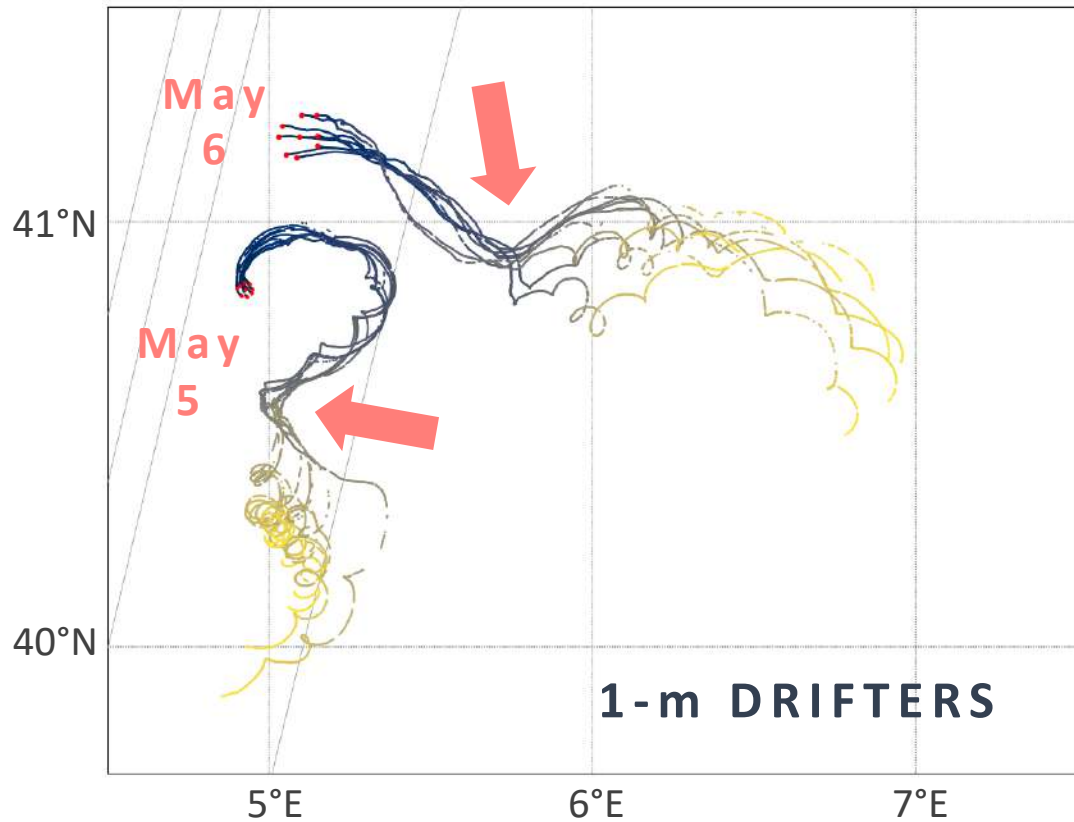
$$D = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \rightarrow \frac{1}{A} \frac{dA}{dt}$$

- orientation: **VORTICITY**
- shape: **STRAIN**
- mass conservation: **VERTICAL VELOCITIES  $w$**

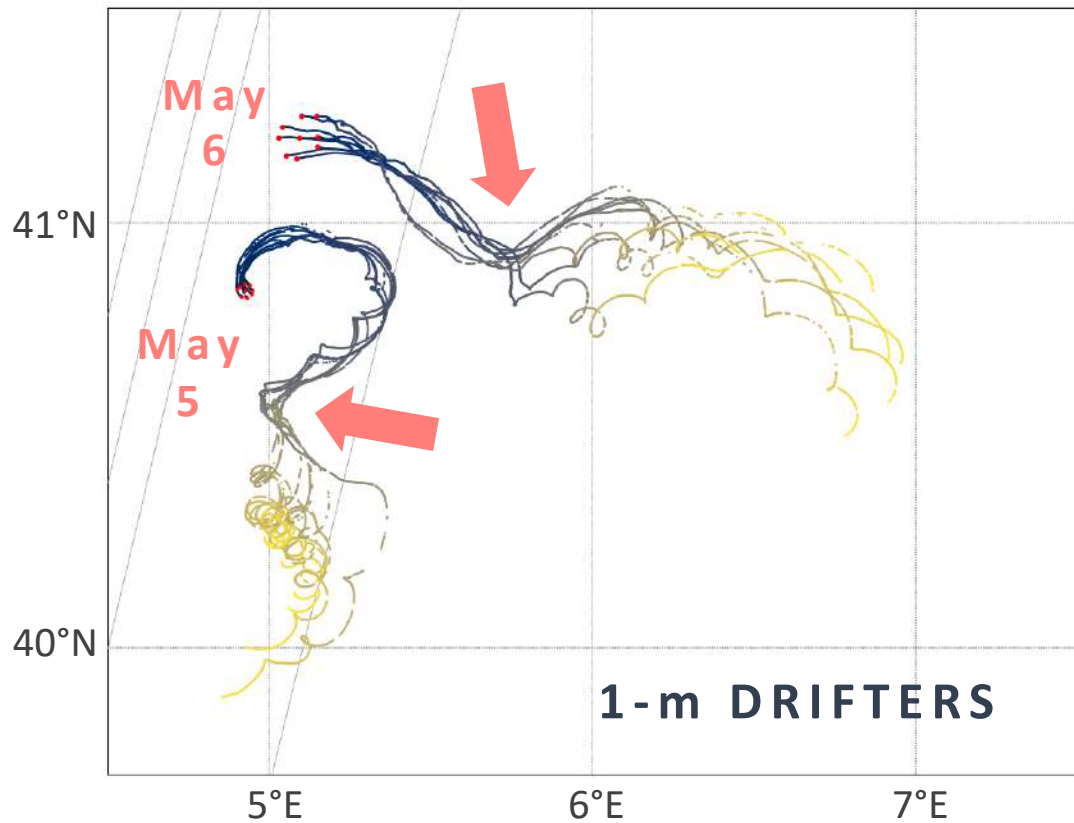
# 3. Drifter's trajectories – May 5 to 12



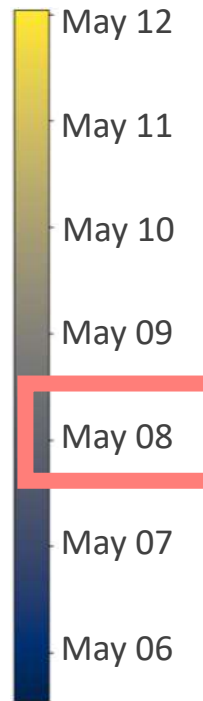
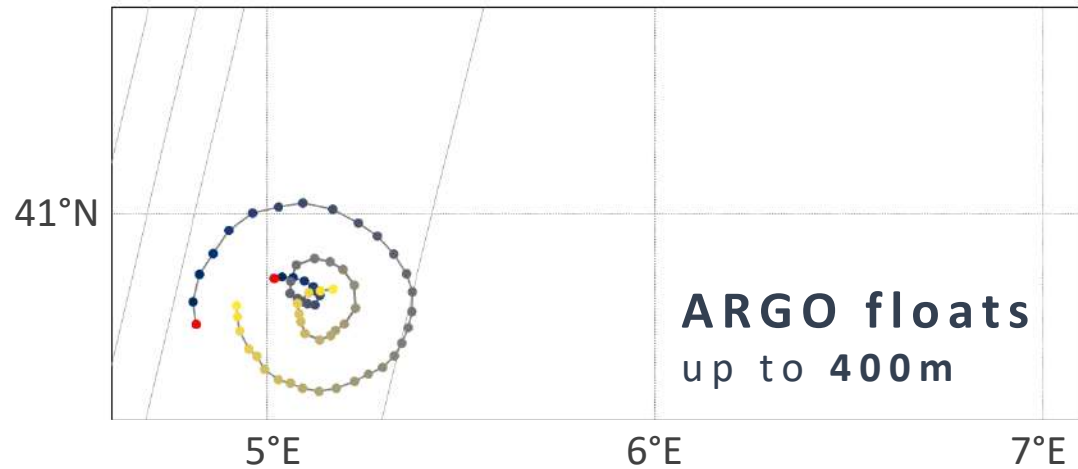
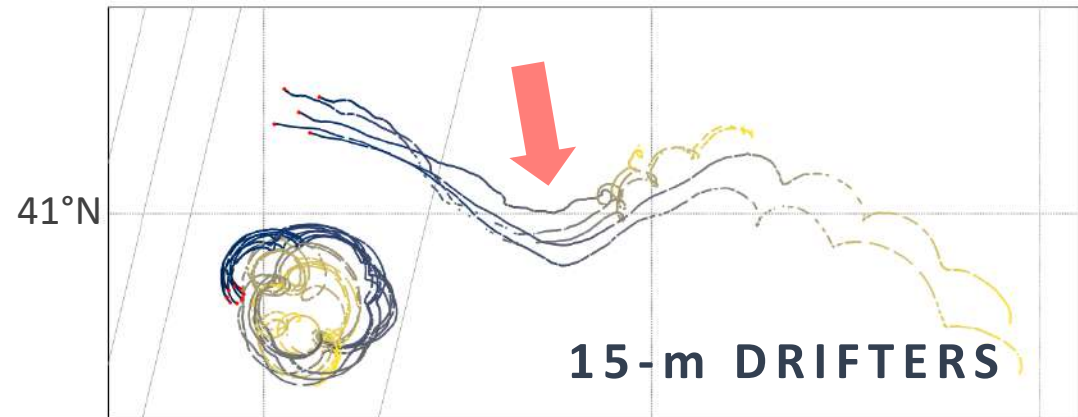
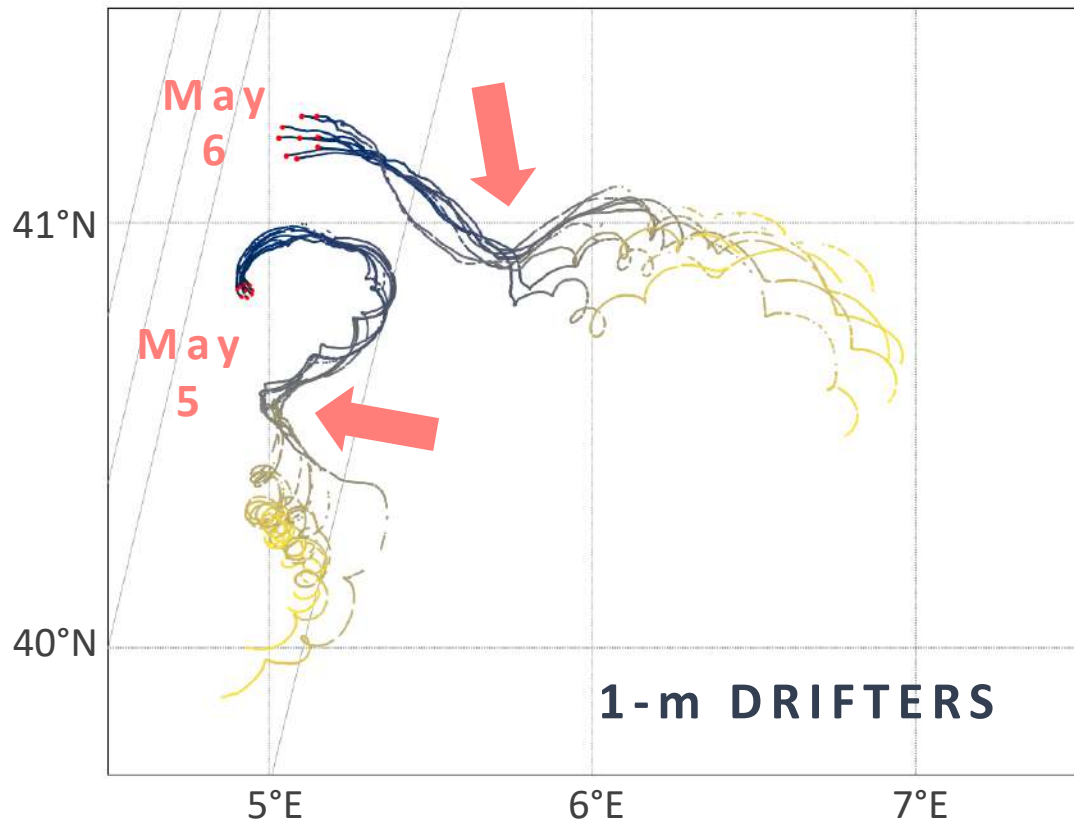
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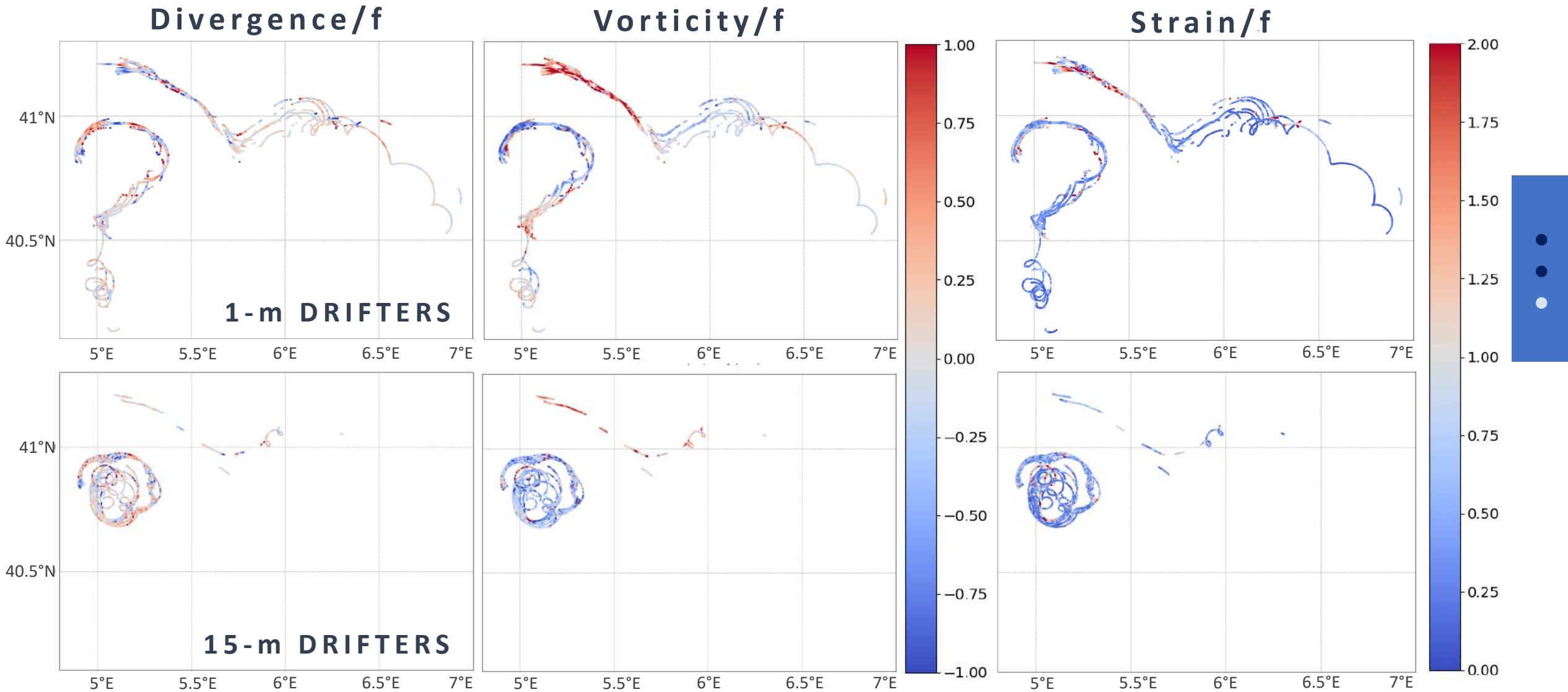
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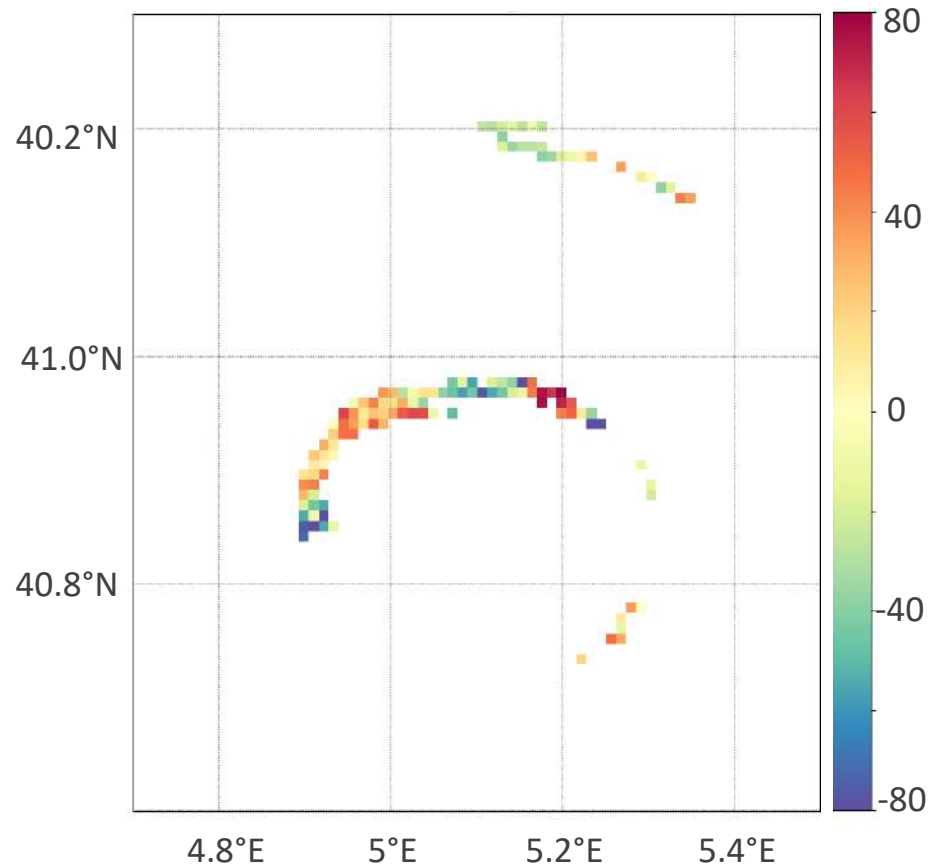


# 3. Lagrangian diagnostics



# 3. Vertical velocities $w$

May 5-12 -  $w$  velocities (m/d)

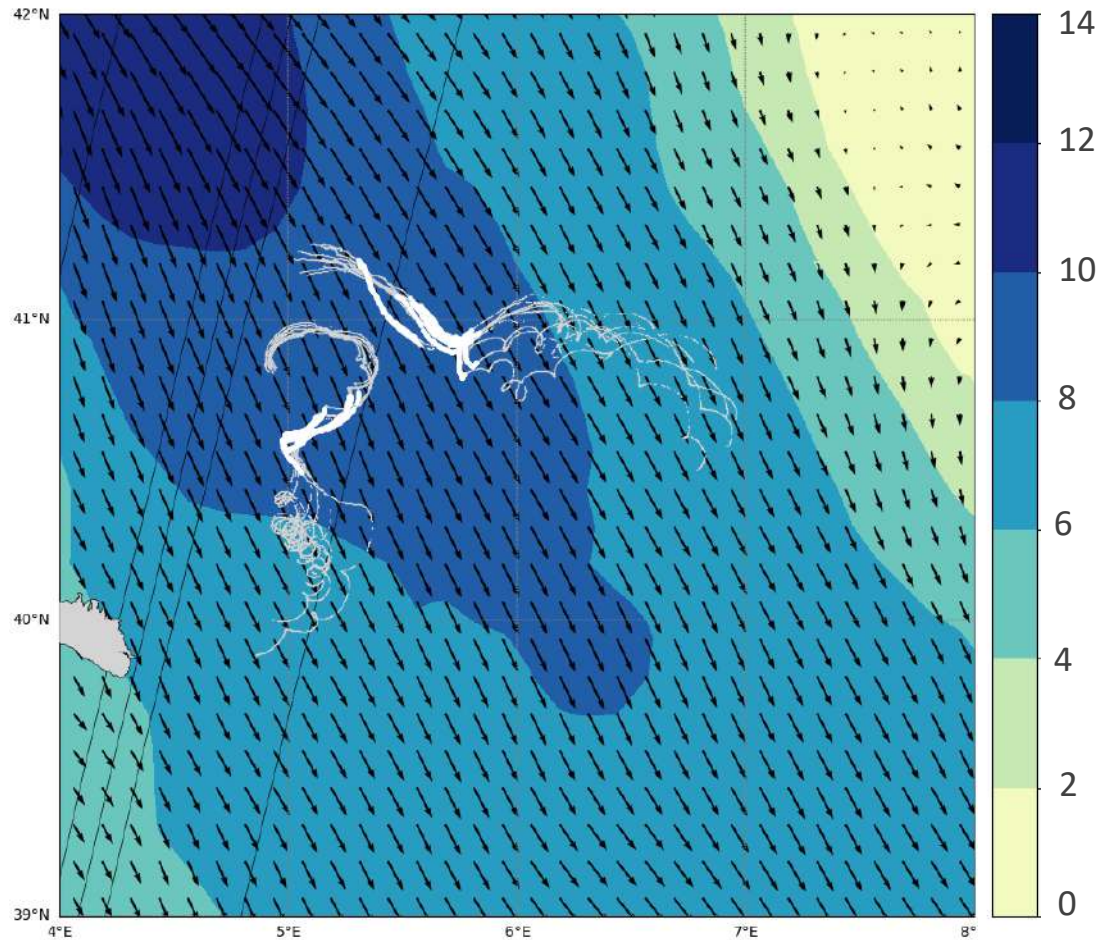


- alternating pattern of positive and negative  $w$
- **+/- 80 m/d**  
(less than 1mm/s)
- Consistent with literature  
(Tarry et al, 2021, 2022,  
Esposito et al. 2023)



# 4. Influence of the wind: friction

May 8 - Wind velocities (m/s)  
1-m drifters trajectories



## May 7:

- moderate wind: 2-6 m/s

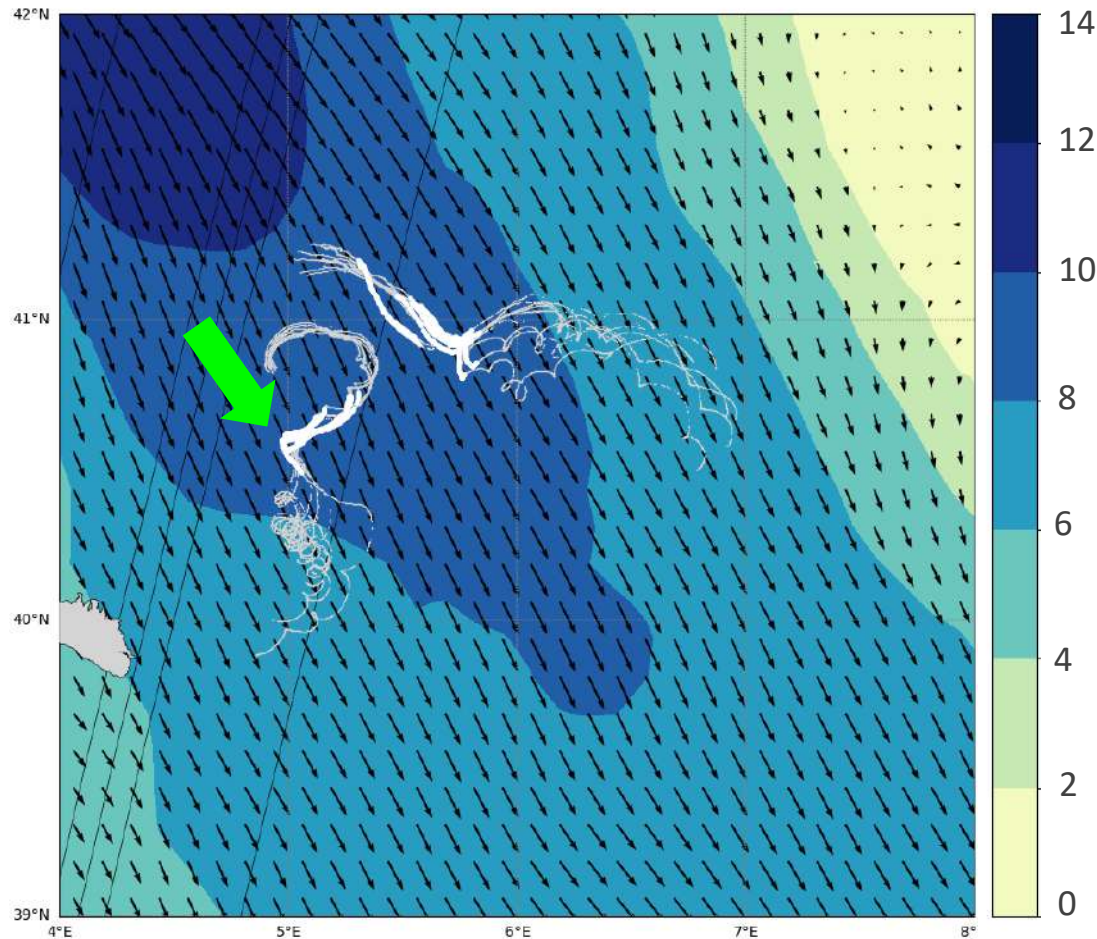
## May 8 afternoon:

- Mistral event: > 14m/s



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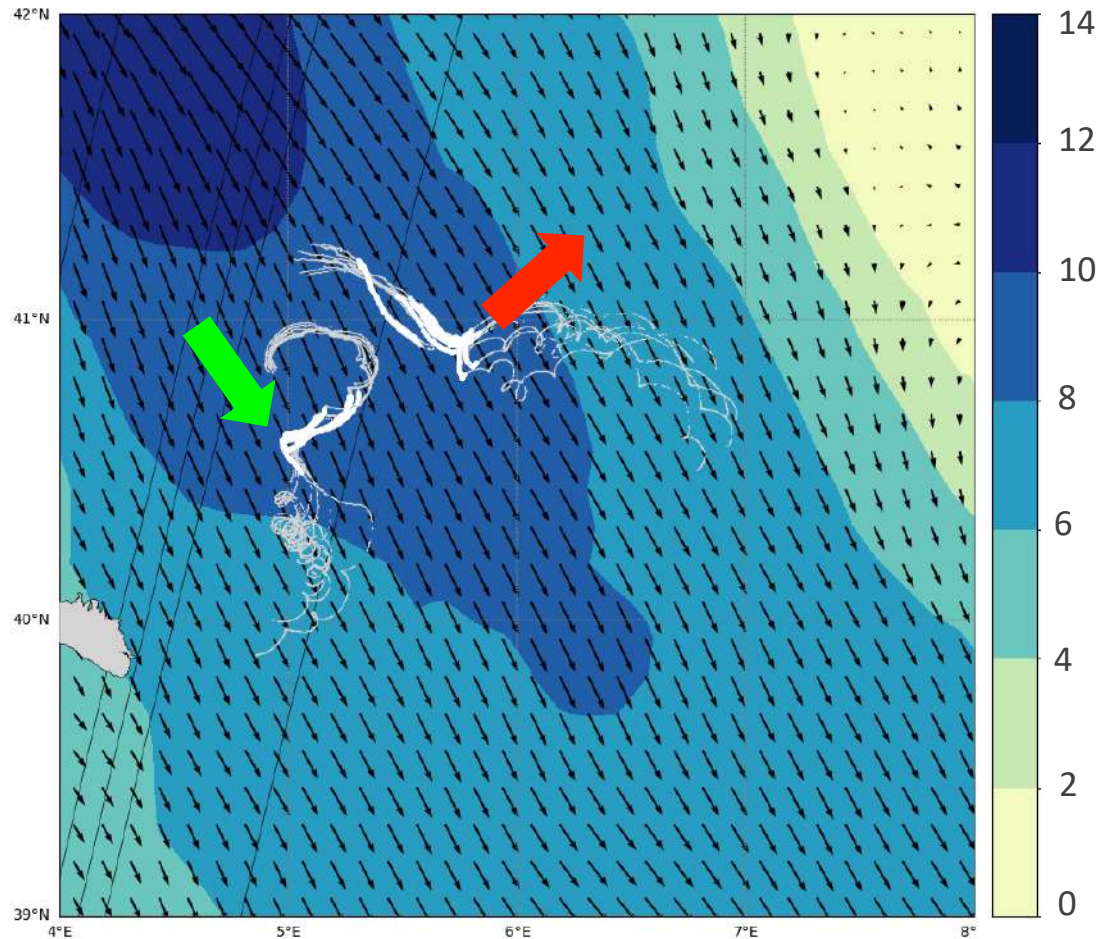
## May 8 afternoon:

- Mistral event: > 14m/s
- 1-m drifters:
  - in eddy: ejected toward SE (as wind)



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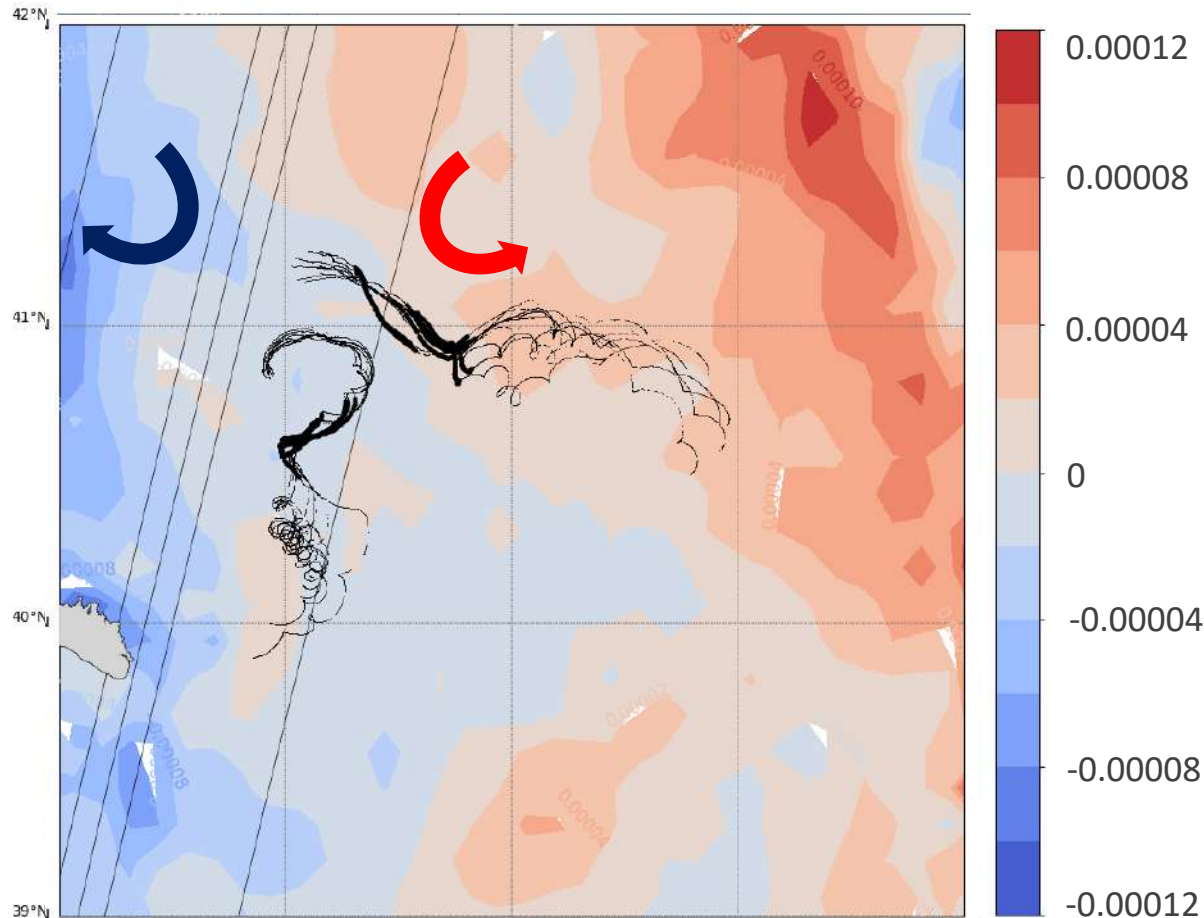
## May 8 afternoon:

- Mistral event: > 14m/s
- 1-m drifters:
  - in eddy: ejected toward SE (as wind)
  - in front: drifters toward NE. Why???



# 4. Influence of the wind: vorticity

May 8 - Wind curl ( $m^{-1}$ )  
1-m drifters trajectories

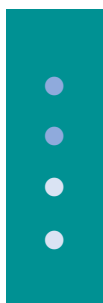


## Before May 8:

- Negative vorticity in drifter's area

## May 8:

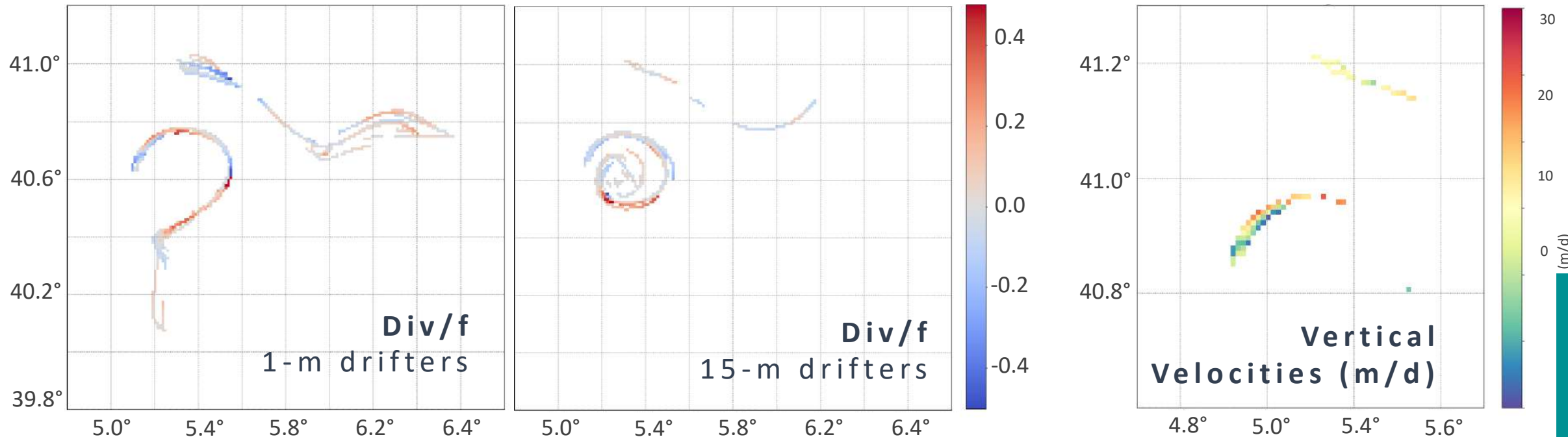
- Shift from negative to positive vorticity coinciding with shift in 1-m drifters trajectories and vorticity



# 4. NIOs filtering and new Lagrangian diag.

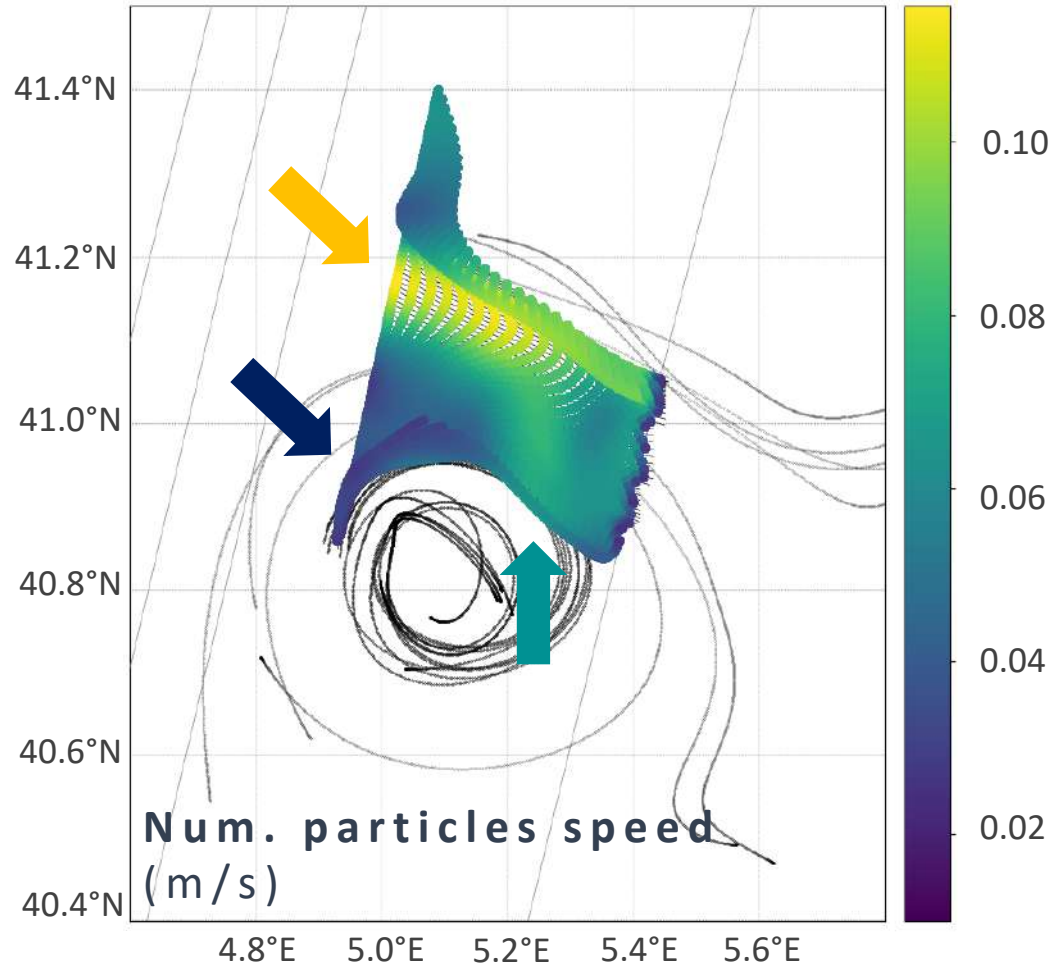
Low-pass Butterworth filter (order = 4, period = 24h)

**FILTERED TRAJECTORIES:**



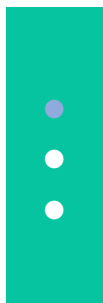
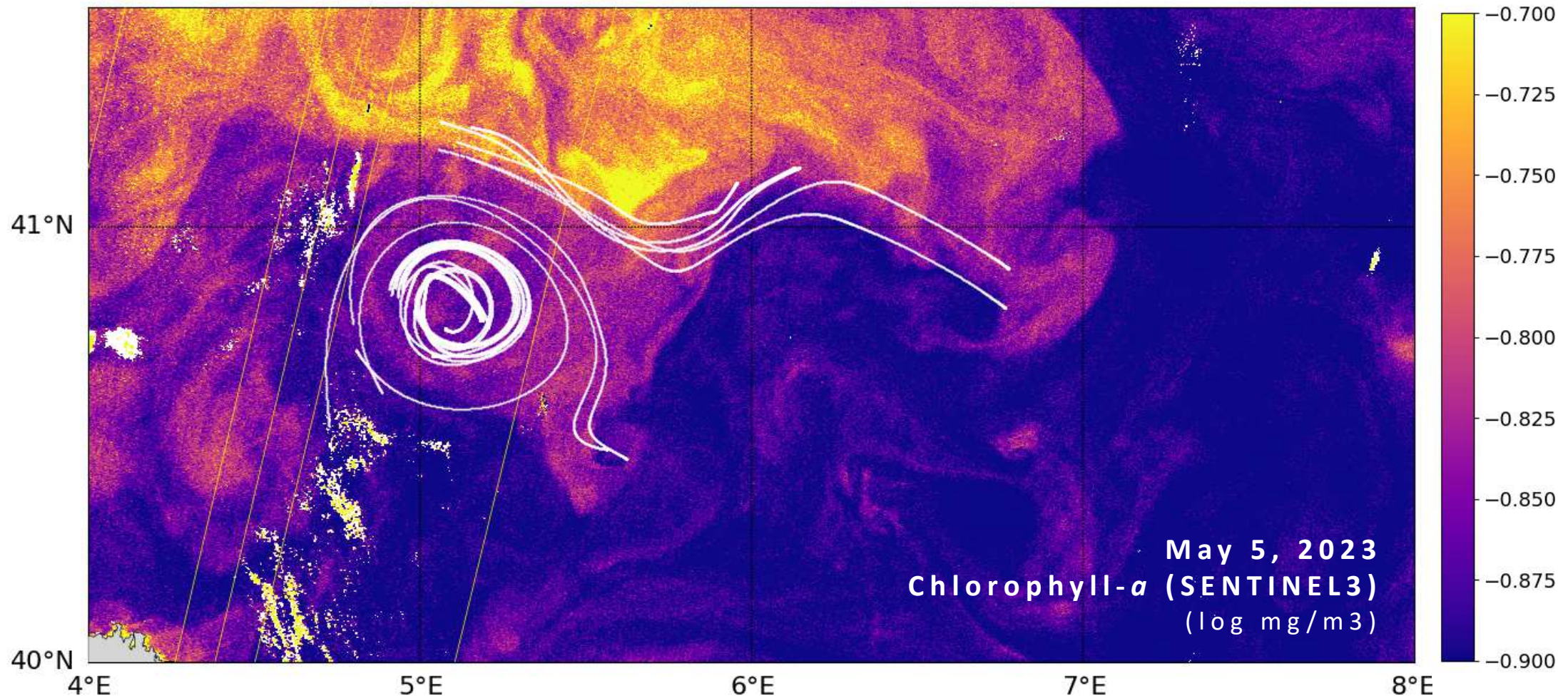
- Secondary vertical ageostrophic circulation (Tarry et al. 2021, 2022, Esposito et al. 2023)
- **Interaction between the eddy and the front?**

## 4. Interactions eddy / front: LAMTA simulations 14



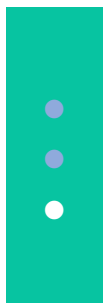
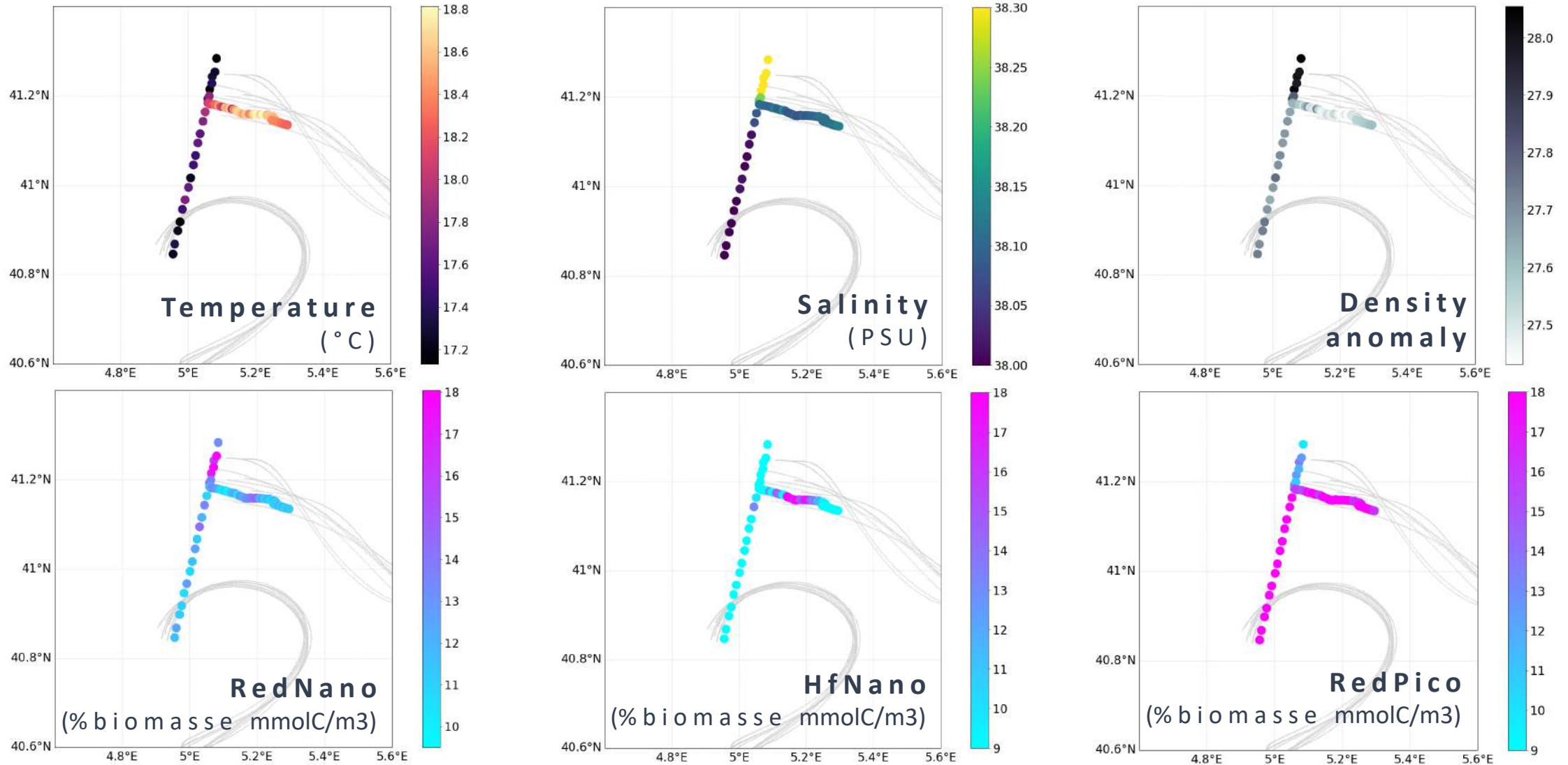
- **3-days simulation: May 6 to 8**
- 60 particles initialized on an array on the drifter deployment's locations in the eddy to the front
- **Day 1:** strong contrasted velocities between the front ( $> 0.1$  m/s) and the anticyclone (0.02-0.03 m/s)
- **Days 2-3: homogeneous velocities** (0.07-0.08 m/s)
- Interactions between the two fine-scale features

# 5. Coupling physics - biogeochemistry



# 5. Coupling physics – biology

(TSG - cytometry data: courtesy of L. Oms and G. Grégori)



# 5. Conclusion and perspectives

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- **Fine-scale features and circulation & Lagrangian diagnostics tools:** in-situ & remote sensing data + numerical tool;
  - **Estimation of Lagrangian properties (divergence,  $w$ ) for BioSWOT-Med** in conformity with literature;
  - **Original results** on interactions between two fine-scale features;
  - **Perspectives:** 3-D exploration of the impact of physical characteristics over biogeochemical processes;
- + Contribution to EGU 2024 abstract and poster (Martelucci et al.) and publication (Demol et al., in prep) on drifter datasets.



(not)  
the end...



# HISTORY OF DRIFTERS

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- 18<sup>th</sup> cent. Franklin / Lagrange
- **1980s:** Coastal Oceanic drifter experience (CODE, Davis et al., 1985)
- **1990s:** Surface Velocity Program (SVP)
- 1992: Friendly Floatees

REWARD

50¢

FOR RETURN OF  
DRIFT BOTTLE CARD OR  
SEABED DRIFTER TAG  
*with*  
WHERE & WHEN  
PLACE DATE  
\* THE BOTTLE OR DRIFTER WAS CAUGHT  
GIVE CARD OR TAG TO  
PORT INTERVIEWER  
OR MAIL TO  
WOODS HOLE OCEANOGRAPHIC INSTITUTION  
WOODS HOLE, MASS. 02543  
\* A LETTER WILL BE SENT YOU DESCRIBING WHERE THEY WERE RELEASED

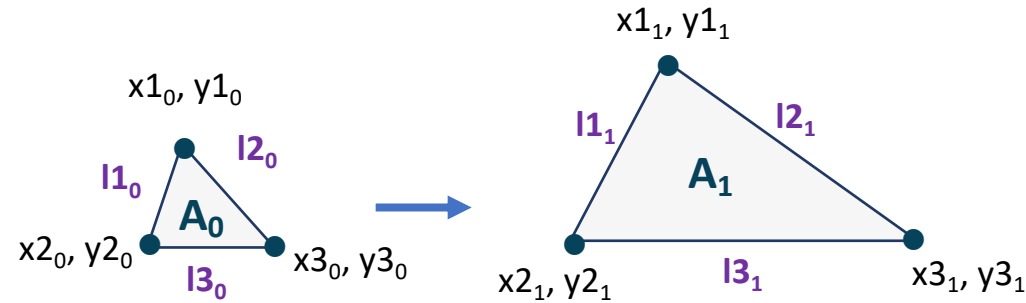
MOBY-DUCK

The True Story of 28,800 Bath Toys  
Lost at Sea and of the Beachcombers,  
Oceanographers, Environmentalists,  
and Fools, Including the Author, Who  
Went in Search of Them

Donovan Hohn

# AREA RATE OF CHANGE Molinari et Kirwan. (1975)

$$D = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \rightarrow \frac{1}{A} \frac{dA}{dt}$$

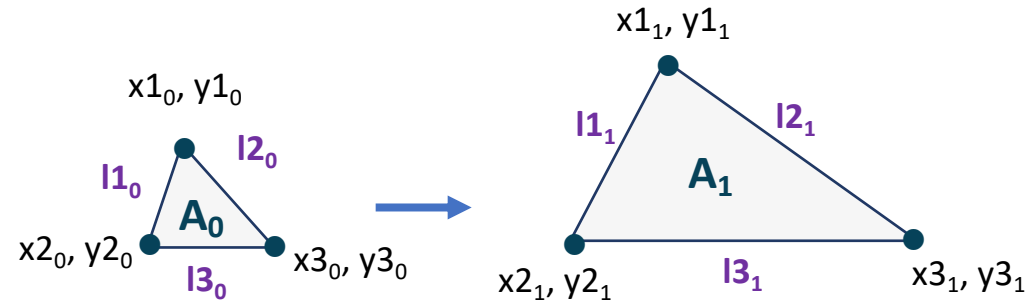


$D > 0 \rightarrow$  divergence  
 $D < 0 \rightarrow$  convergence

Heron's formula for  $A_0 / A_1$ :  $\sqrt{c \cdot (c-l_1) \cdot (c-l_2) \cdot (c-l_3)}$   
with  $c$  = half-perimeter

# AREA RATE OF CHANGE Molinari et Kirwan. (1975)

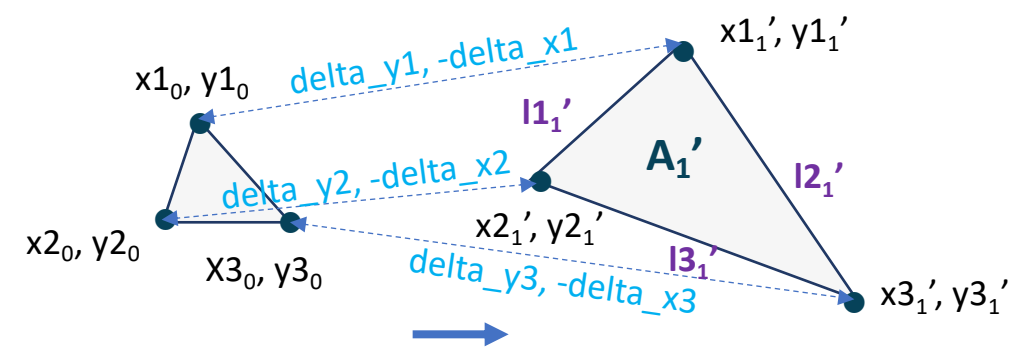
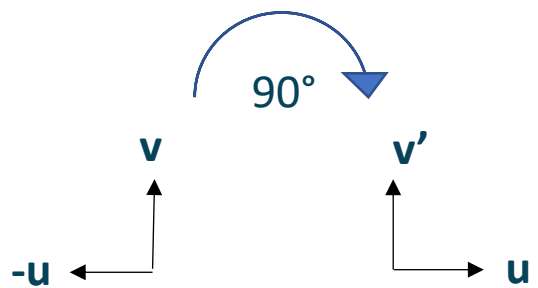
$$D = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \rightarrow \frac{1}{A} \frac{dA}{dt}$$



$D > 0 \rightarrow$  divergence  
 $D < 0 \rightarrow$  convergence

Heron's formula for  $A_0 / A_1$ :  $\sqrt{c \cdot (c-l_1) \cdot (c-l_2) \cdot (c-l_3)}$   
 with  $c$  = half-perimeter

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} \rightarrow \frac{1}{A'} \frac{dA'}{dt}$$



$x_{1_1'}, y_{1_1}' = x_{1_0}, y_{1_0}, \text{delta\_y}_1, -\text{delta\_x}_1$   
 $\zeta > 0 \rightarrow$  cyclonique  
 $\zeta < 0 \rightarrow$  anticyclonique

# AREA RATE OF CHANGE Molinari et Kirwan. (1975)

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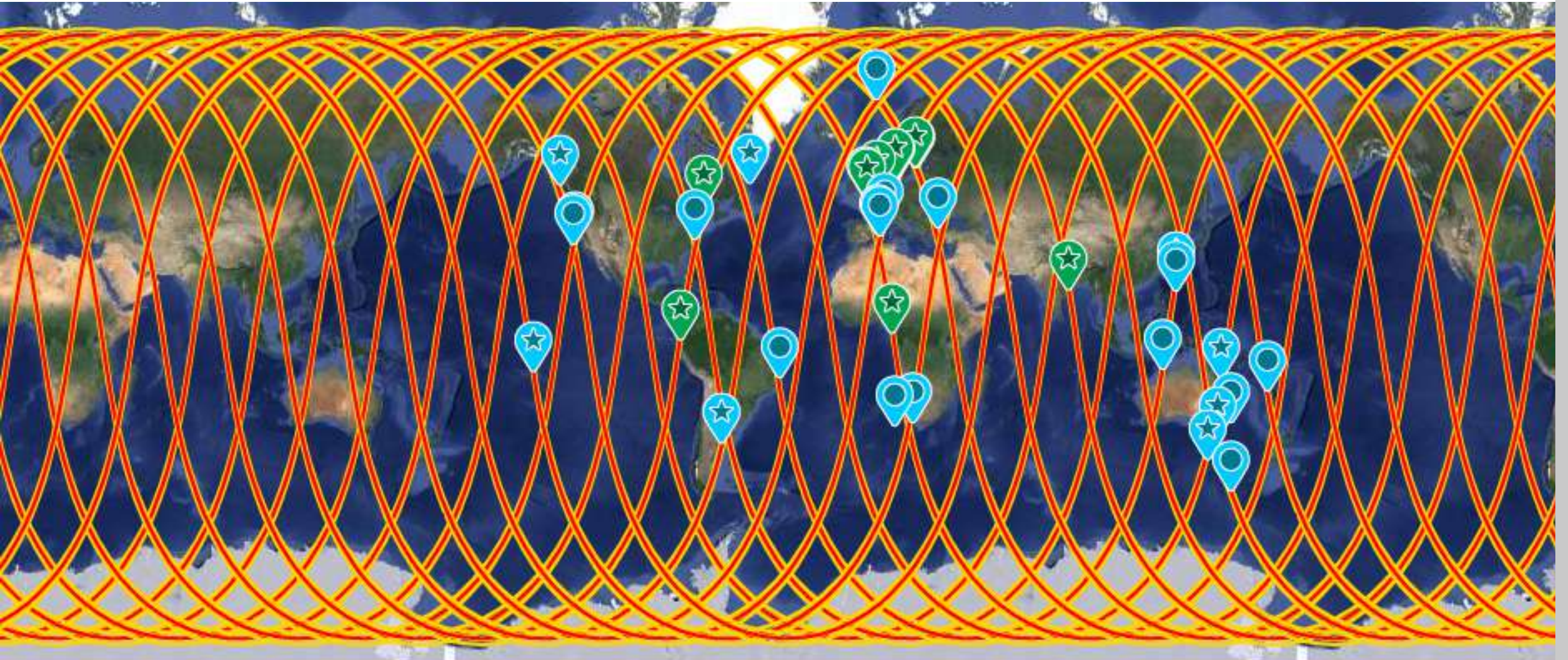
$$N_{st} = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = \frac{\partial u''}{\partial x} + \frac{\partial v''}{\partial y} \rightarrow \frac{1}{A''} \frac{dA''}{dt}$$

$$\text{Strain} = \sqrt{N_{st}^2 + S_{st}^2}$$

$$S_{st} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = \frac{\partial u'''}{\partial x} + \frac{\partial v'''}{\partial y} \rightarrow \frac{1}{A'''} \frac{dA'''}{dt}$$

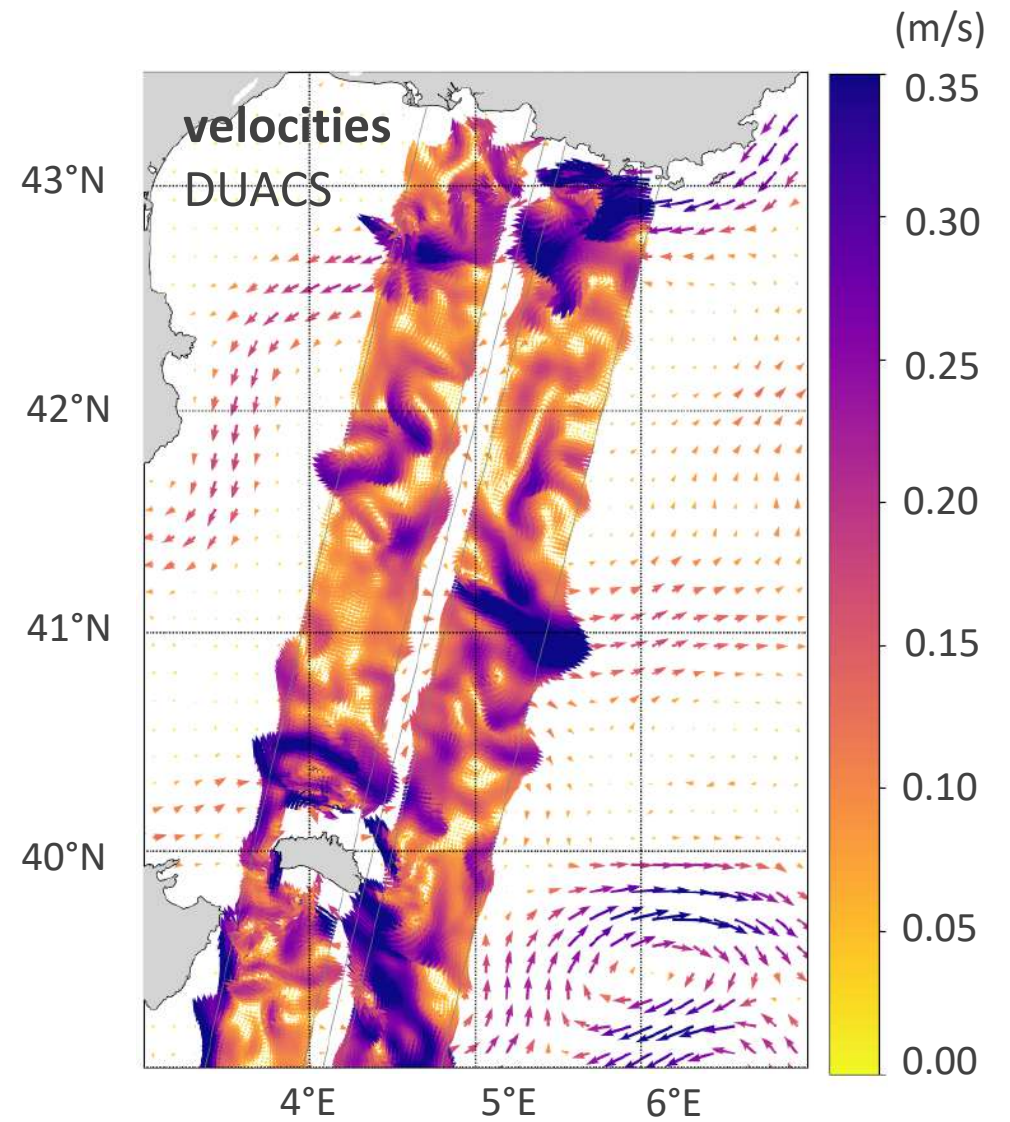
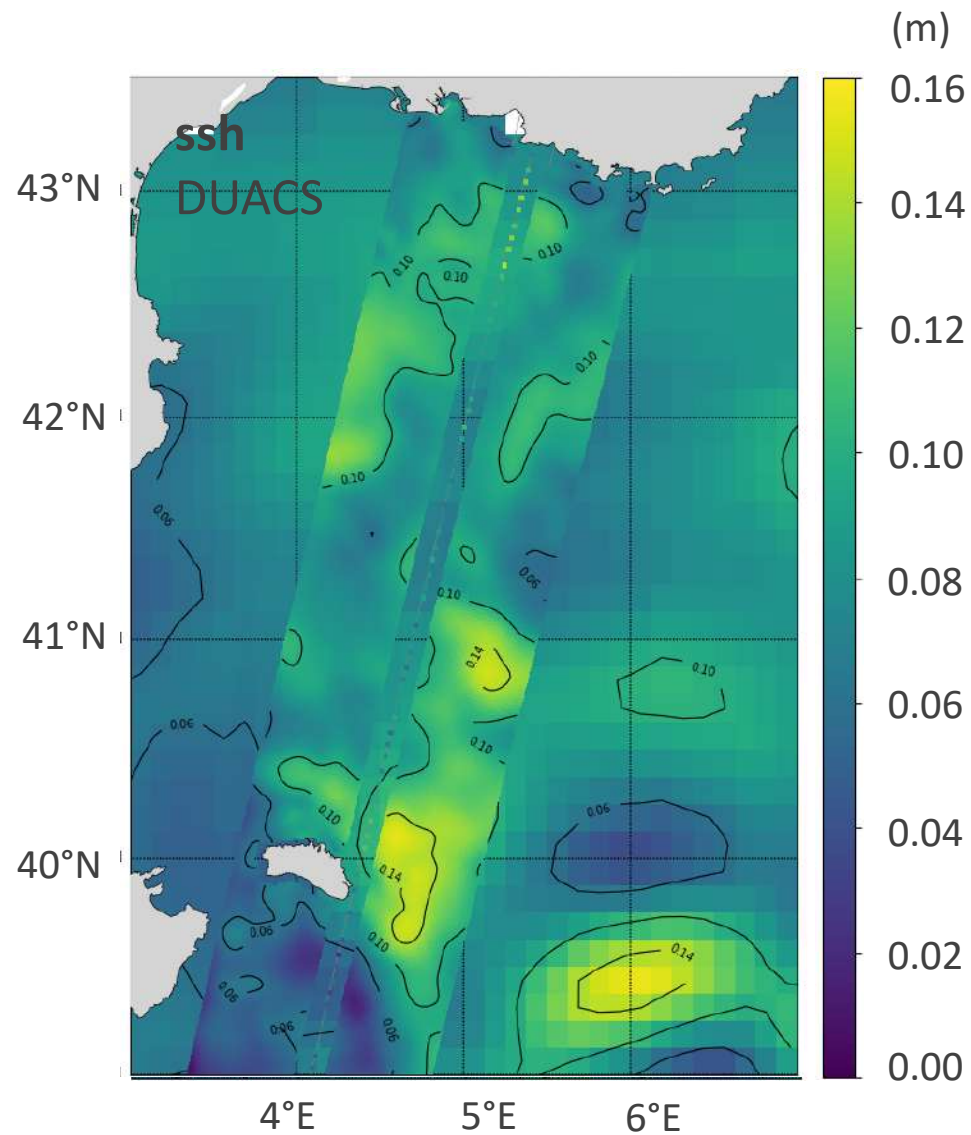
# SWOT vs DUACS

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Source: SWOT-AdAc

# SWOT vs DUACS



# OCEANIC VARIABILITY SEEN BY DUACS / SWOT

May 5

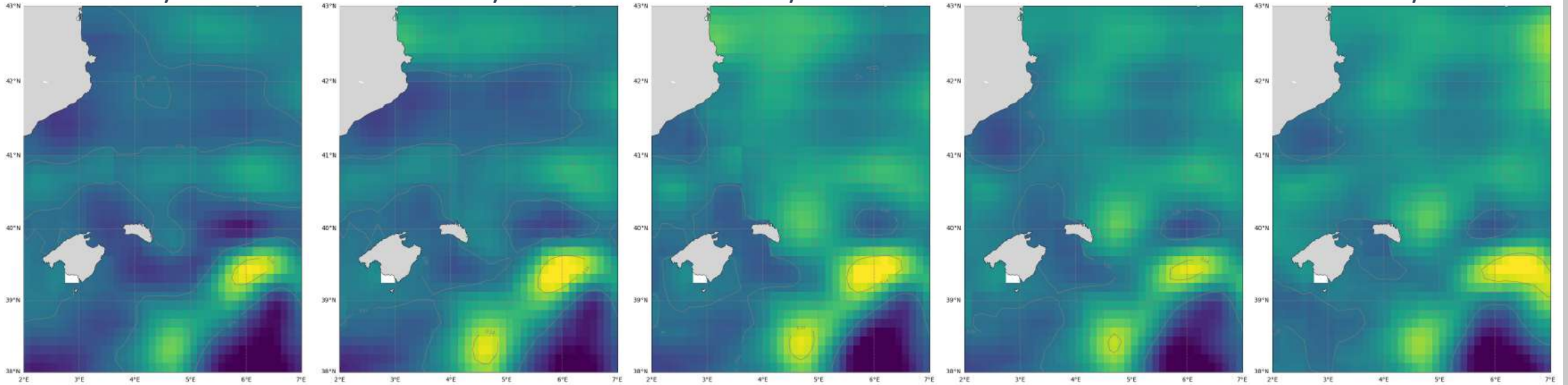
May 6

May 7

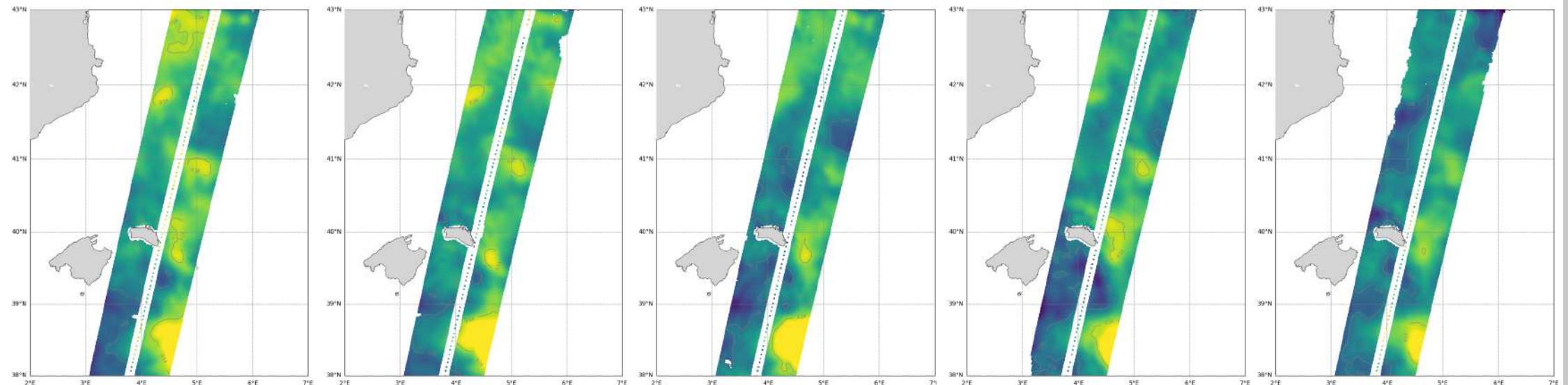
May 8

May 9

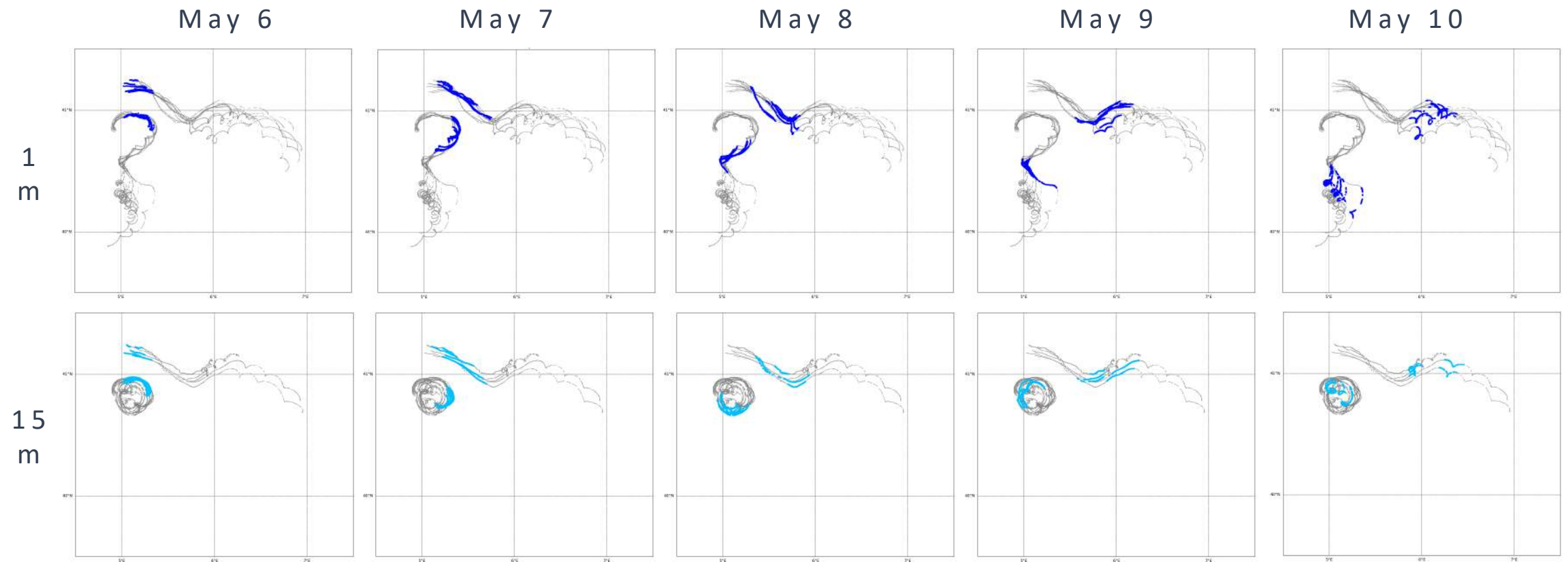
DUACS



SWOT



# DRIFTER'S TRAJECTORIES



# VERTICAL VELOCITY $w$ (1/2)

*Assumption of an incompressible fluid*

$$\text{Equation of continuity : } \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \Rightarrow \quad -\frac{\partial w}{\partial z} = D$$

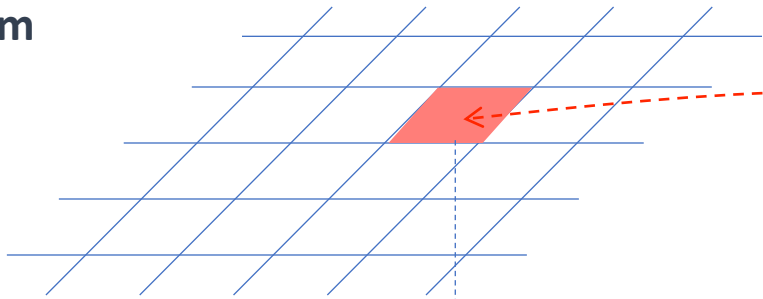
$$\Rightarrow \int_{-15m}^{0m} -\frac{\partial w}{\partial z} dz = \int_{-15m}^{0m} D dz \quad \Rightarrow \quad [-w]_{-15m}^{0m} = D \int_{-15m}^{0m} dz$$

$$\Rightarrow -\underbrace{w_{0m}}_{=0} - (-w_{-15m}) = D [z]_{-15m}^{0m}$$

$$\Rightarrow w_{-15m} = \boxed{D} z_{-15m}$$

# VERTICAL VELOCITY $w$ (2/2)

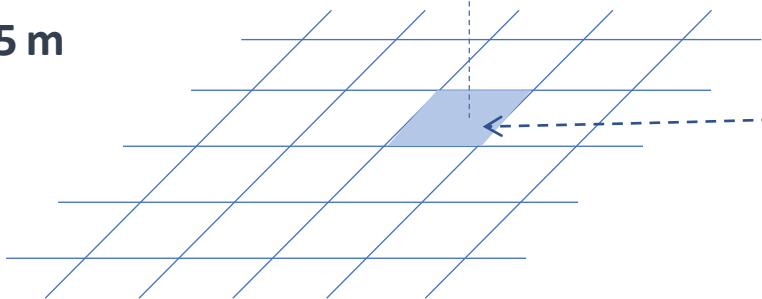
1 m



Grid resolution: 1 km x 1km, per day  
In each bin:

$$D = \frac{D_{mean0m} + D_{mean15m}}{2}$$

15 m



→  $w$ : smaller value at 15m  
most conservative method of estimation

# GEOSTROPHIC VELOCITY (1/2)

Navier – Stokes equations :  $\frac{d\vec{v}}{dt} = -\frac{1}{\rho}\vec{\Delta}p + \vec{g} - 2\Omega\vec{v} + \nu\vec{\Delta}^2\vec{v}$

**X** Neglected: stationary fluid, no friction

**□** Geostrophic approximation (pressure, gravitational & Coriolis forces)

$$v = \frac{1}{f\rho} \frac{\partial p}{\partial x} \quad u = -\frac{1}{f\rho} \frac{\partial p}{\partial y} \quad \frac{\partial p}{\partial z} = -\rho g \quad (\text{éq. hydrostatique})$$

# GESTROPHIC VELOCITY (2/2)

$$\Rightarrow \int_{-z_0}^{\eta} dp = -\rho_0 g \int_{-z_0}^{\eta} dz \quad (\eta = \text{sea - surface height})$$

$$\Rightarrow [p]_{-z_0}^{\eta} = -\rho_0 g [z]_{-z_0}^{\eta} \quad \Rightarrow \quad p_{\eta} - p_{-z_0} = -\rho_0 g(\eta + z_0)$$

$$\Rightarrow p_{-z_0} = p_{\eta} + \rho_0 g(\eta + z_0) \quad (p_{\eta} = \text{pression atm.})$$

$$\Rightarrow v = \frac{1}{f\rho} \frac{\partial(\rho_0 g(\eta + z_0))}{\partial x} \quad u = -\frac{1}{f\rho} \frac{\partial(\rho_0 g(\eta + z_0))}{\partial y}$$

$$\Rightarrow v = \frac{g}{f} \frac{\partial \eta}{\partial x} \quad u = -\frac{g}{f} \frac{\partial \eta}{\partial y} \quad \eta = \text{ssh measured by satellite}$$

# SATELLITE ALTIMETRY

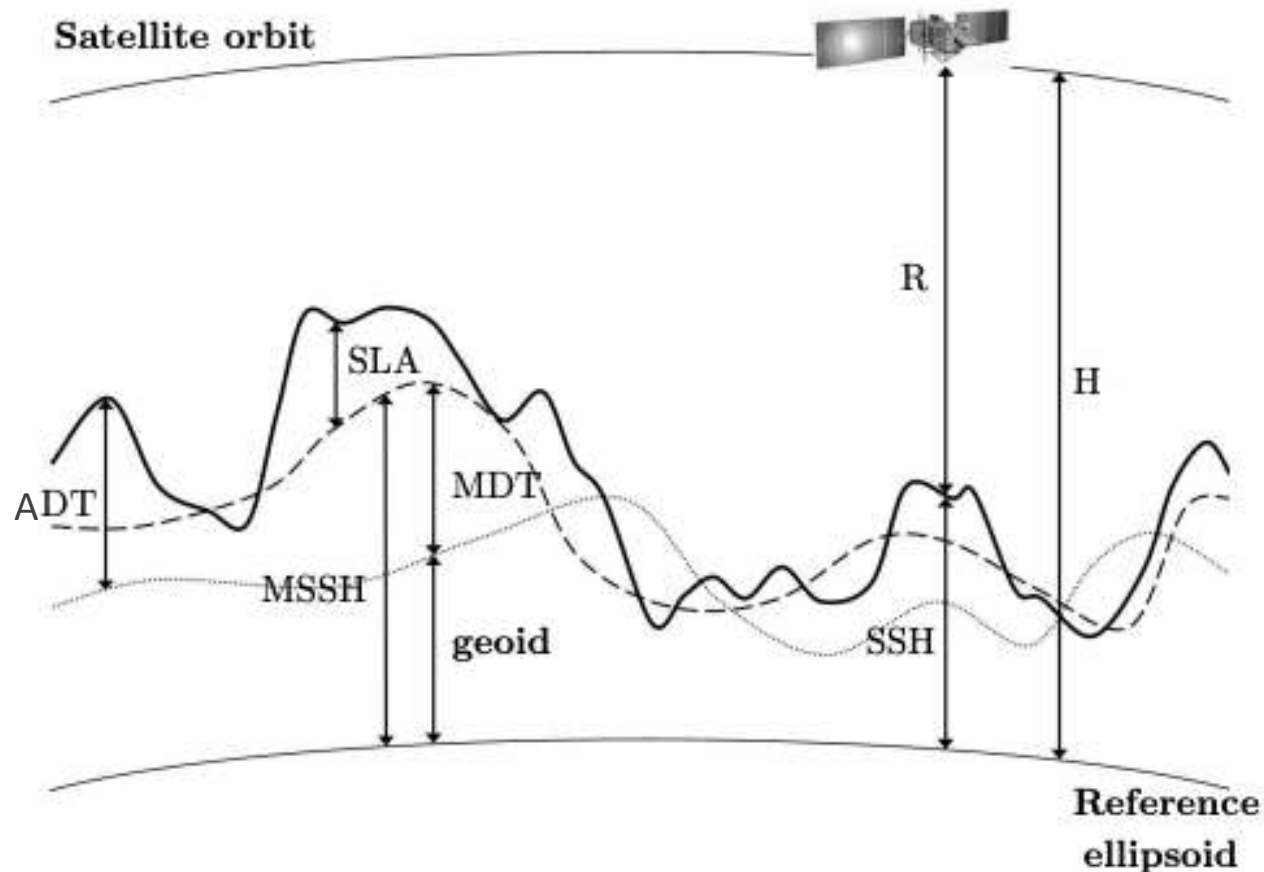


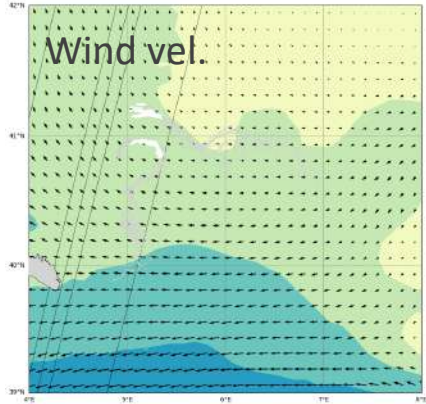
FIG. 1.1 – Principe de la mesure altimétrique. D'après Castruccio (2006).

THESE Clément Ubelmann. Etude de scénarios d'altimétrie satellitaire pour le contrôle de la circulation océanique dans l'océan Atlantique tropical par assimilation de données. Océan, atmosphère. Université Joseph-Fourier - Grenoble I, 2009. Français. ffNNT : ff. fftel-00408763f

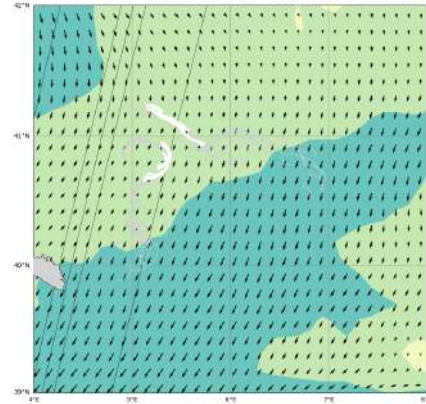
- **Géoïde** = surface équipotentielle du champ de gravité terrestre
  - **Ellipsoïde de référence** = approximation du géoïde
  - **H** = distance satellite – ellipsoïde
  - **R** = distance satellite – surface de l'eau
  - **SSH** (Sea Surface High) =  $H - R$ .
- La SSH a deux composantes :
- **MSSH** = Mean SSH (SSH moyenne sur un temps long)
  - **SLA** (Sea Level anomaly, due à la circulation océanique) =  $SSH - MSSH$
  - **ADT** (Absolute dynamique topographie, circulation océanique variable en fonction du géoïde) =  $SLA + MDT$
  - **MDT** =  $MSSH - \text{géoïde}$

# WIND FIELDS: May 06 to 10

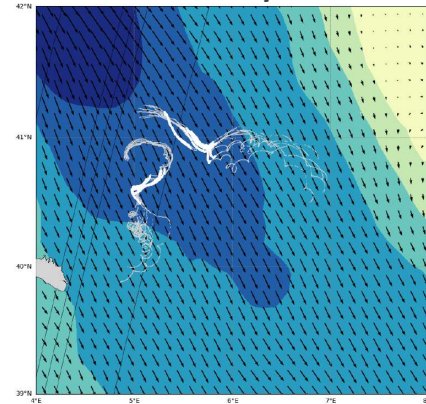
May 6



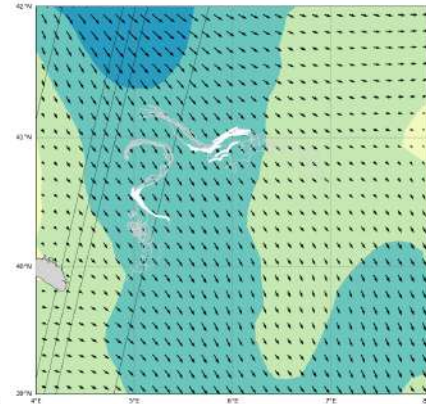
May 7



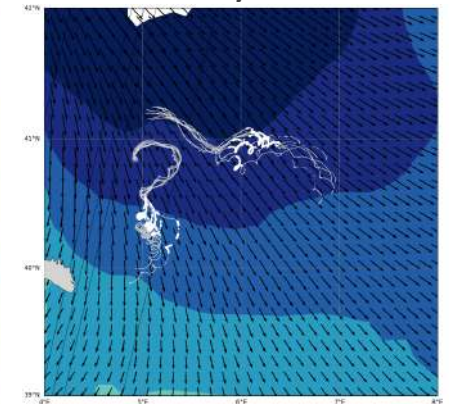
May 8



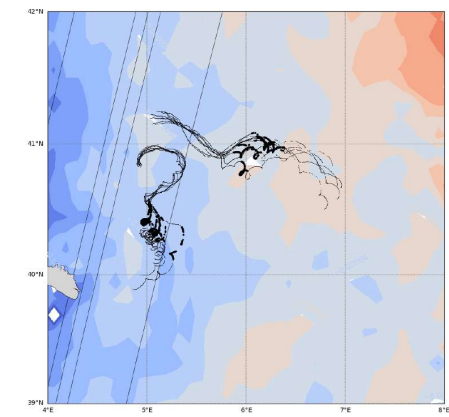
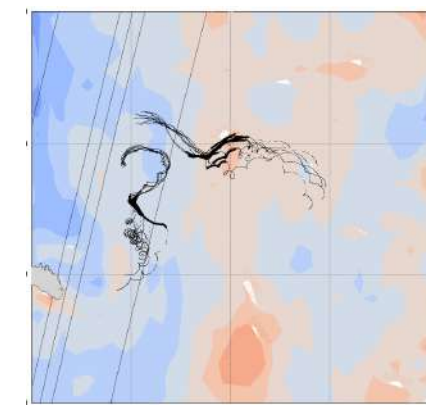
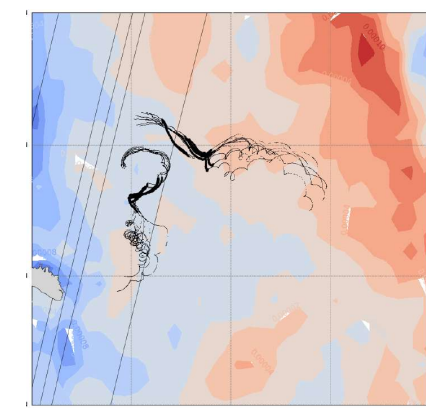
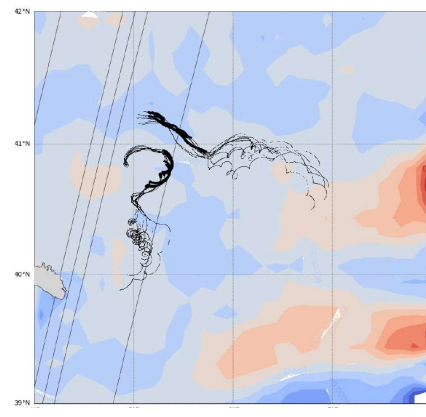
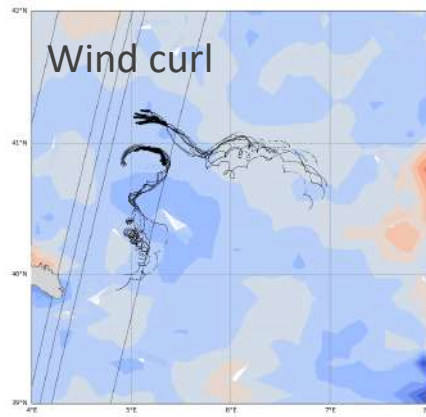
May 9



May 10



Wind curl



# INFLUENCE OF ssh

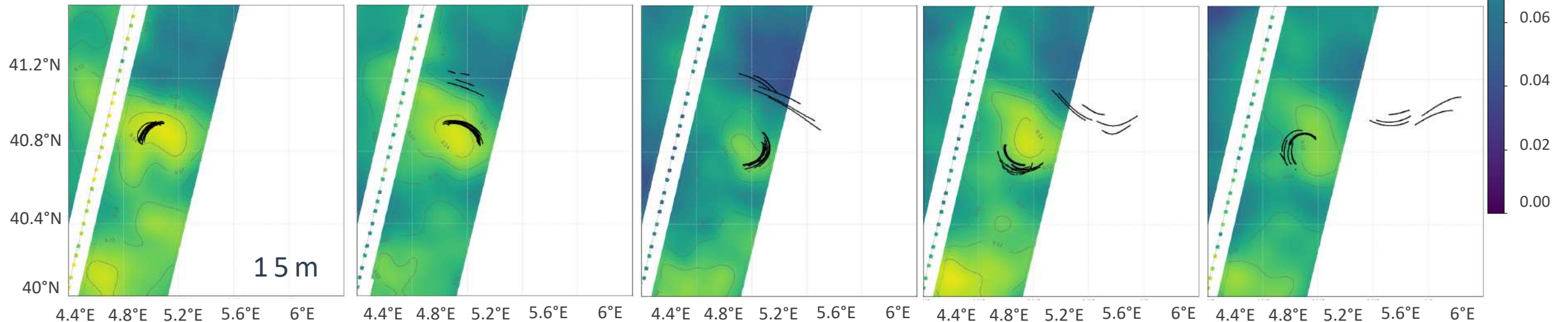
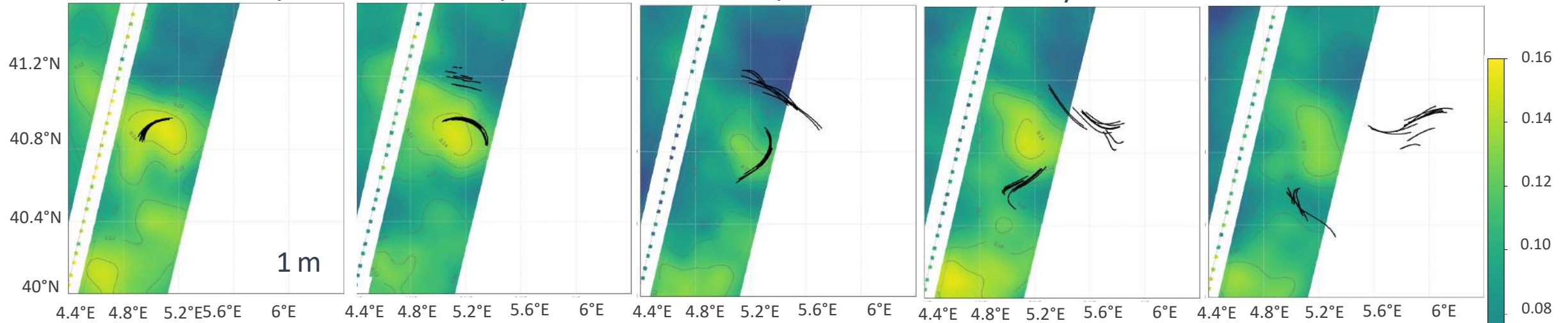
ssh (m) May 5

May 6

May 7

May 8

May 9



# LAMTA – METHODE NUM. RUNGE-KUTTA ORDRE 4

Solving Ordinary Differential Equations (ODE):

$$\frac{dy}{dt} = f(t, y)$$

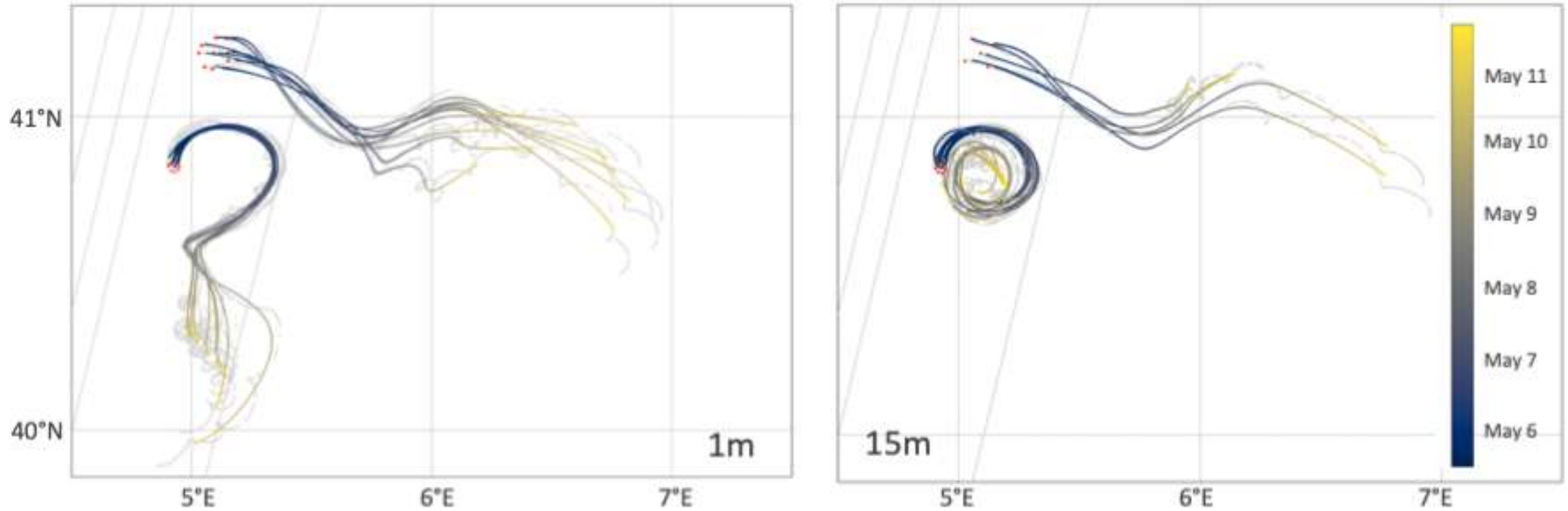
avec une condition initiale  $y(t_0) = y_0$ .

$$Y_{n+1} = Y_n + \frac{h}{6} (K_1 + 2K_2 + 2K_3 + K_4)$$

$$\begin{aligned} K_1 &= f(x_n, Y_n) \\ K_2 &= f\left(x_n + \frac{h}{2}, Y_n + \frac{h}{2}K_1\right) \\ K_3 &= f\left(x_n + \frac{h}{2}, Y_n + \frac{h}{2}K_2\right) \\ K_4 &= f(x_n + h, Y_n + hK_3) \end{aligned}$$

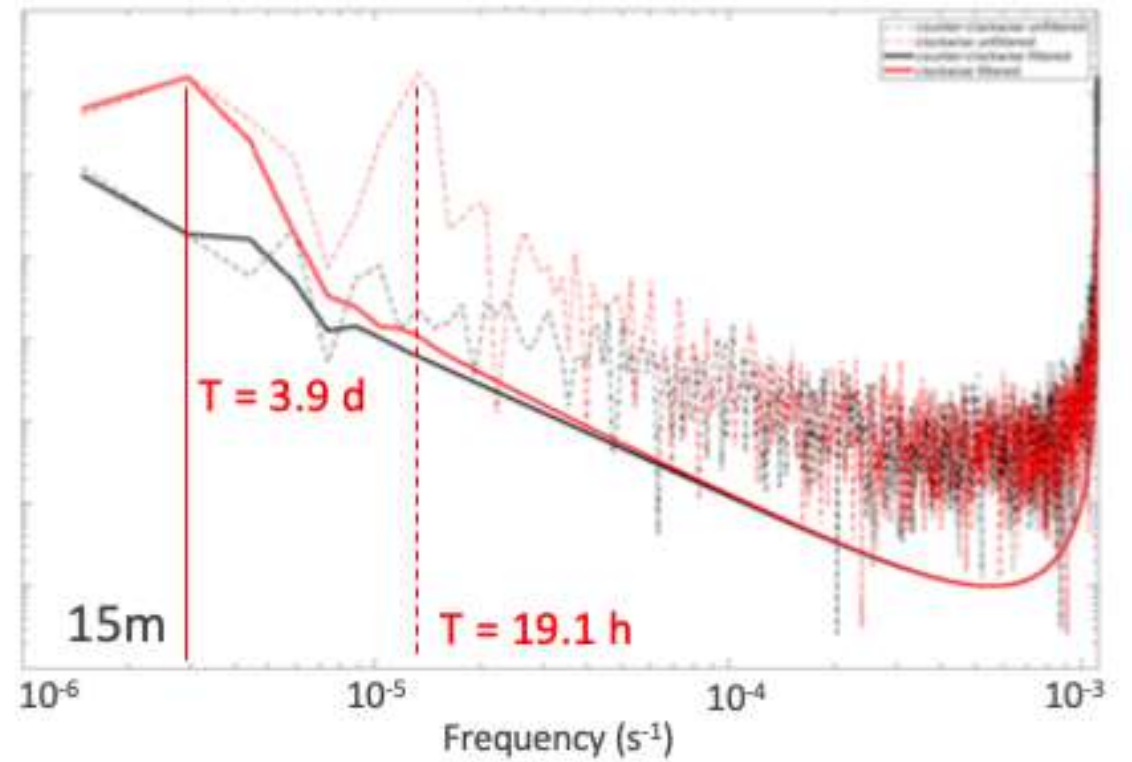
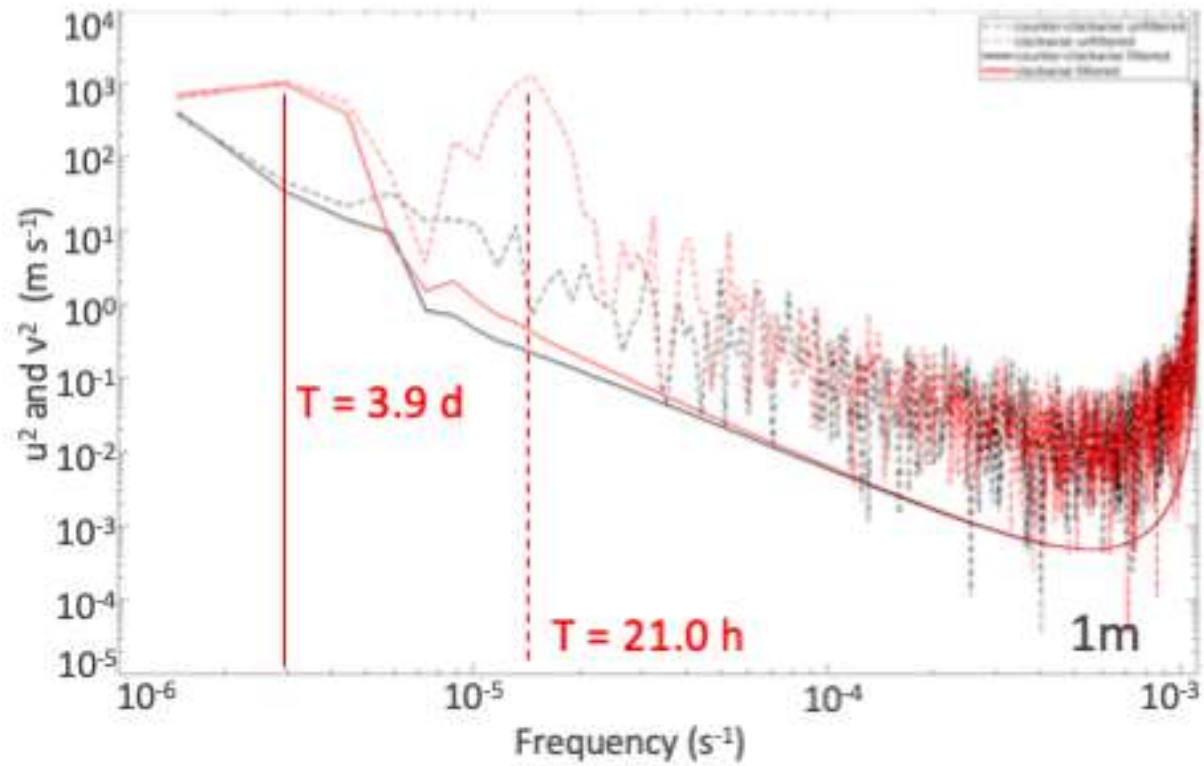
$$\begin{array}{c|cccc} 0 & & & & \\ \frac{1}{2} & \frac{1}{2} & & & \\ \frac{1}{2} & 0 & \frac{1}{2} & & \\ 1 & 0 & 0 & 1 & \\ \hline & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} \end{array}$$

# BUTTERWORTH FILTER

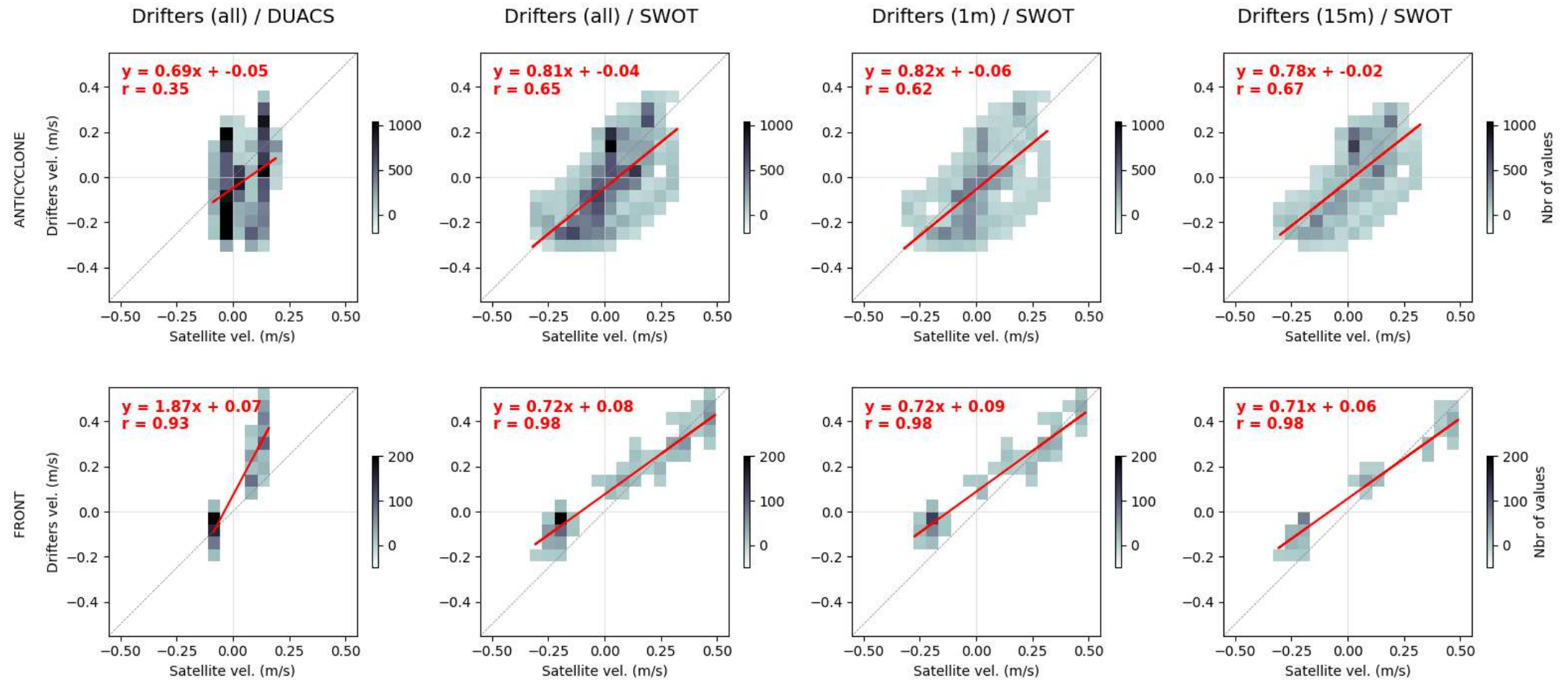


$$|H(\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^{2n}}}$$

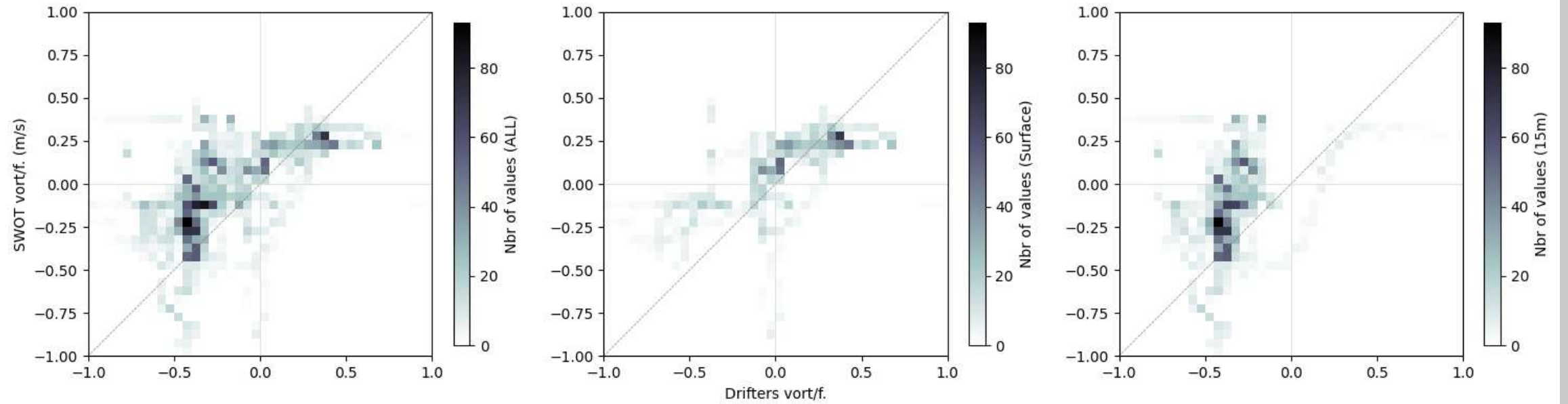
# ROTARY SPECTRA



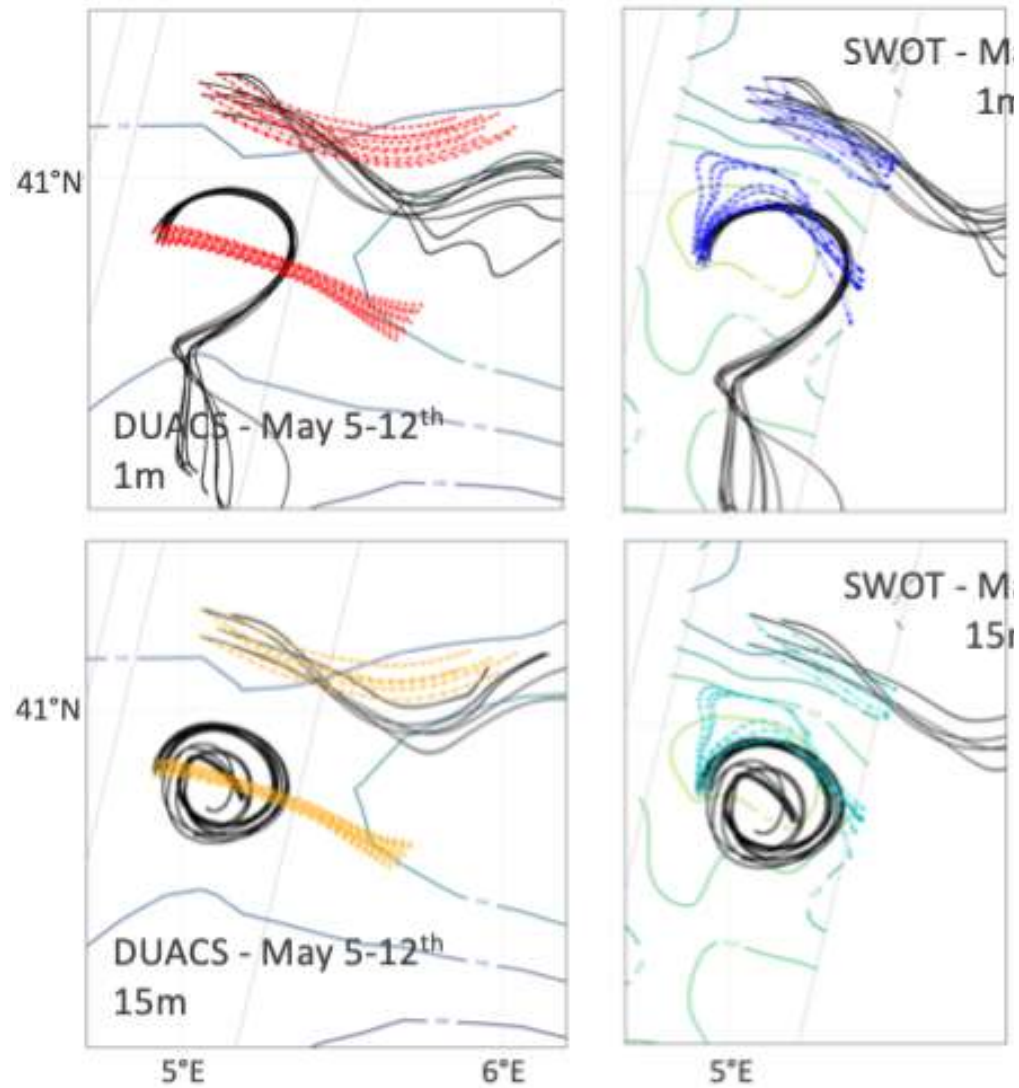
# COMPARISON DUACS / SWOT / DRIFTERS



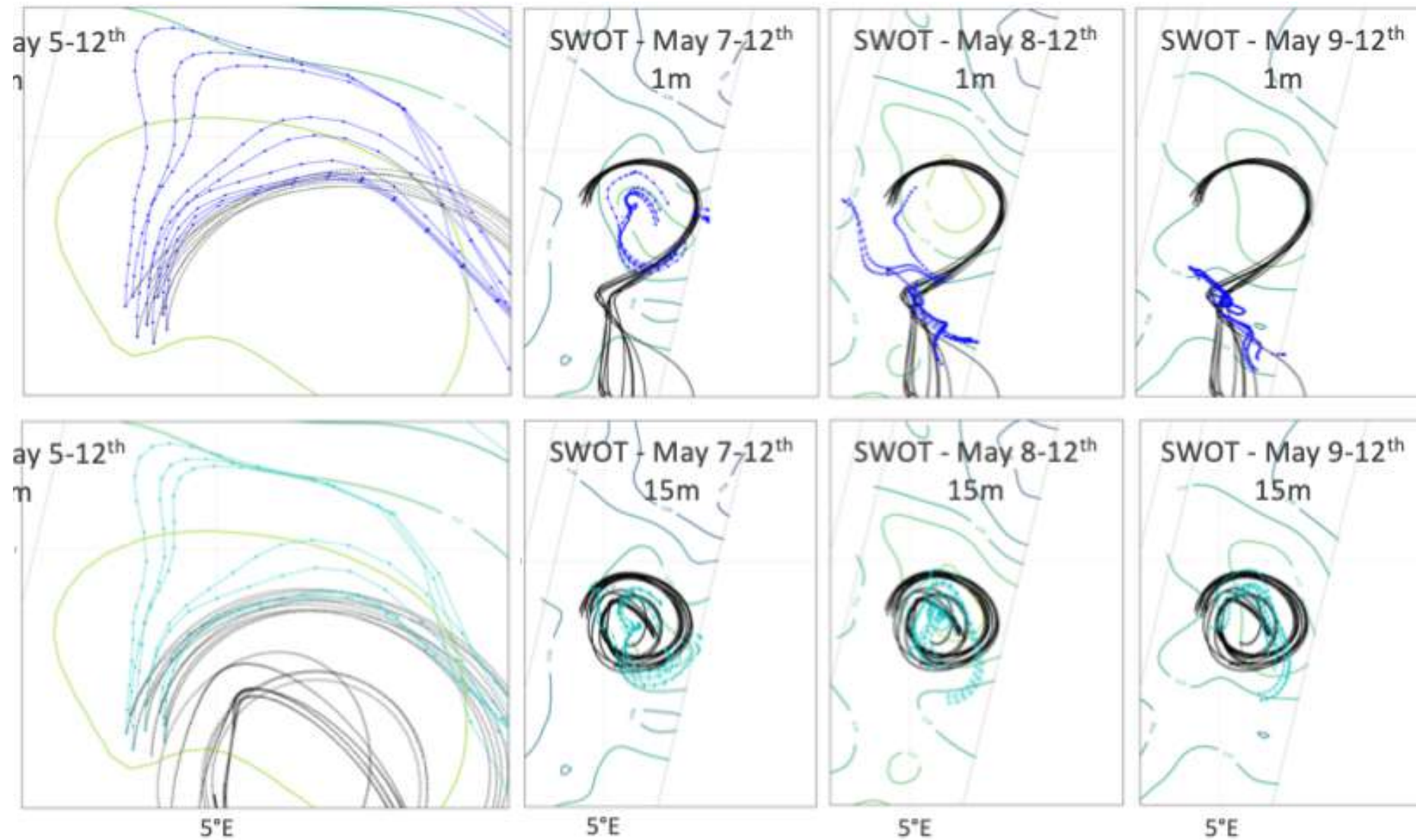
# COMPARISON DUACS / SWOT / DRIFTERS



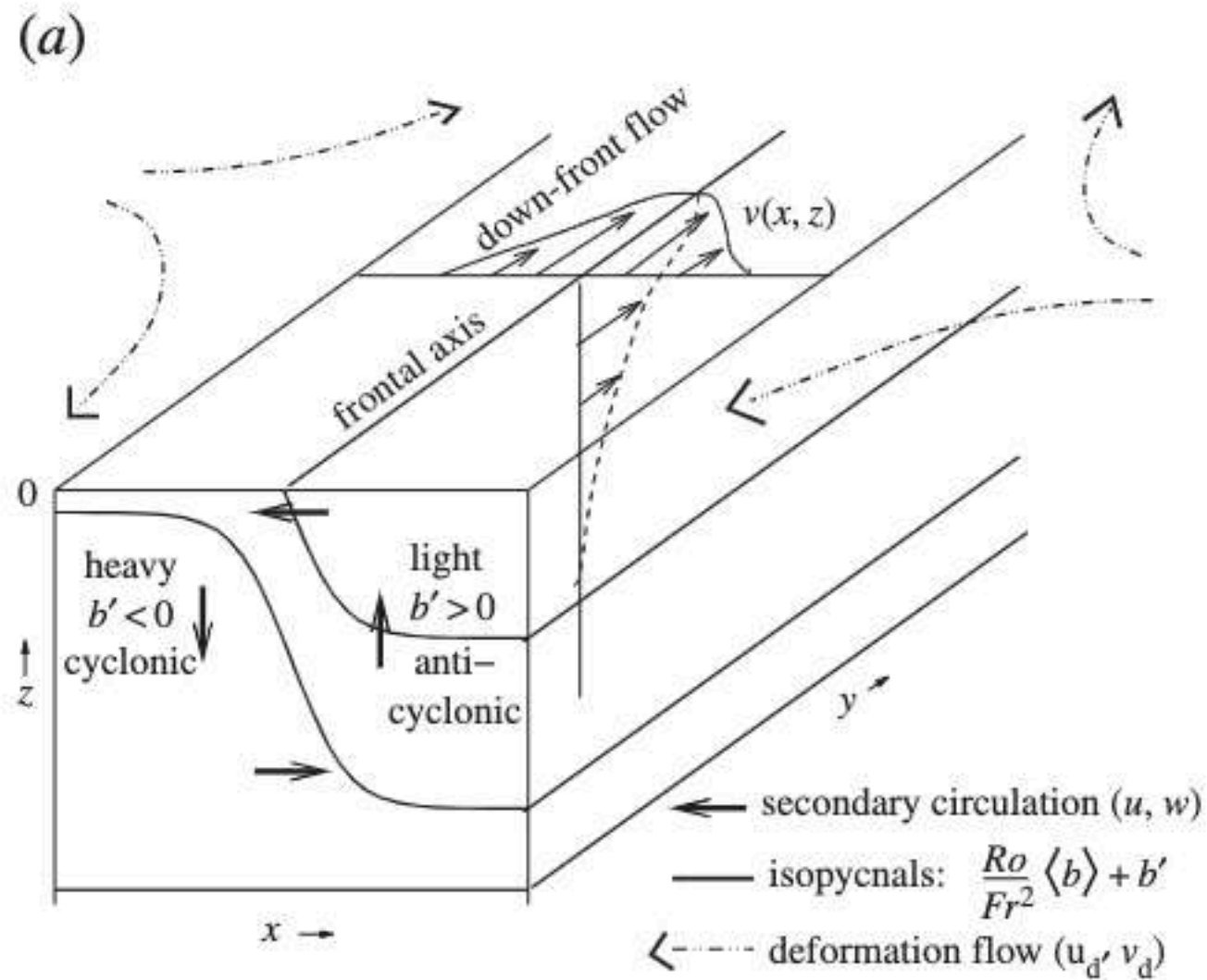
# COMPARISON DUACS / SWOT / DRIFTERS



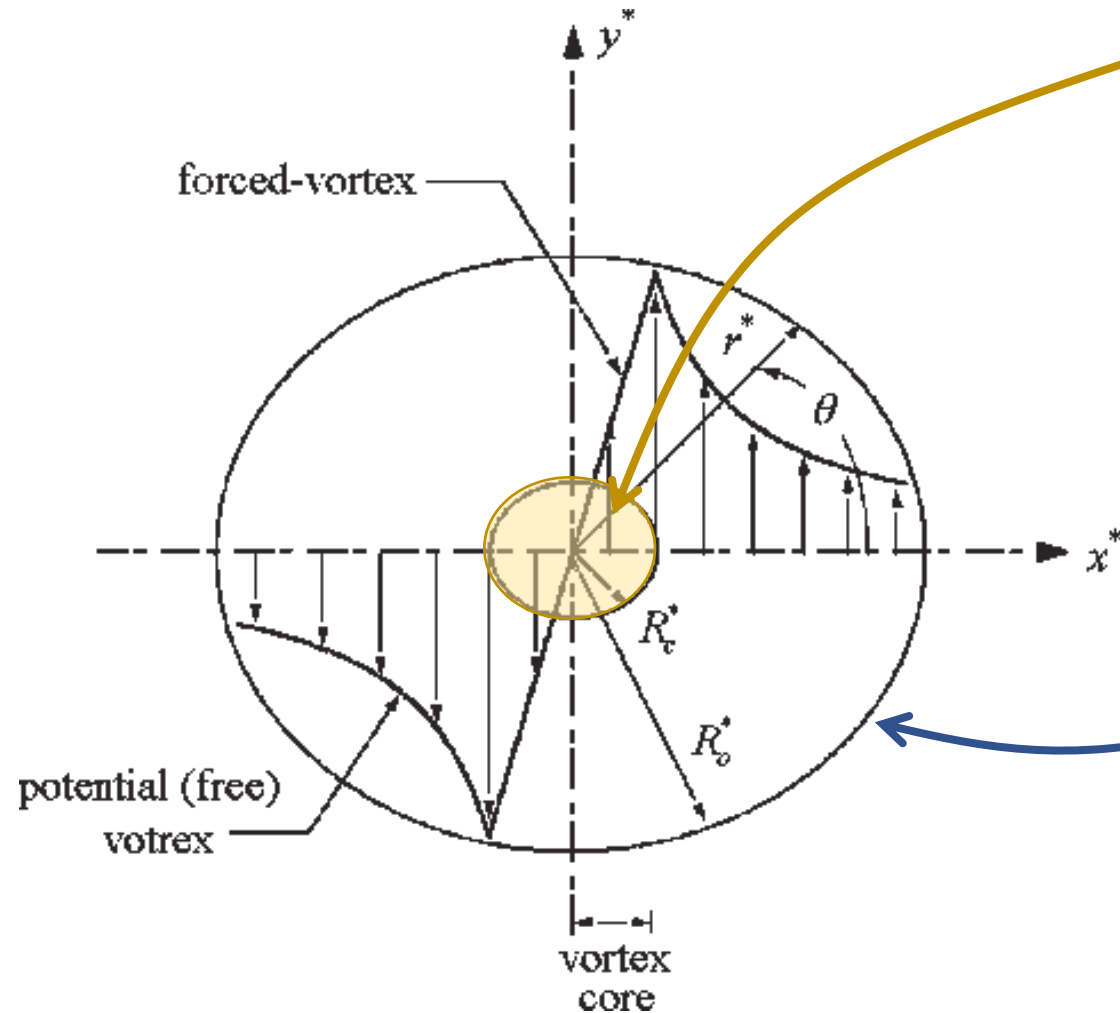
# COMPARISON DUACS / SWOT / DRIFTERS



# SECONDARY VERTICAL AGEOSTROPHIC CIRCULATION



# RANKINE MODEL



## Solid-body rotation

- Angular velocity: constant
- Tangential velocity: increase with distance to center

## Irrotational eddy

- Angular velocity: increase with proximity to the center) constant

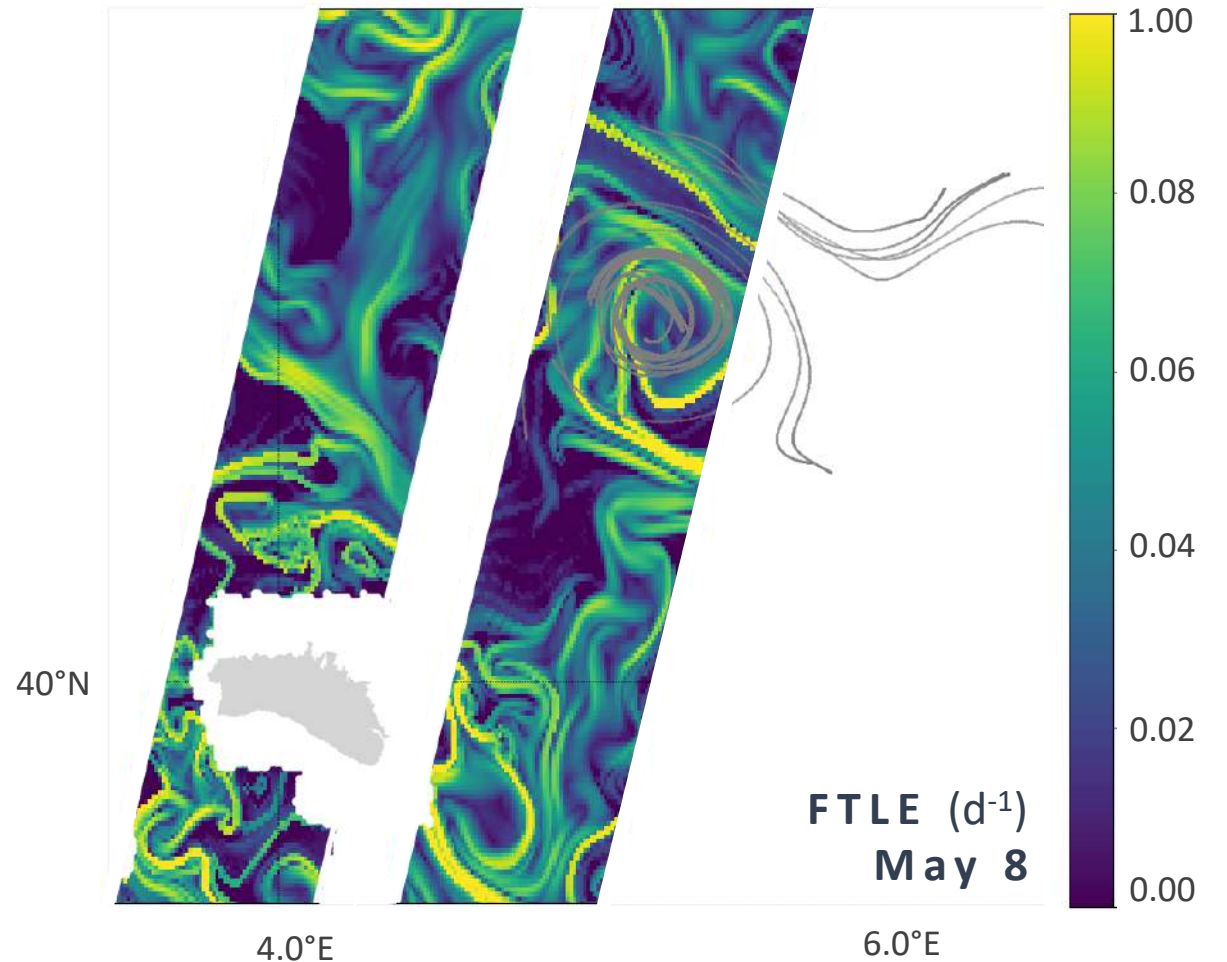
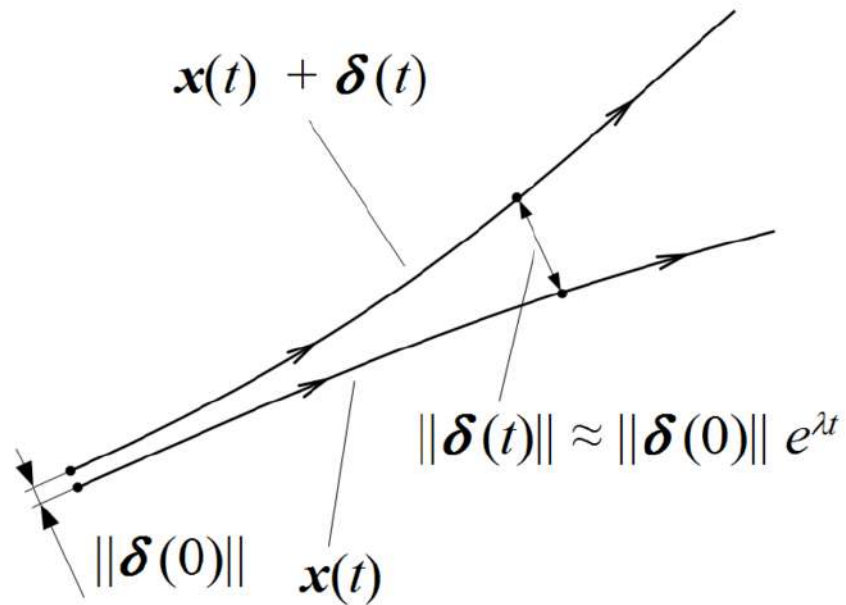
# FINITE-TIME LYAPUNOV EXPONENT (FTLE)

$$\lambda(t) = \lim_{\delta(0) \rightarrow 0} \frac{1}{t} \ln \frac{\|\delta(t)\|}{\|\delta(0)\|}$$

in  $s^{-1}$

$\delta(0)$  initial distance between 2 particles

$\delta(t)$  final distance after time  $t$



# CHL- $\alpha$ SENTINEL 3 – May 5

