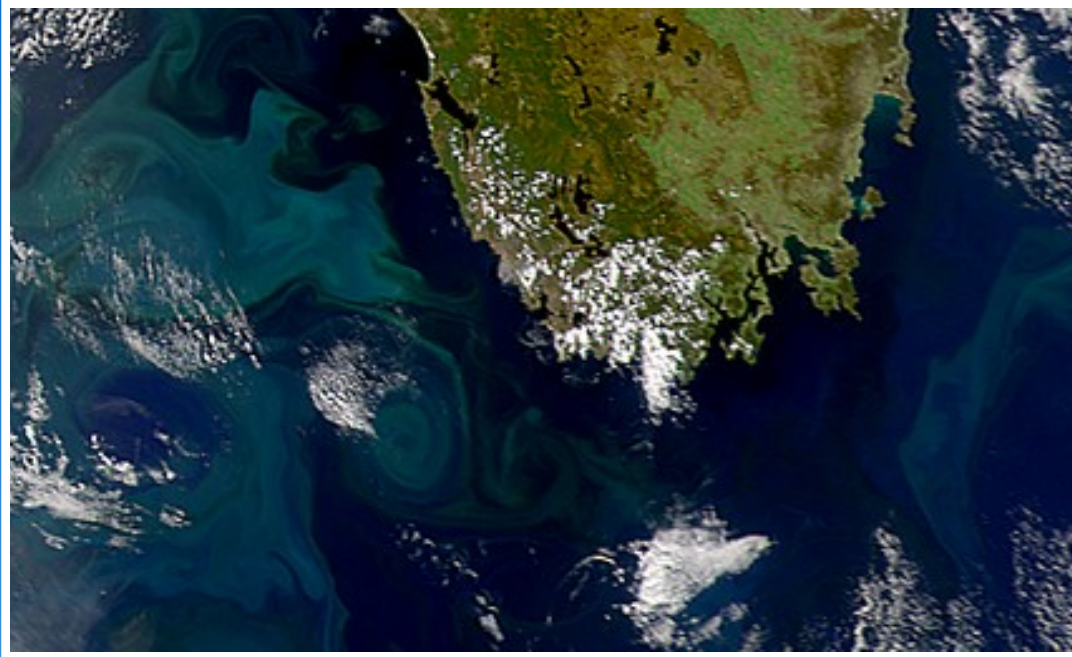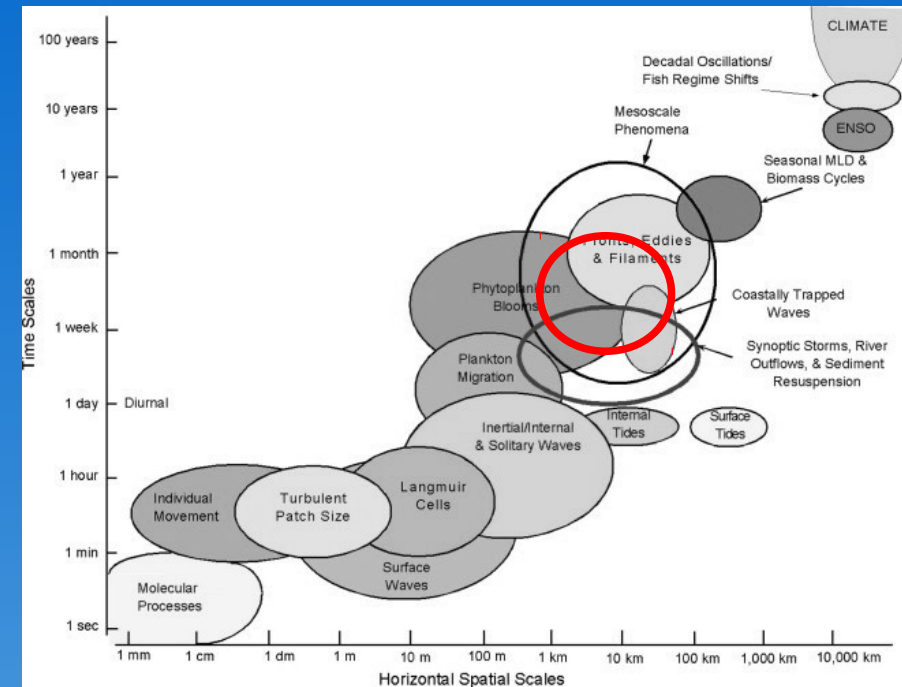# *In-situ* estimate of submesoscale horizontal eddy diffusion coefficients across a front

F. Nencioli [1], F. d'Ovidio [2], A.M. Doglioli [1], A.A. Petrenko [1]

(1) Aix-Marseille Université, Mediterranean Institute of Oceanography (MIO), 13288, Marseille, Cedex 9, France ; Université du Sud Toulon-Var; CNRS-INSU/IRD UM 110

(2) Laboratoire d'Océanographie et du Climat: Experimentation et Approches Numeriques, IPSL, Paris, France

MIO
Institut Méditerranéen
d'Océanologie

Aix*Marseille
université

cnrs

IRD
Institut de recherche
pour le développement

UNIVERSITÉ
DU SUD TOULON VAR

Submesoscale structures

(from http://oceanservice.noaa.gov/)

(from Dickey et al., J. Mar. Syst,.2003)

➔ **Fronts, jets and eddies: Horizontal scales ~(100m - 10km)**

Time scales ~(days - week)

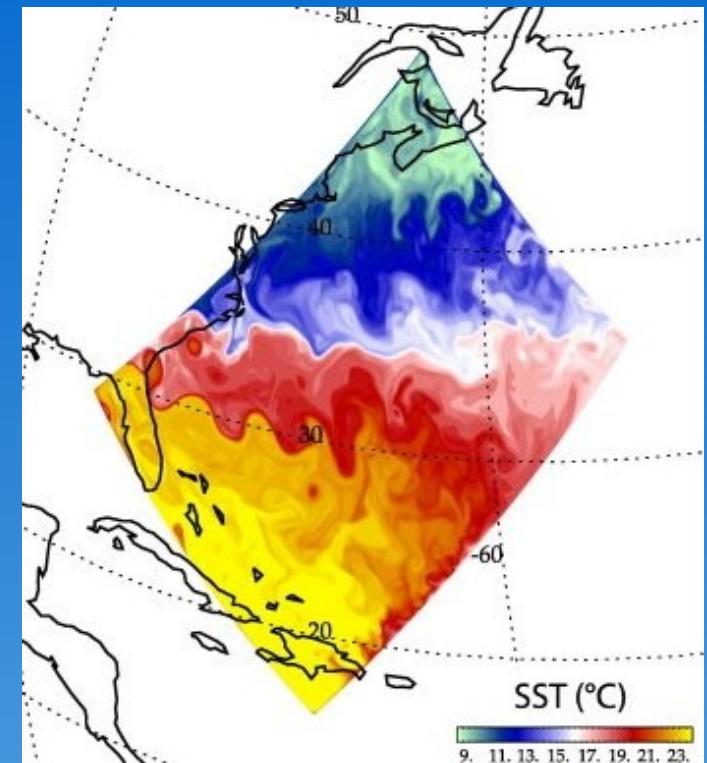➔ **Key role for : Energy transfer**

Horizontal and vertical transport

Biogeochemical cycles

Submesoscale structures

➔ **Numerical model studies**

    Favored by:

    Advances in computational power

    Development of regional models

(from Levy et al., Ocean Model., 2012)

Submesoscale structures

➜ **Numerical model studies**

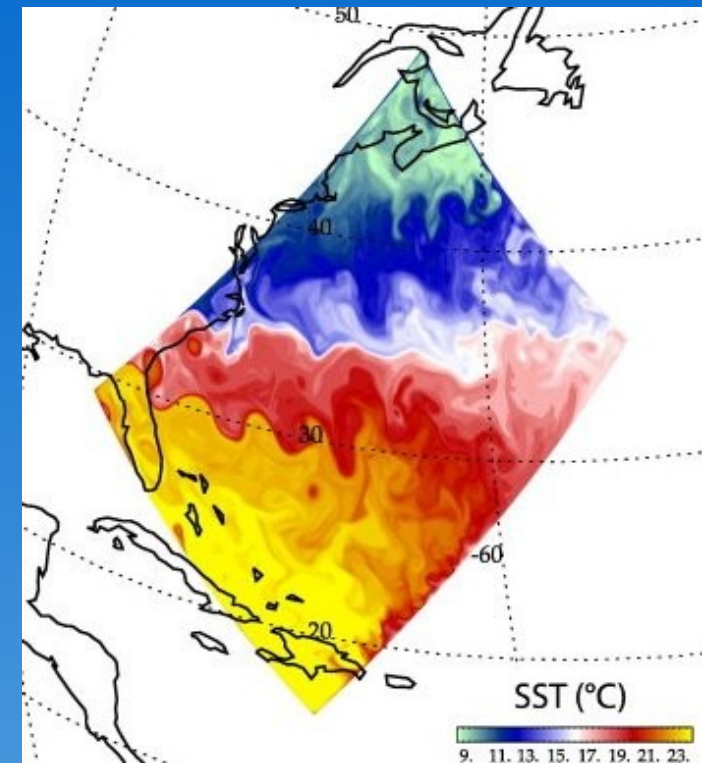**Favored by:**

Advances in computational power

Development of regional models

➜ **In-situ observations**

Challenging due to small spatial and temporal scales

⬇

Limited estimates of key physical parameters

(from Levy et al., Ocean Model., 2012)

Submesoscale structures

➔ **Numerical model studies**

     Favored by:

     Advances in computational power

     Development of regional models

➔ **In-situ observations**

     Challenging due to small spatial and temporal scales

⬇

(from Levy et al., Ocean Model., 2012)

Limited estimates of key physical parameters

Focus of this study

New approach to estimate horizontal eddy diffusion coefficients (Kh) from in-situ observations

Lagrangian Transport EXperiment
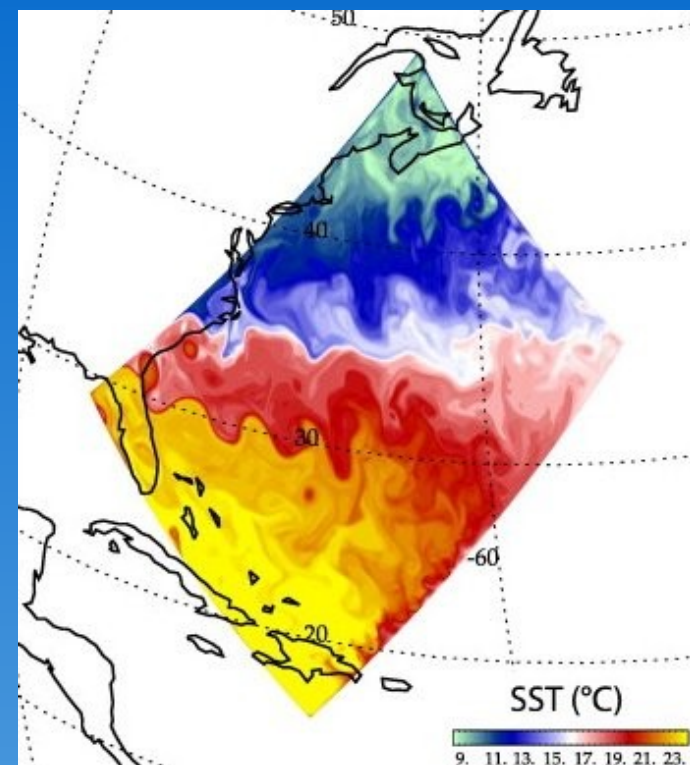
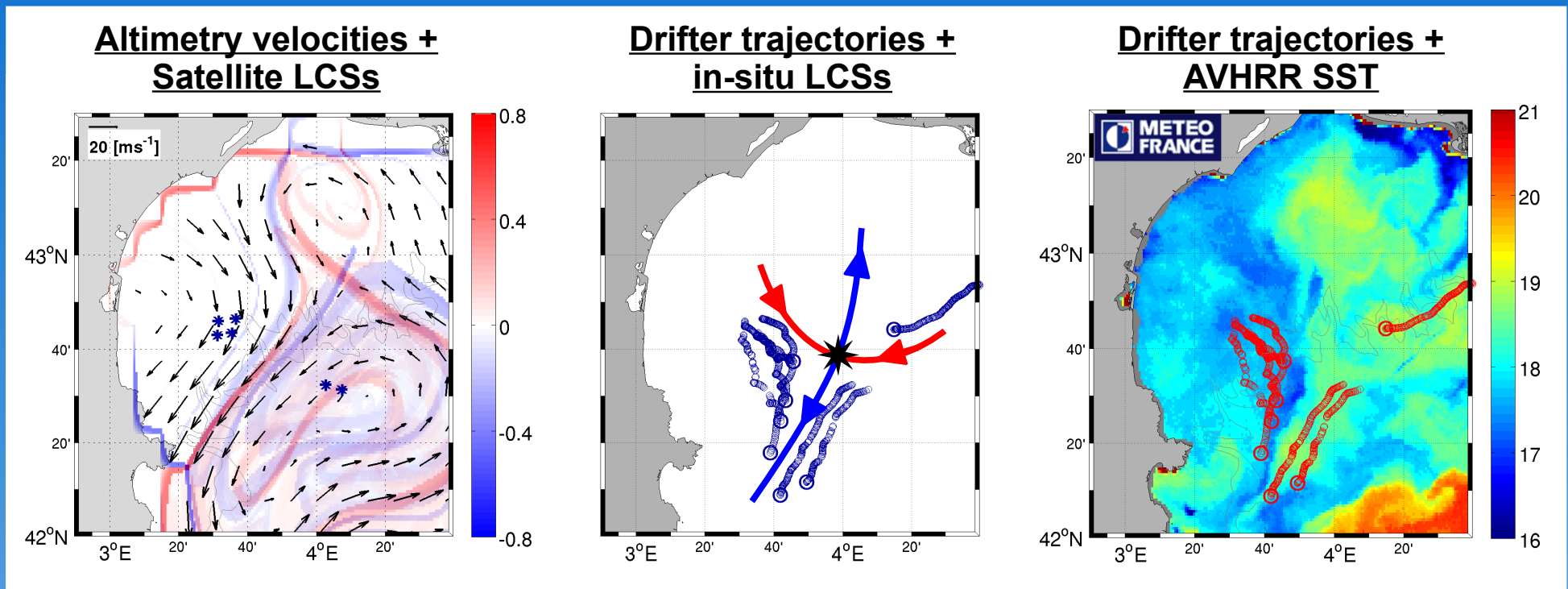➔ **Latex10 campaign** *(September 1-24, 2010)*
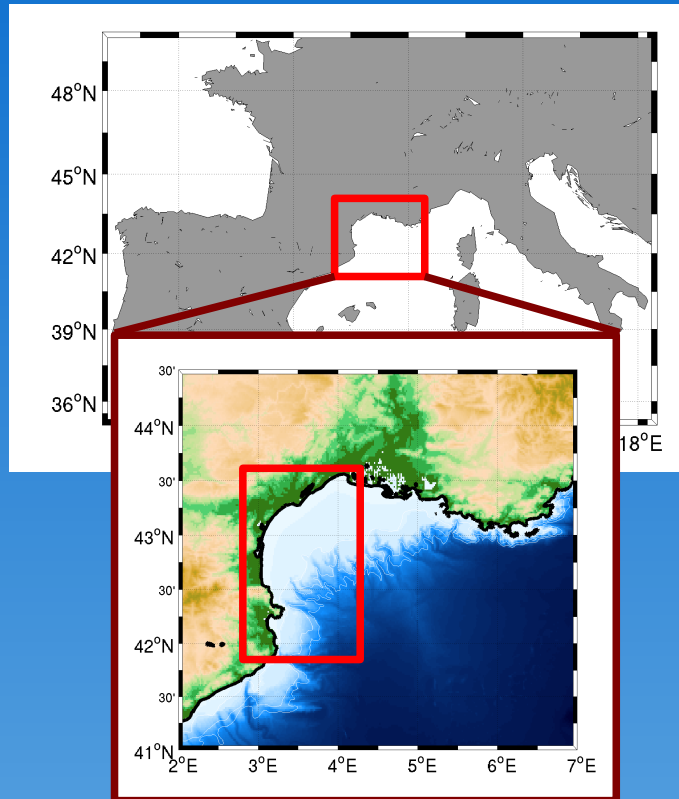
- **Western part of Gulf of Lion**
  (NW Mediterranean)

- **<u>Main goals</u> : (Sub)mesoscale dynamics**
  Cross-shelf exchanges

- **Adaptive sampling strategy based on :**
  - **Satellite data**
  - **Ship-based current measurements**
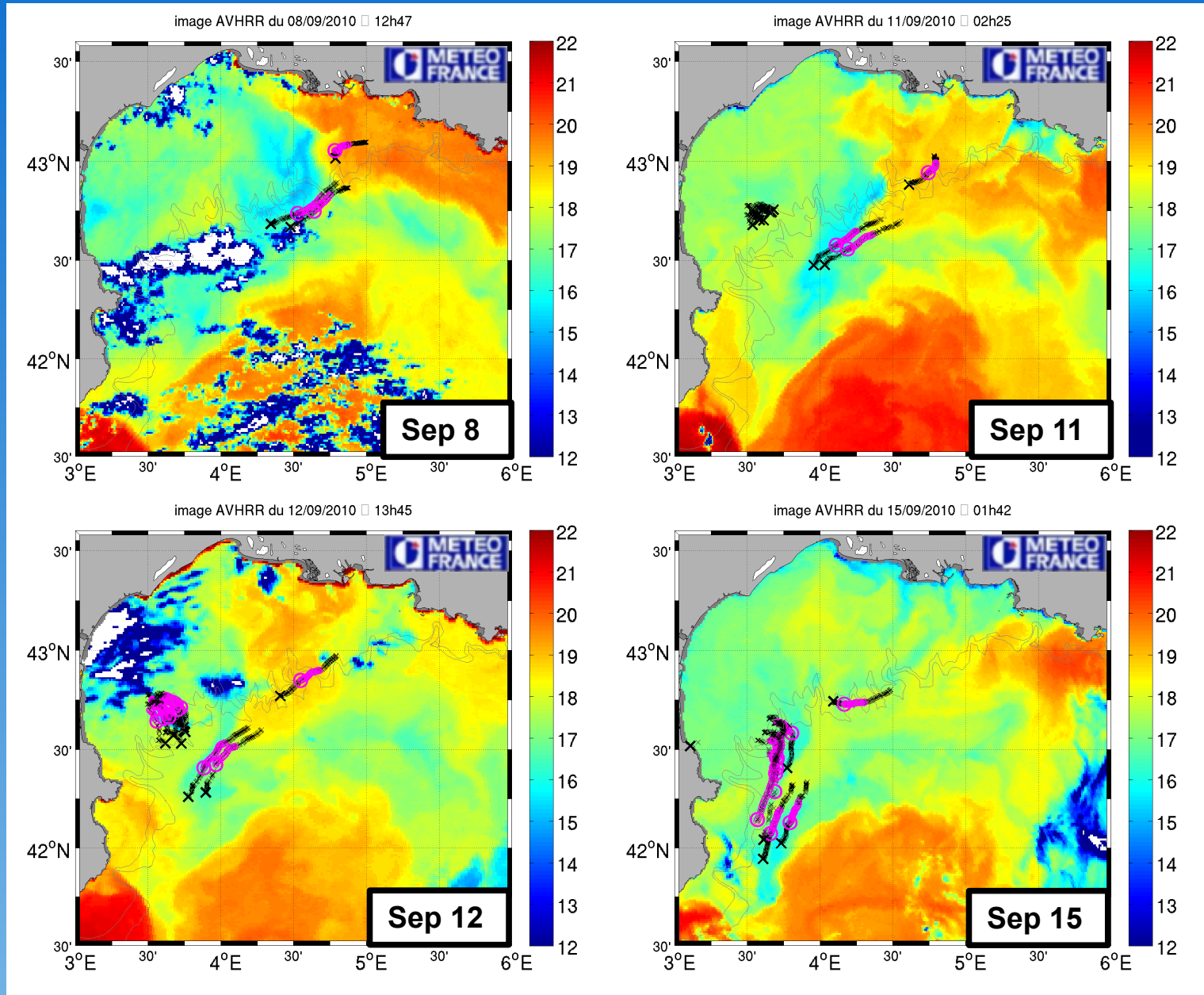 - **Iterative Lagrangian drifter releases**

Example:

➔ **Drifter array deployment "Lyap01"** *(Sep. 15, 2010)*

**Altimetry velocities +
Satellite LCSs**

**Drifter trajectories +
in-situ LCSs**

**Drifter trajectories +
AVHRR SST**

(from Nencioli et al., GRL, 2011)

- **Identified in-situ Lagrangian Coherent Structures (LCSs)**

- **Evidenced inaccuracy of altimetry in coastal region**

- **LCS associated with a frontal structure**

Front development

➔ **AVHRR SST + 3-day drifter trajectories** *(Sep. 8 to 15, 2010)*

image AVHRR du 08/09/2010 12h47 — Sep 8

image AVHRR du 11/09/2010 02h25 — Sep 11

image AVHRR du 12/09/2010 13h45 — Sep 12

image AVHRR du 15/09/2010 01h42 — Sep 15

Front development

➔ **AVHRR SST + 3-day drifter trajectories** *(Sep. 8 to 15, 2010)*

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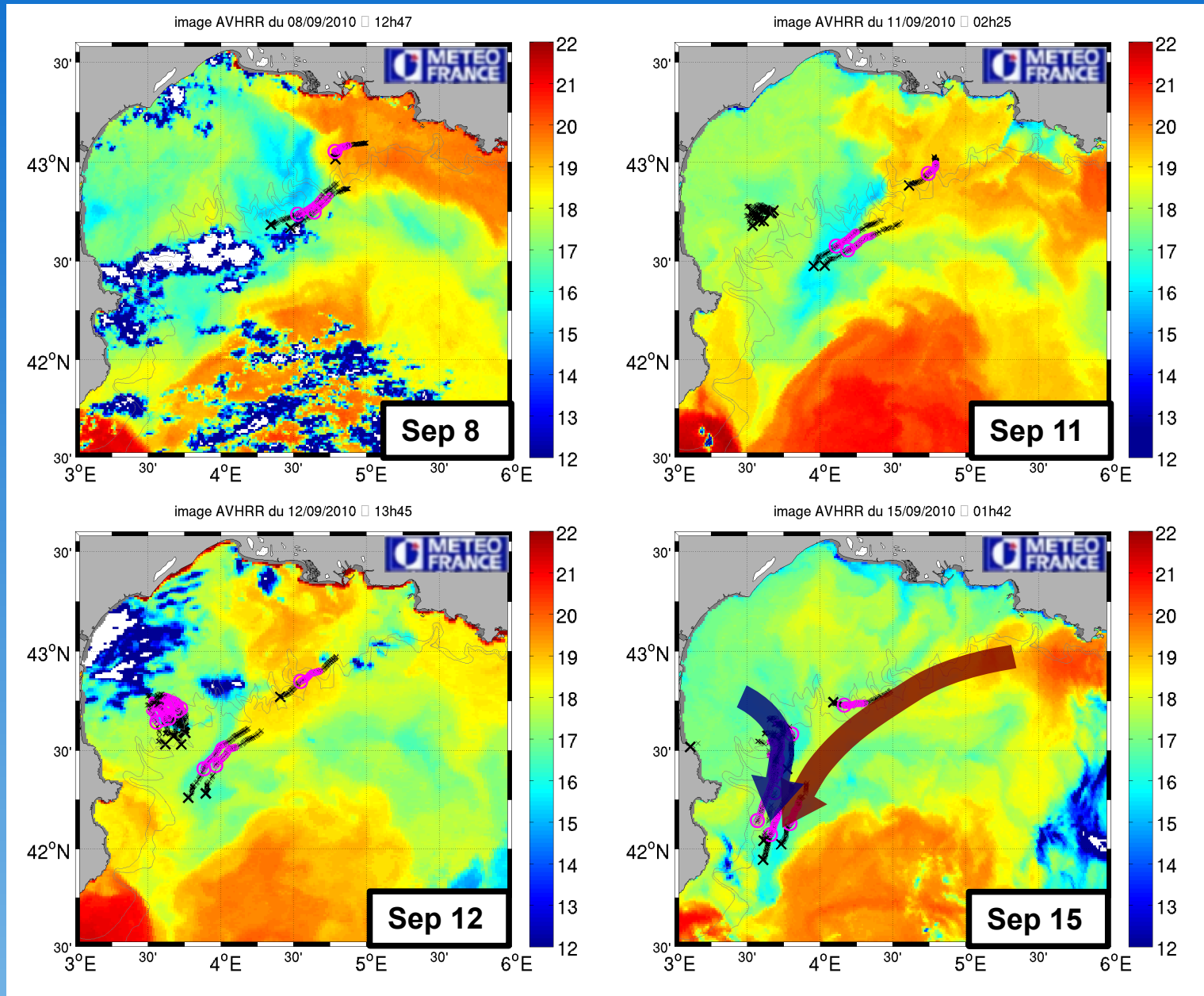
Convergence of warmer (eastern outer shelf) and colder (western inner shelf) water masses
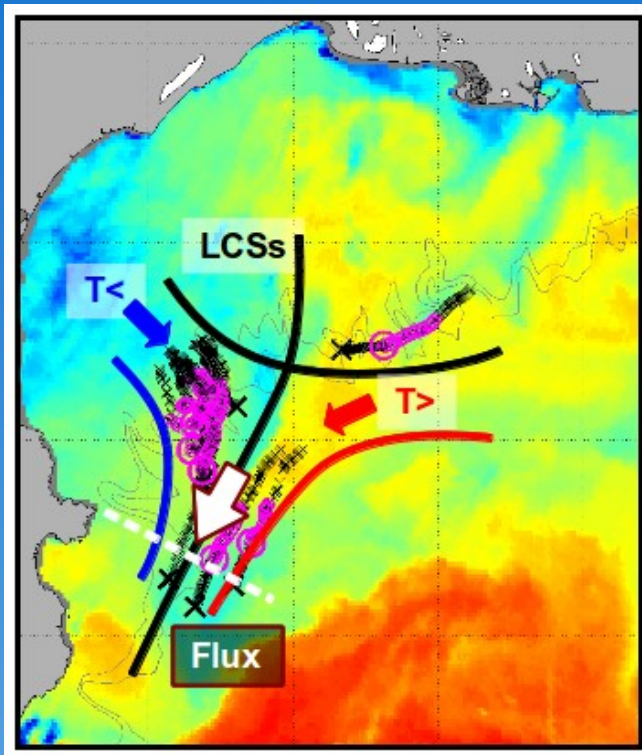
In-situ approach

➔ **Colder and warmer water masses converging along attractive LCS**

In-situ approach

➔ **Colder and warmer water masses converging along attractive LCS**

• **Shape of T and S profile across the front results from balance between convergence and horizontal mixing**

In-situ approach
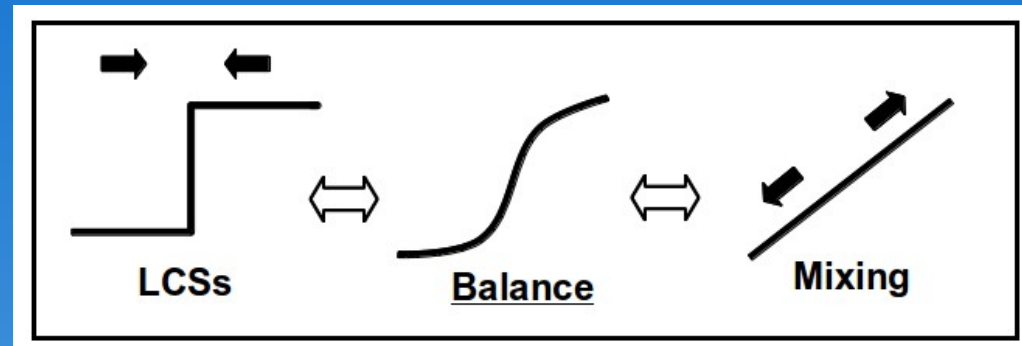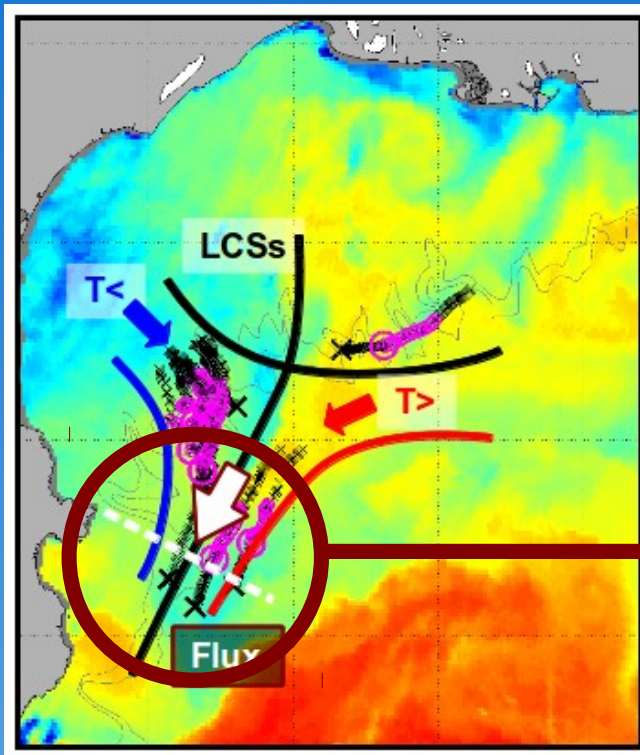
➔ **Colder and warmer water masses converging along attractive LCS**
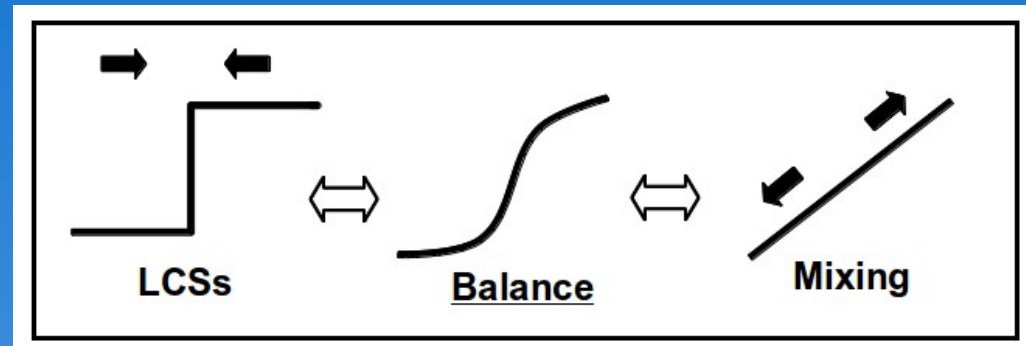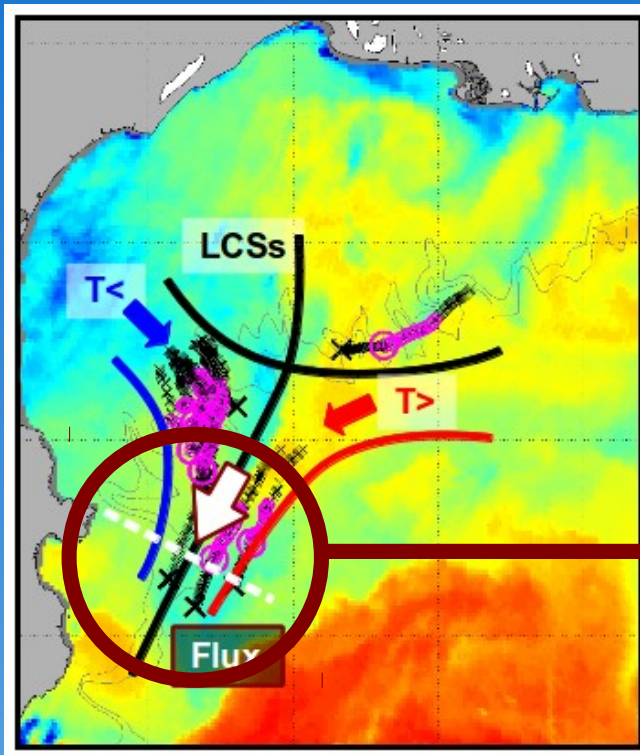
- **Shape of T and S profile across the front results from balance between convergence and horizontal mixing**

$$\frac{\partial T}{\partial t} + u(x)\frac{\partial T}{\partial x} = K_H \frac{\partial^2 T}{\partial x^2}$$

- **Analytical solution to 1D advection diffusion equation for a tracer *T***

➔ **Analogous to Flament et al. 1985, Ledwell et al. 1998 (from satellite)**

Advection-diffusion equation

1D equation for a tracer *T*

$$\frac{\partial T}{\partial t} + u(x)\frac{\partial T}{\partial x} = K_H \frac{\partial^2 T}{\partial x^2}$$

Advection-diffusion equation

1D equation for a tracer *T*
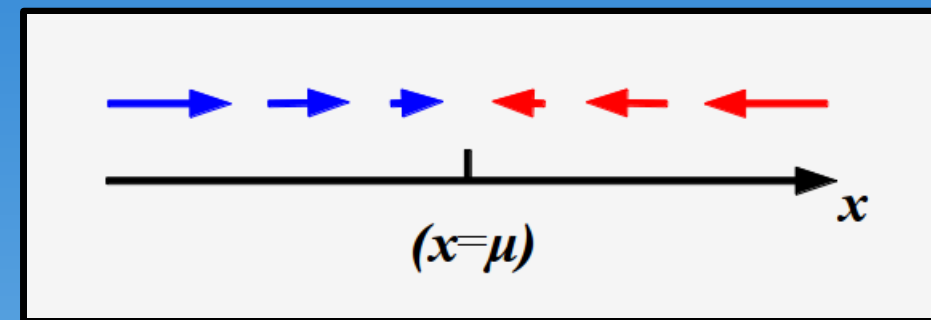
$$\frac{\partial T}{\partial t} + u(x)\frac{\partial T}{\partial x} = K_H \frac{\partial^2 T}{\partial x^2}$$

$$-\gamma(x-\mu)\frac{dT}{dx} = K_H \frac{d^2 T}{dx^2}$$

Assumptions:

- **Front is at equilibrium (steady state)**

- **x is the across LCS direction**

Advection-diffusion equation

1D equation for a tracer *T*

$$\cancel{\frac{\partial T}{\partial t}} + u(x)\frac{\partial T}{\partial x} = K_H \frac{\partial^2 T}{\partial x^2}$$

$$-\gamma(x-\mu)\frac{dT}{dx} = K_H \frac{d^2 T}{dx^2}$$

with

Assumptions:

• **Front is at equilibrium (steady state)**

• **x is the across LCS direction**

γ : **Strain rate (Lyapunov exponent)**

μ : **Position of LCS axis**

$(x=\mu)$

x

Advection-diffusion equation

1D equation for a tracer *T*

$$\frac{\partial T}{\partial t} + u(x)\frac{\partial T}{\partial x} = K_H \frac{\partial^2 T}{\partial x^2}$$

$$-\gamma(x-\mu)\frac{dT}{dx} = K_H \frac{d^2 T}{dx^2}$$

with

Boundary Conditions

$$T(x = -\infty) = T_1;$$
$$T(x = +\infty) = T_2;$$

Assumptions:

- **Front is at equilibrium (steady state)**

- **x is the across LCS direction**

γ : **Strain rate (Lyapunov exponent)**

μ : **Position of LCS axis**

$(x=\mu)$

$$T(x) = \frac{T_2 + T_1}{2} + \frac{T_2 - T_1}{2} \operatorname{erf}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{\gamma}{K_H}}(x-\mu)\right)$$

$$T(x) = \frac{T_2 + T_1}{2} + \frac{T_2 - T_1}{2} \operatorname{erf}\left(\frac{1}{\sqrt{2}} \sqrt{\frac{\gamma}{K_H}} \, (x - \mu)\right)$$

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \mathrm{d}t$$

Analytical solution

$$T(x) = \frac{T_2 + T_1}{2} + \frac{T_2 - T_1}{2}\,\mathrm{erf}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{\gamma}{K_H}}\,(x - \mu)\right)$$

C1

C1

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\mathrm{d}t$$

$$T(x) = \frac{T_2 + T_1}{2} + \frac{T_2 - T_1}{2} \, \mathrm{erf}\left(\frac{1}{\sqrt{2}} \sqrt{\frac{\gamma}{K_H}} \, (x - \mu)\right)$$
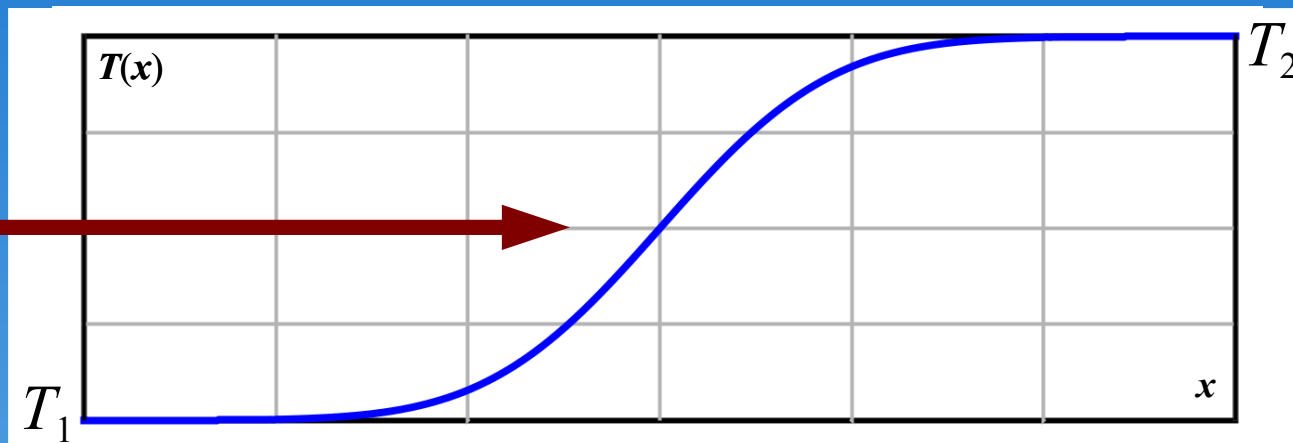
C1   C2

C1

C2

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

$$T(x) = \frac{T_2 + T_1}{2} + \frac{T_2 - T_1}{2} \, \mathrm{erf}\left(\frac{1}{\sqrt{2}} \sqrt{\frac{\gamma}{K_H}} \, (x - \mu)\right)$$

C1   **C2**   **C4**

C1

C2

$T(x)$

T_2

T_1

x

C4

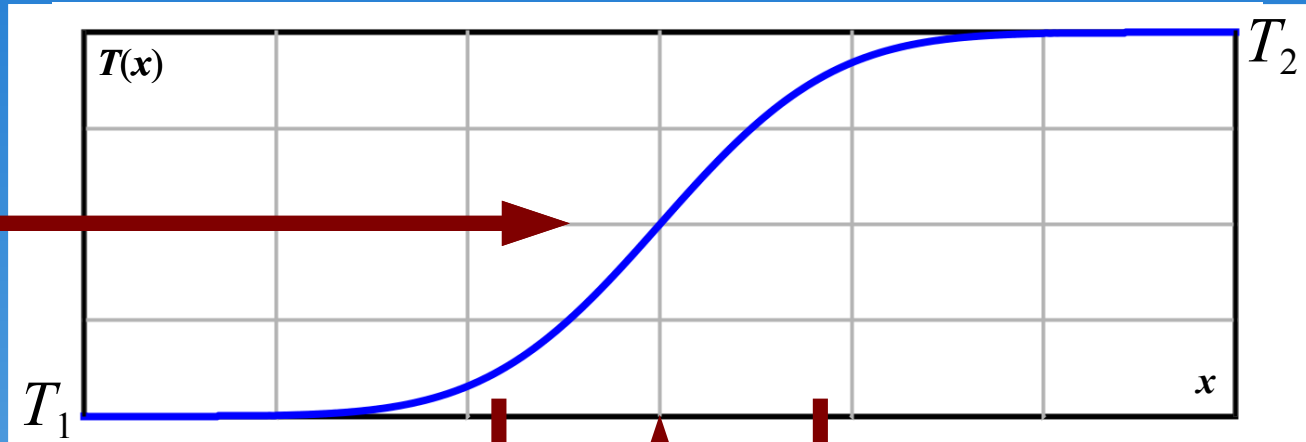$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \mathrm{d}t$$

Analytical solution

$$T(x) = \underbrace{\frac{T_2 + T_1}{2}}_{} + \underbrace{\frac{T_2 - T_1}{2}}_{} \operatorname{erf}\left(\underbrace{\frac{1}{\sqrt{2}}\sqrt{\frac{\gamma}{K_H}}}_{} \underbrace{(x - \mu)}_{}\right)$$
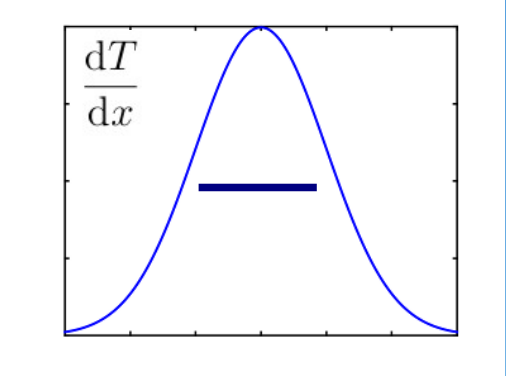
C1 **C2** **C3** **C4**

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,\mathrm{d}t$$

C1 **C2** **C3** **C4**

$$T(x) = \frac{T_2 + T_1}{2} + \frac{T_2 - T_1}{2} \, \text{erf}\left(\frac{1}{\sqrt{2}} \sqrt{\frac{\gamma}{K_H}} \, (x - \mu)\right)$$

C1   **C2**   **C3**   **C4**

C1

C2

C3

C4
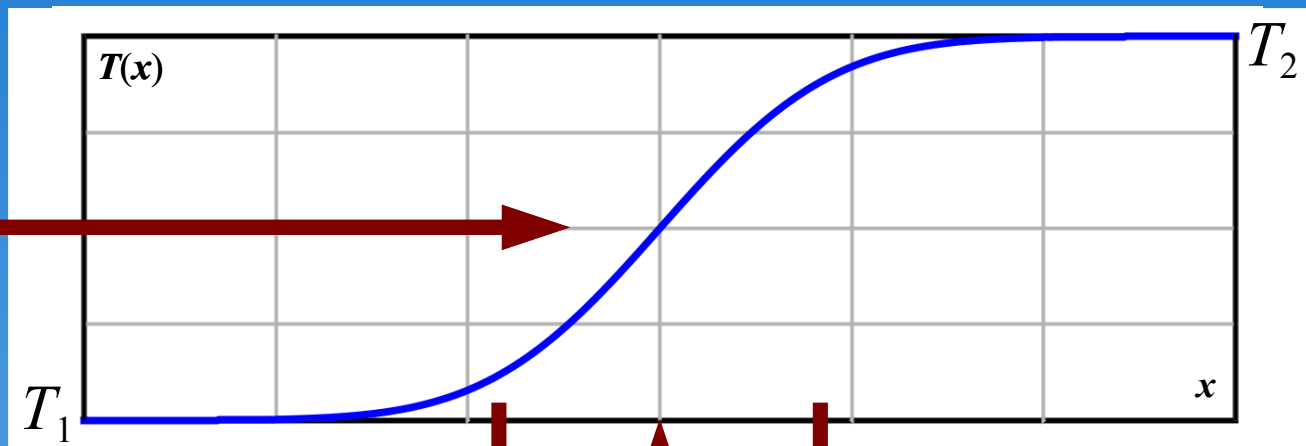
$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \mathrm{d}t$$
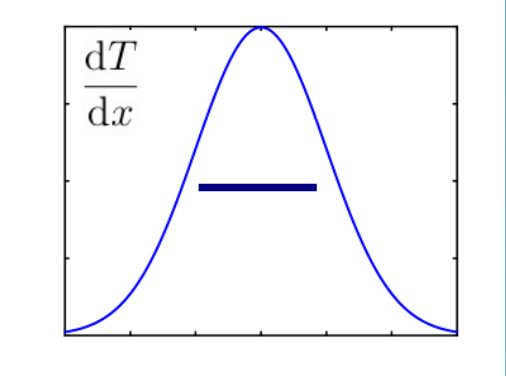
$$W_{front} = 2\sqrt{\frac{K_H}{\gamma}}$$

68% of T variation

$$T(x) = \frac{T_2 + T_1}{2} + \frac{T_2 - T_1}{2}\, \mathrm{erf}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{\gamma}{K_H}}\,(x - \mu)\right)$$

C1   **C2**   **C3**   **C4**

C1

C2
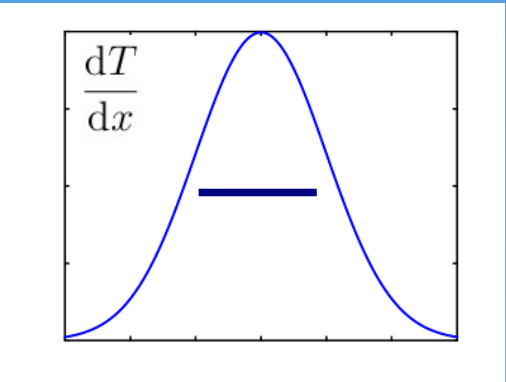
$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,\mathrm{d}t$$

C3   **C4**

$$W_{front} = 2\sqrt{\frac{K_H}{\gamma}}$$

68% of T variation

• **Coefficients computed by best fitting T and S sections**

$$K_H = \frac{\gamma}{\left(2\,\mathrm{C}3^2\right)}$$

$$T(x) = \frac{T_2 + T_1}{2} + \frac{T_2 - T_1}{2} \, \text{erf}\left(\frac{1}{\sqrt{2}} \sqrt{\frac{\gamma}{K_H}} (x - \mu)\right)$$

C1   **C2**   **C3**   **C4**

C1

C2

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \mathrm{d}t$$

$T(x)$ ... T_2 ... T_1 ... x

C3   **C4**

$$\frac{\mathrm{d}T}{\mathrm{d}x}$$

$$W_{front} = 2\sqrt{\frac{K_H}{\gamma}}$$

68% of T variation

- **Coefficients computed by best fitting T and S sections**

$$K_H = \frac{\gamma}{\left(2 \, C3^2\right)}$$

from drifter dispersion!!

Cross-front transects

September 15, 2013   - SST -   - SSS -

SST and SSS from ship thermosalinograph

Cross-front transects
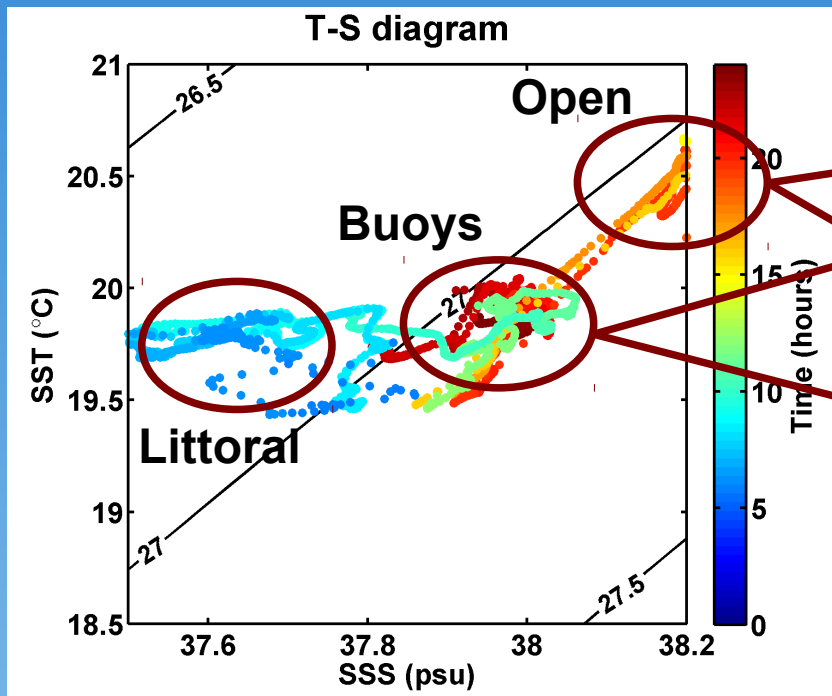
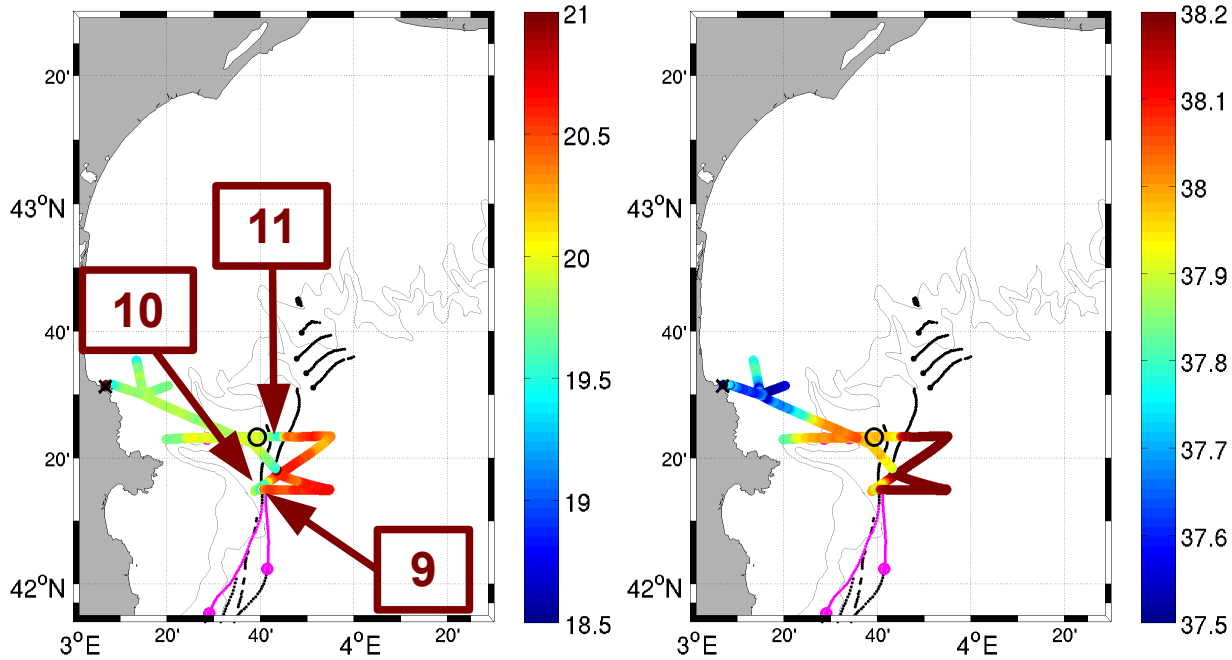September 15, 2013 - SST - - SSS -

SST and SSS from ship thermosalinograph

T-S diagram

September 15, 2013   - SST -

- SSS -

SST and SSS from ship thermosalinograph

T-S diagram

September 15, 2013   - SST -   - SSS -

SST and SSS from ship thermosalinograph

T-S diagram

Open

Buoys

Littoral

SST t-series

SSS t-series

September 15, 2013
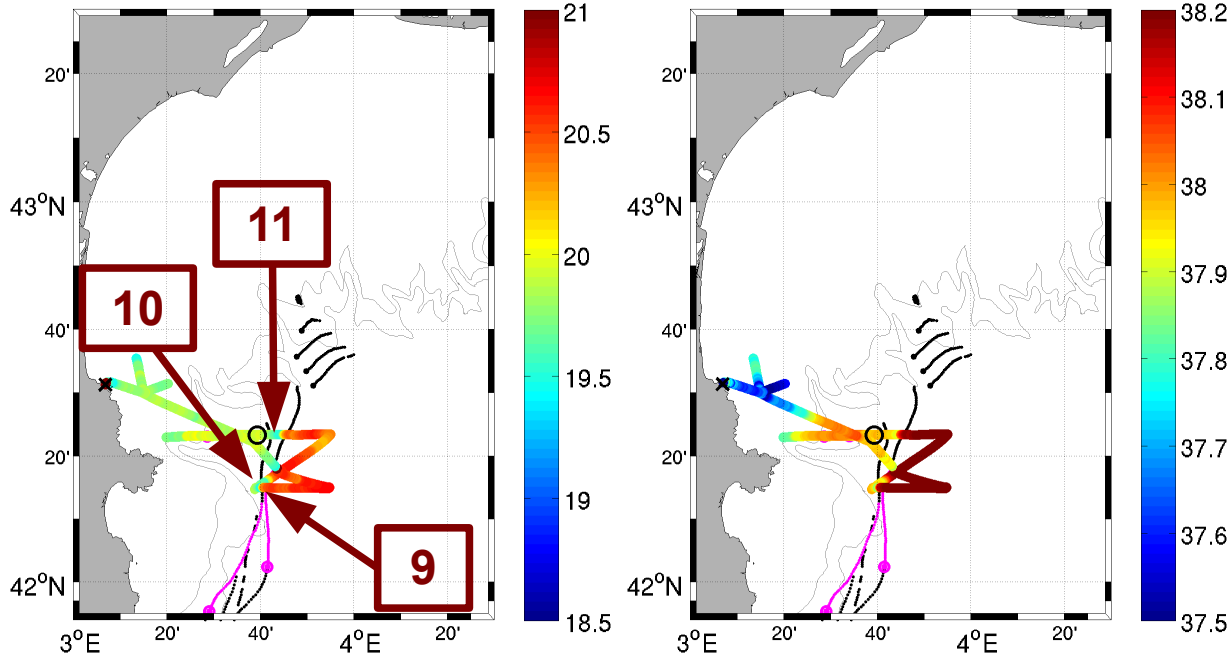
SST and SSS from ship thermosalinograph
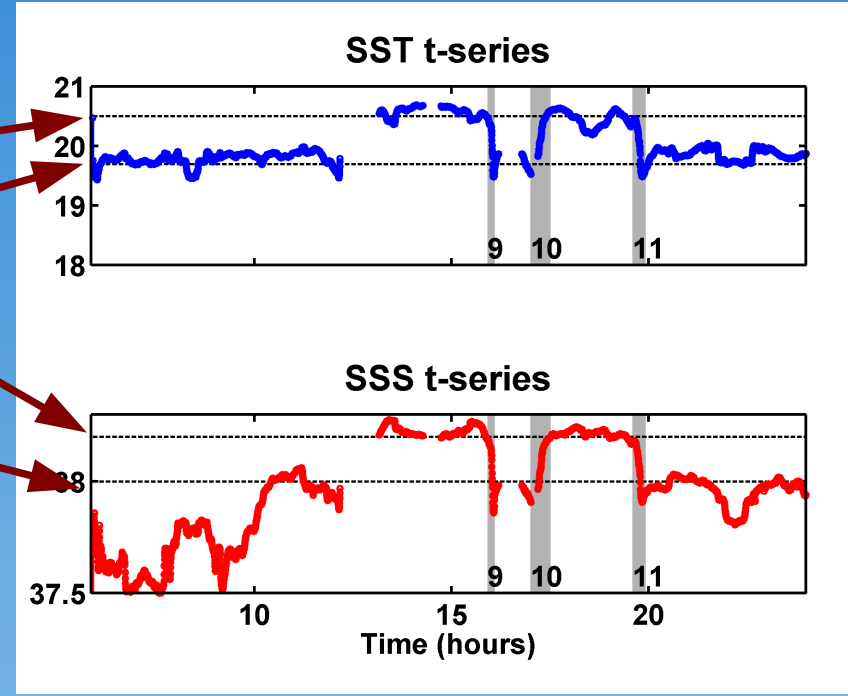
September 15, 2013 — - SST - / - SSS -

SST and SSS from ship thermosalinograph

- **Identified 30 cross-front transects**
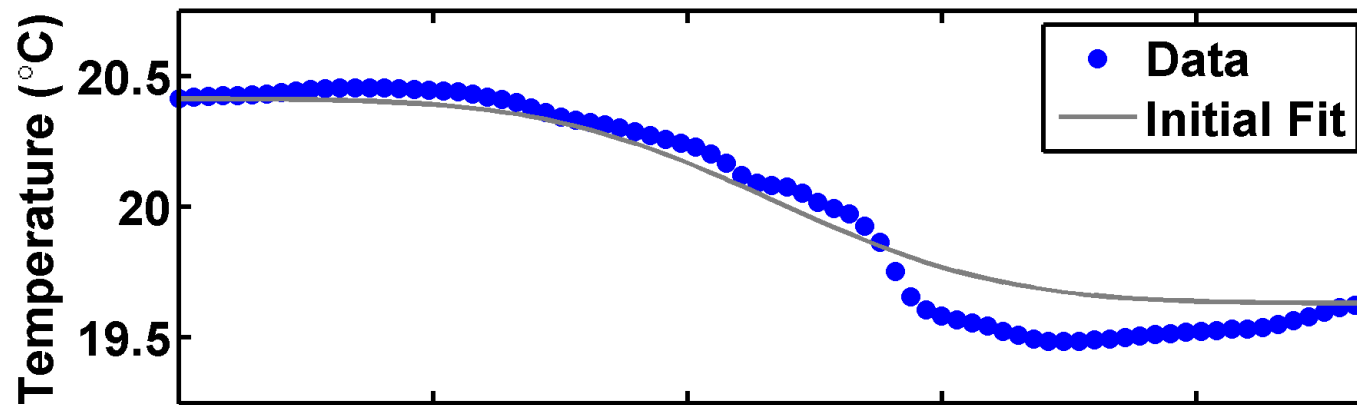- **Transects projected on the direction normal to the LCS axis**

T-S diagram

Open

Buoys

Littoral

SST t-series

SSS t-series

Example: Transect 11

Curve fitting
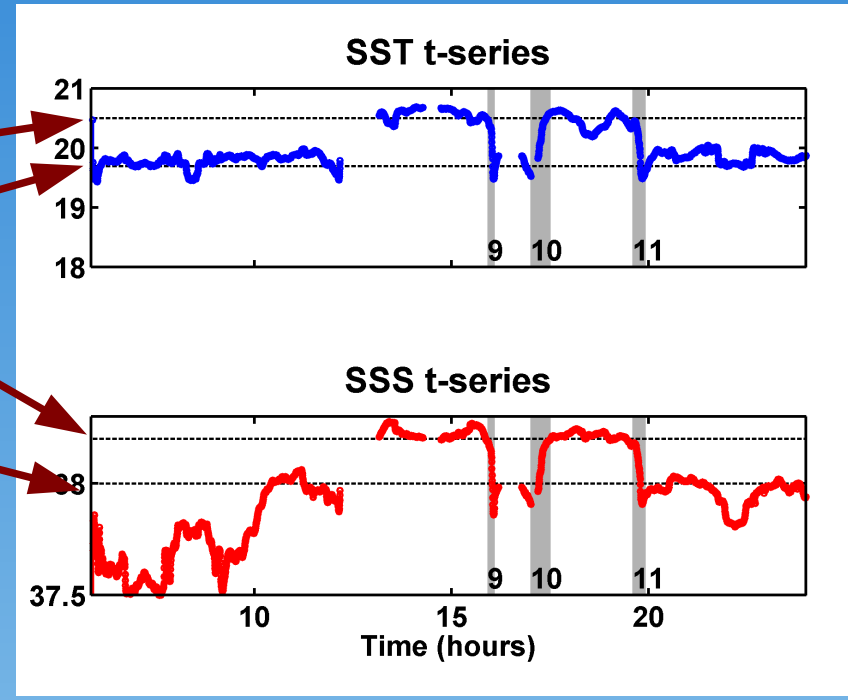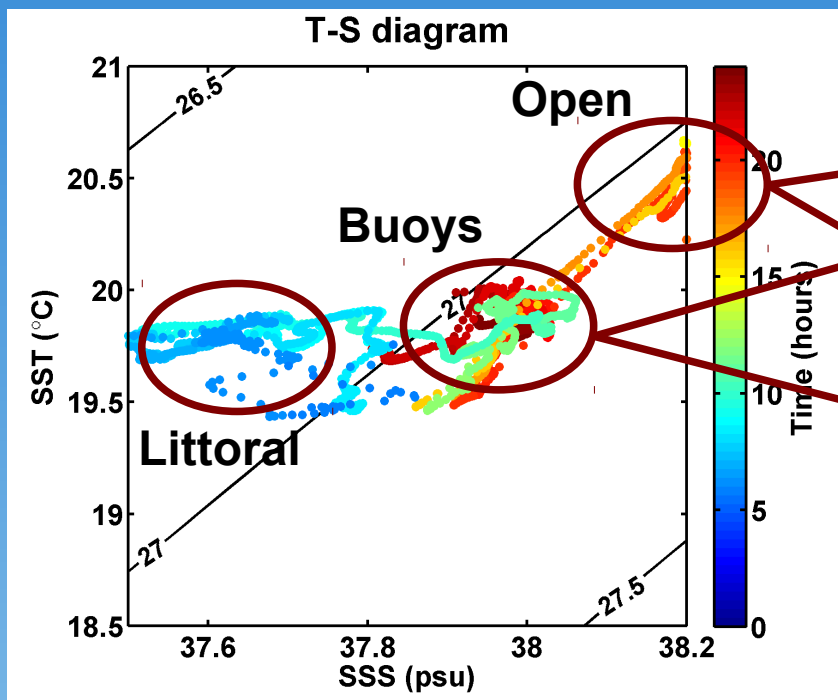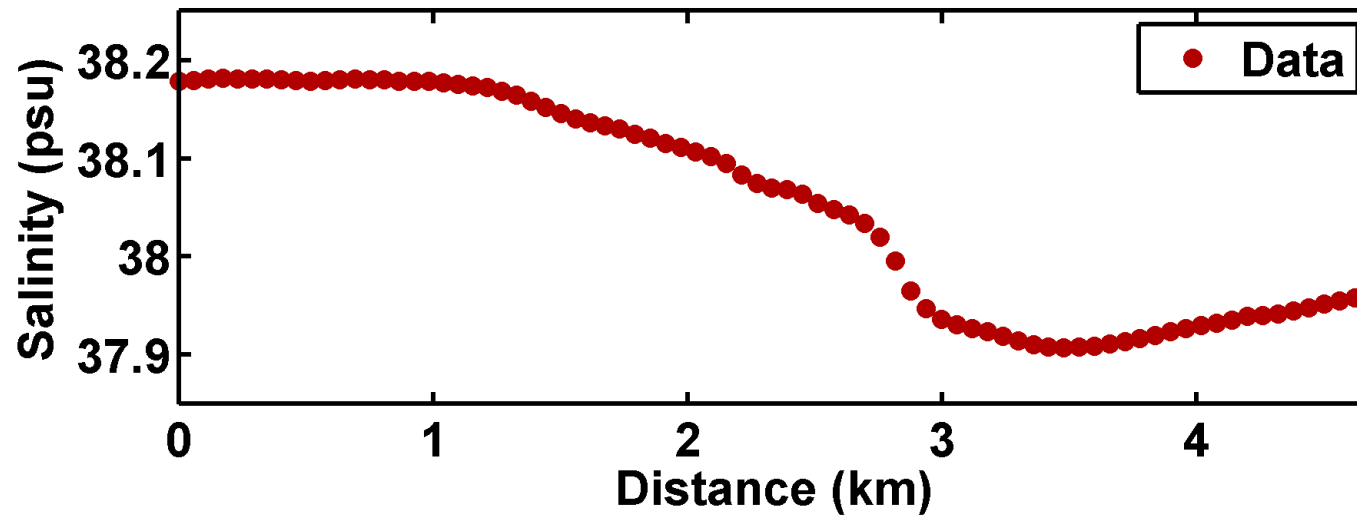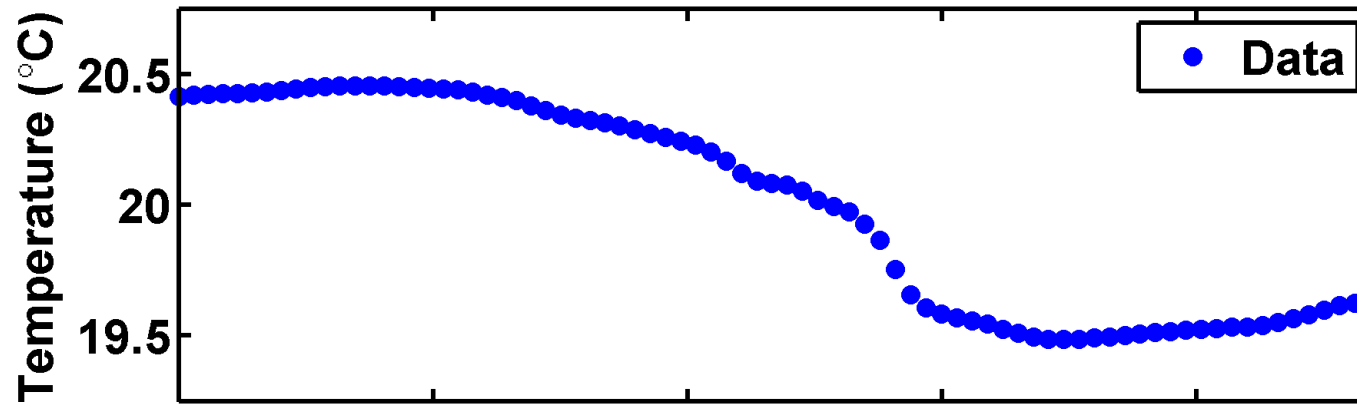
Example: Transect 11

Front equation

$$T(x) = C_1 + C_2 \operatorname{erf}(C_3 \ (x - C_4))$$
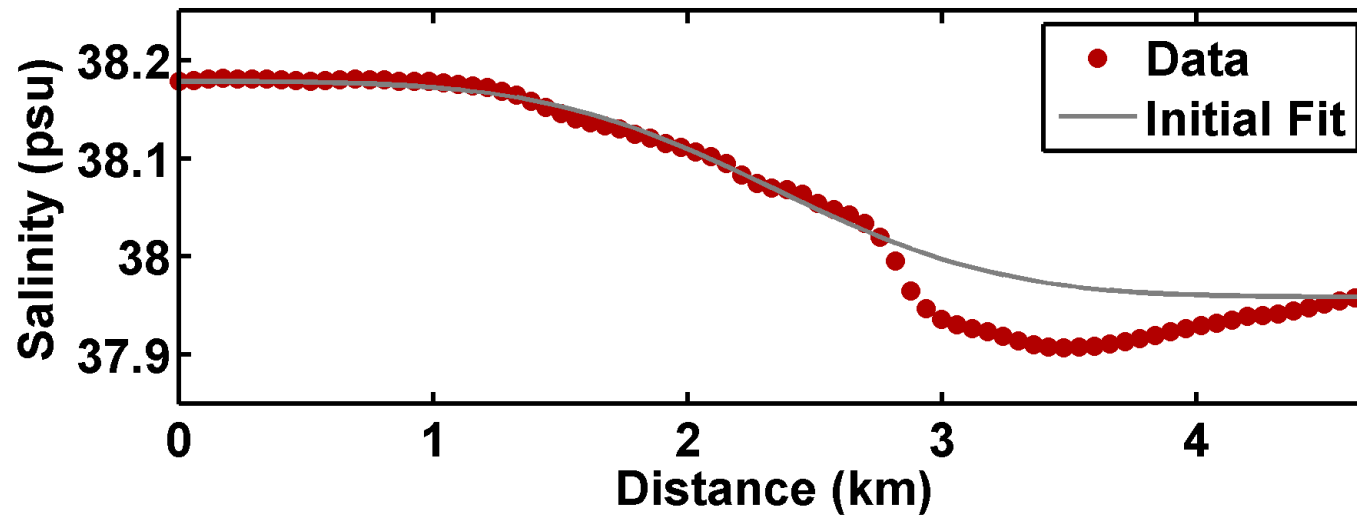
Initial fit

$$C_1 = \frac{T_2 + T_1}{2}$$

$$C_2 = \frac{T_2 - T_1}{2}$$
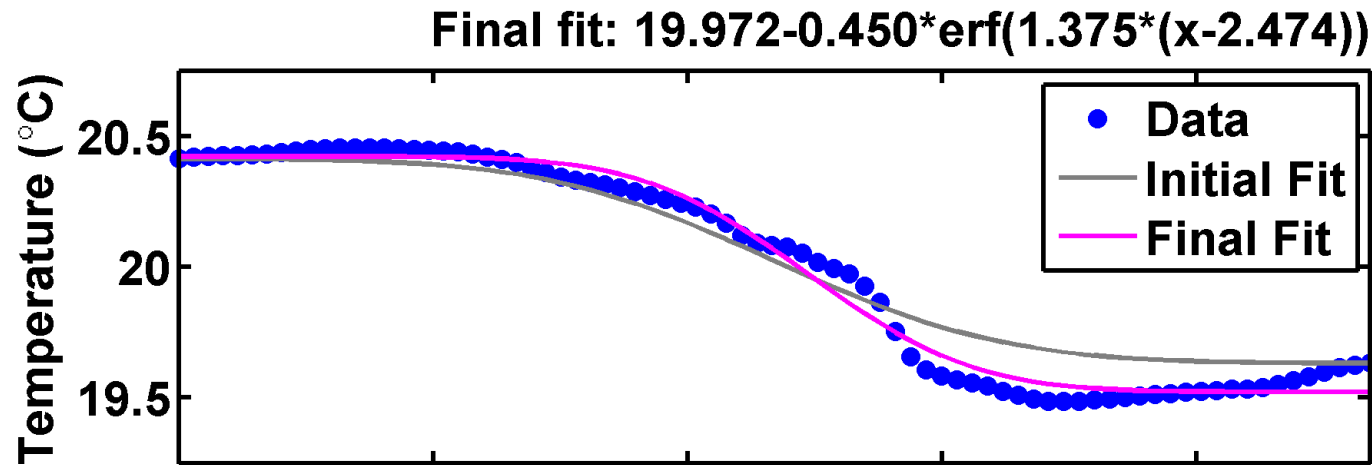
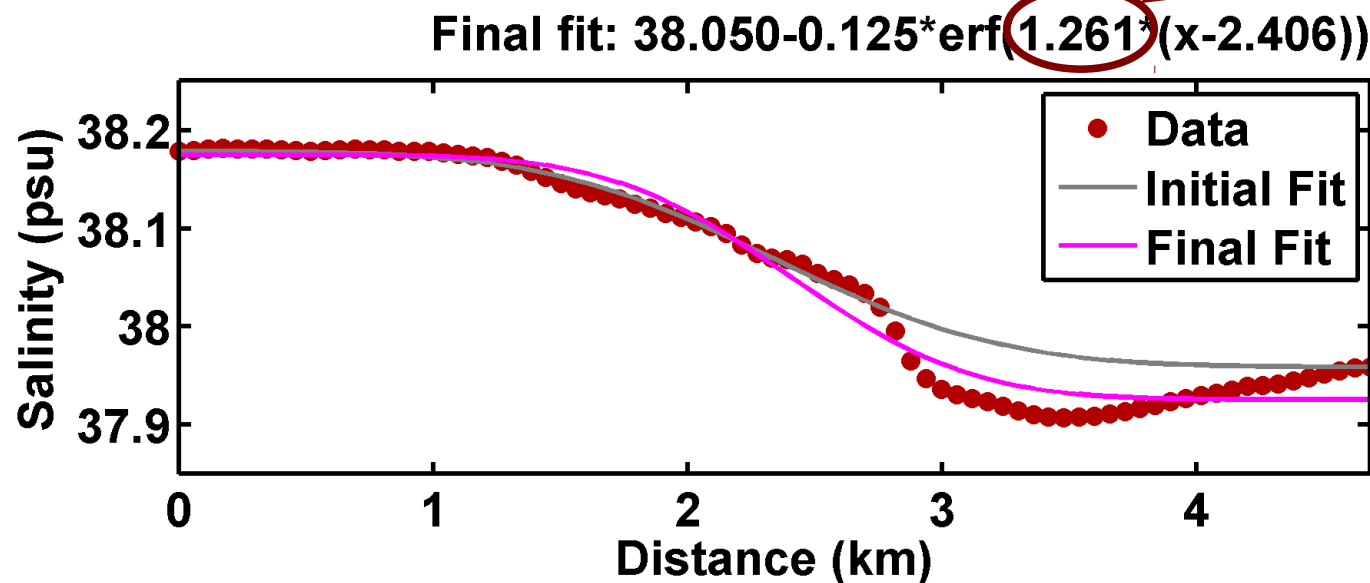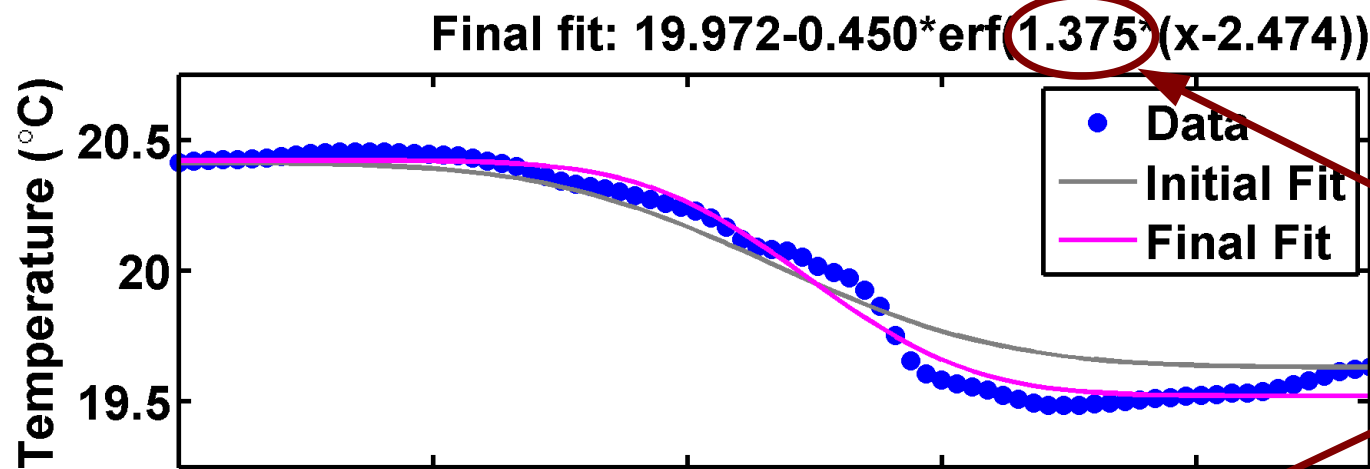$$C_3 = 1$$

$$C_4 = \frac{Dist}{2}$$

Example: Transect 11

➔ Parameters from least square estimation
   using Nelder-Mead simplex direct search

Front equation

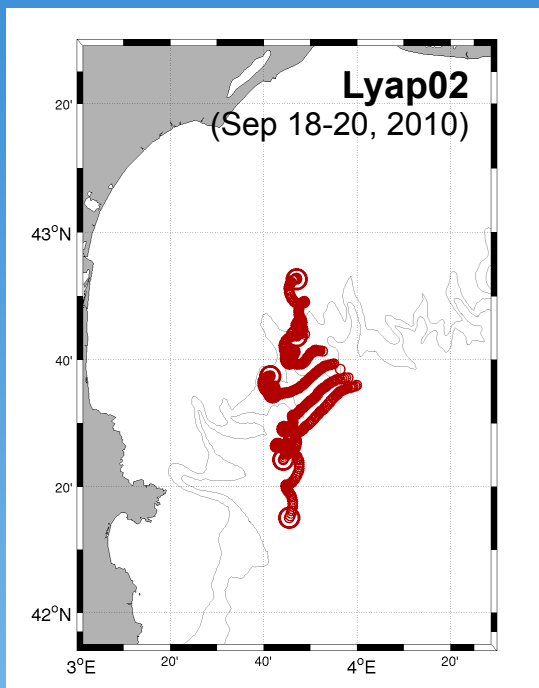$$T(x) = C_1 + C_2 \operatorname{erf}(C_3 \ (x - C_4))$$

Final fit: 19.972-0.450*erf(1.375*(x-2.474))

Final fit: 38.050-0.125*erf(1.261*(x-2.406))

Example: Transect 11

➔ Parameters from least square estimation using Nelder-Mead simplex direct search

Front equation

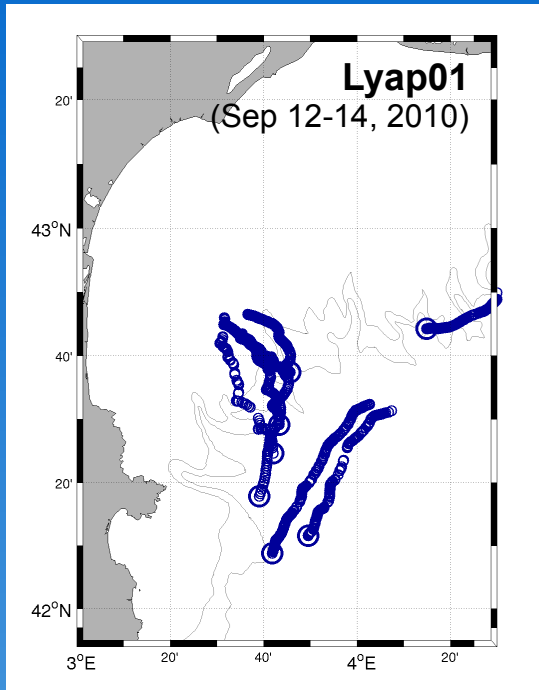$$T(x) = C_1 + C_2 \operatorname{erf}(C_3 \ (x - C_4))$$

Final fit: 19.972-0.450*erf(1.375*(x-2.474))

- Data
- Initial Fit
- Final Fit

Temperature (°C)

Final fit: 38.050-0.125*erf(1.261*(x-2.406))

- Data
- Initial Fit
- Final Fit

Salinity (psu)

Distance (km)
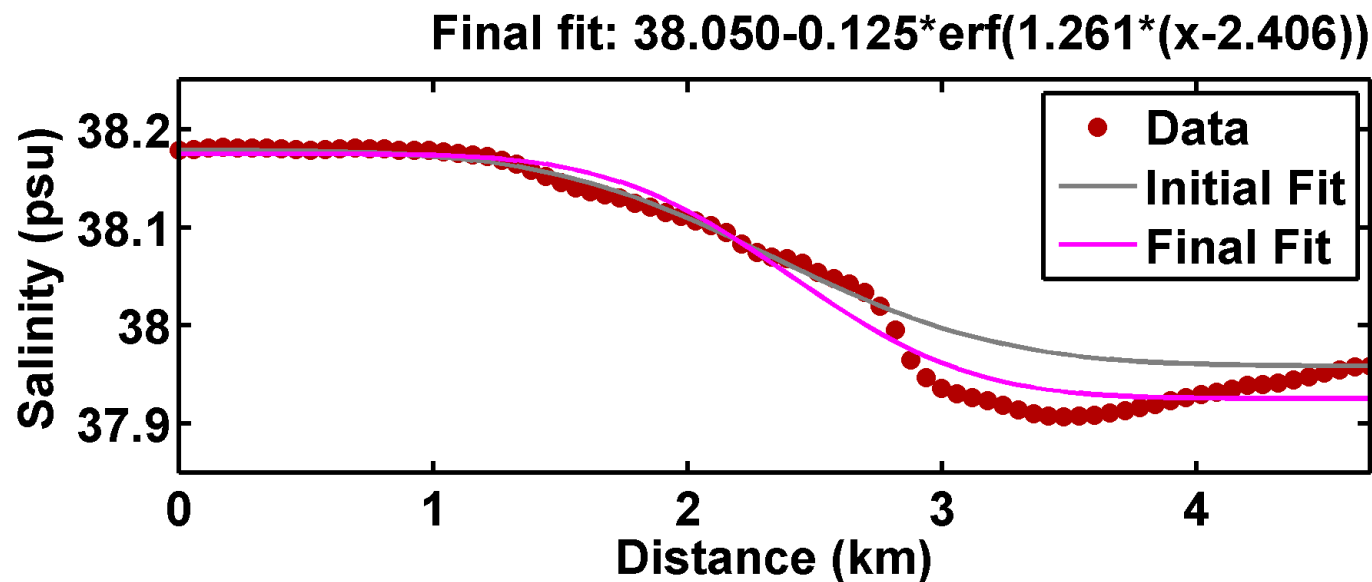
$$C_3 = \frac{1}{\sqrt{2}} \sqrt{\frac{\gamma}{K_H}}$$

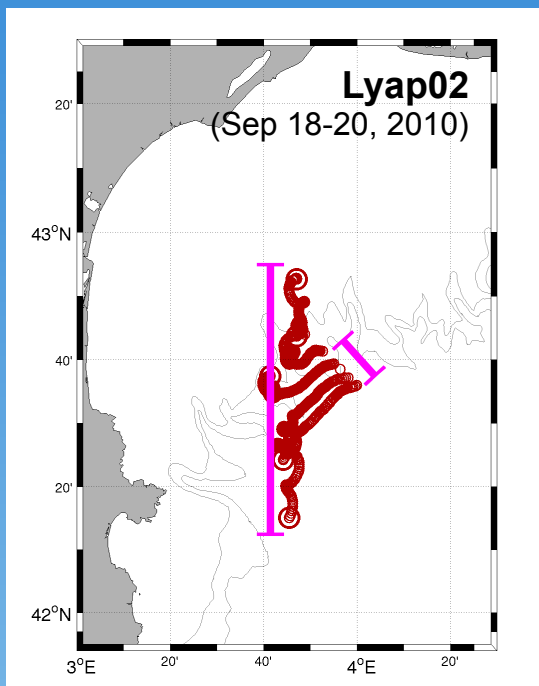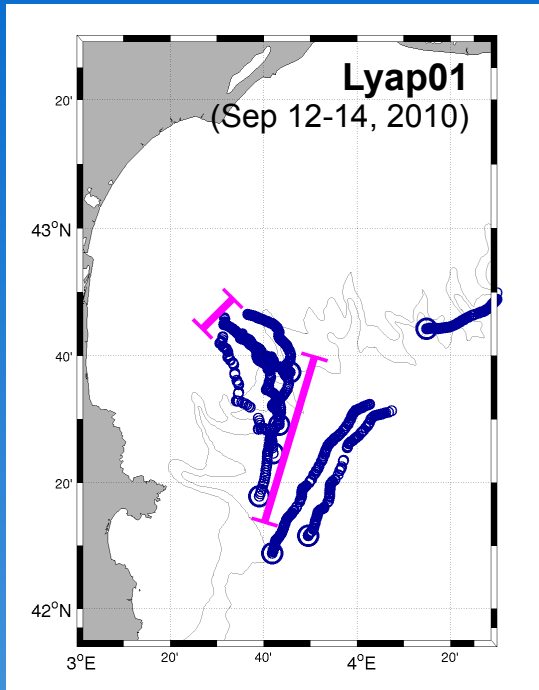- No fit for 11 out of 30 transects: limits of starting assumptions

i.e. impact of vertical dynamics

Dispersion patterns of drifter arrays

Lyap01
(Sep 12-14, 2010)

Lyap02
(Sep 18-20, 2010)

Strain rate

Lyap01
(Sep 12-14, 2010)
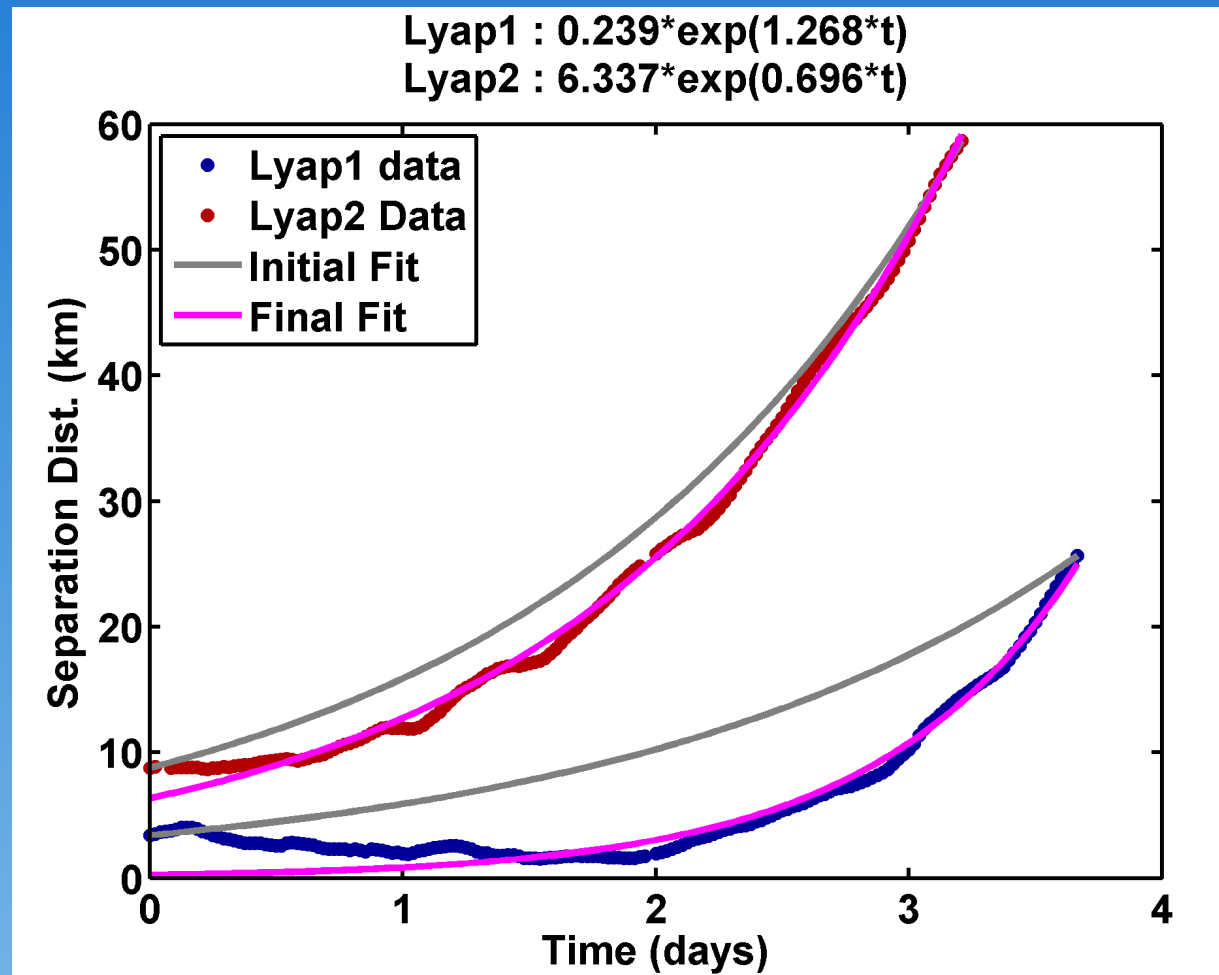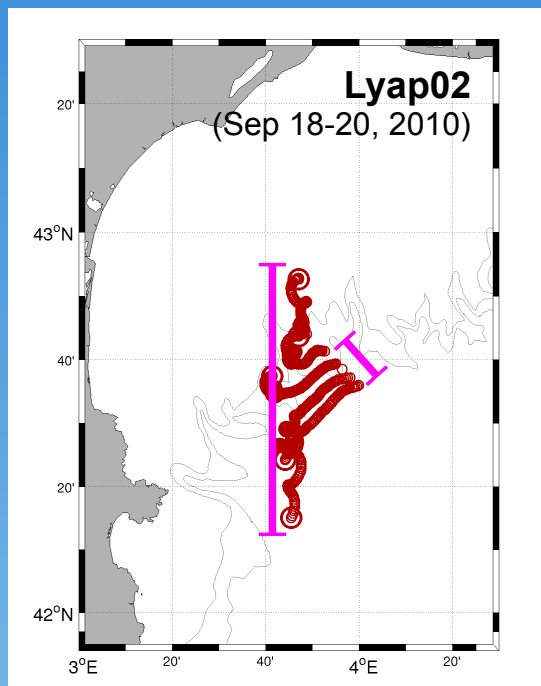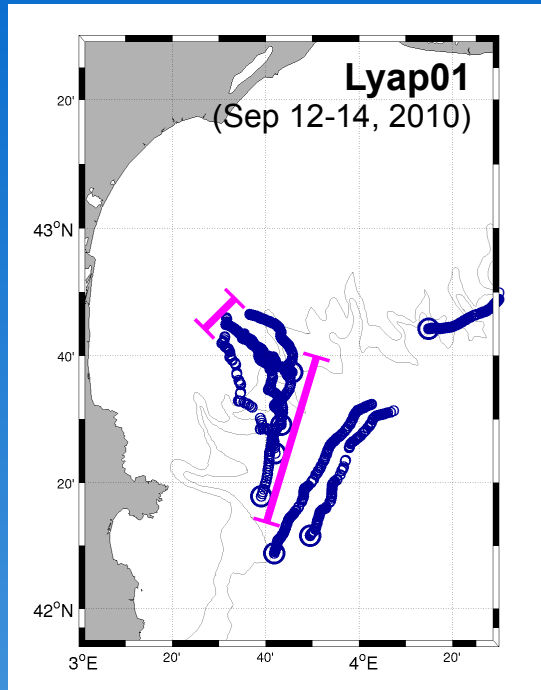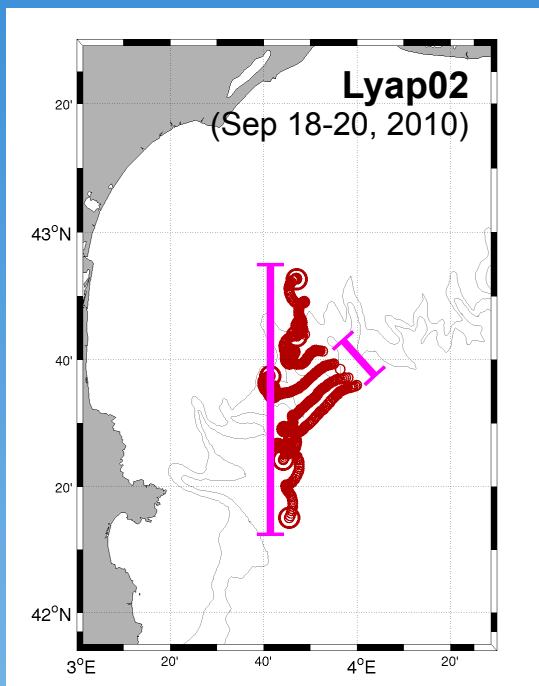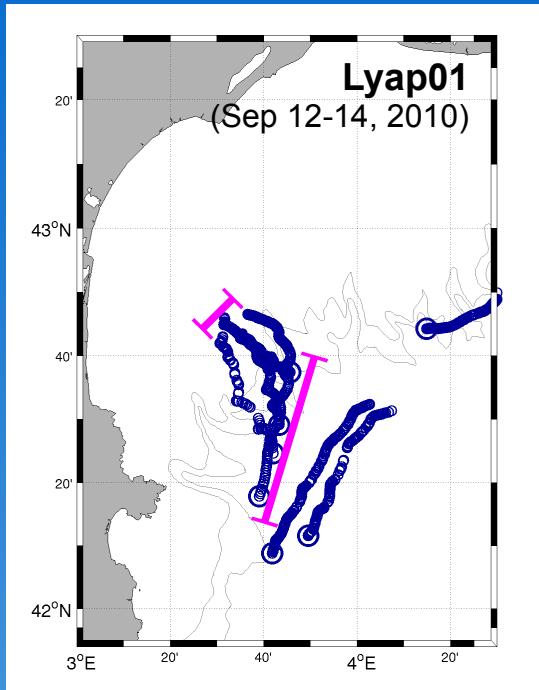

Lyap02
(Sep 18-20, 2010)

Dispersion patterns of drifter arrays

- **For each deployment, computed fastest separation rate between buoy couples (analogous to Lyapunov exponent)**

Strain rate

Lyap01
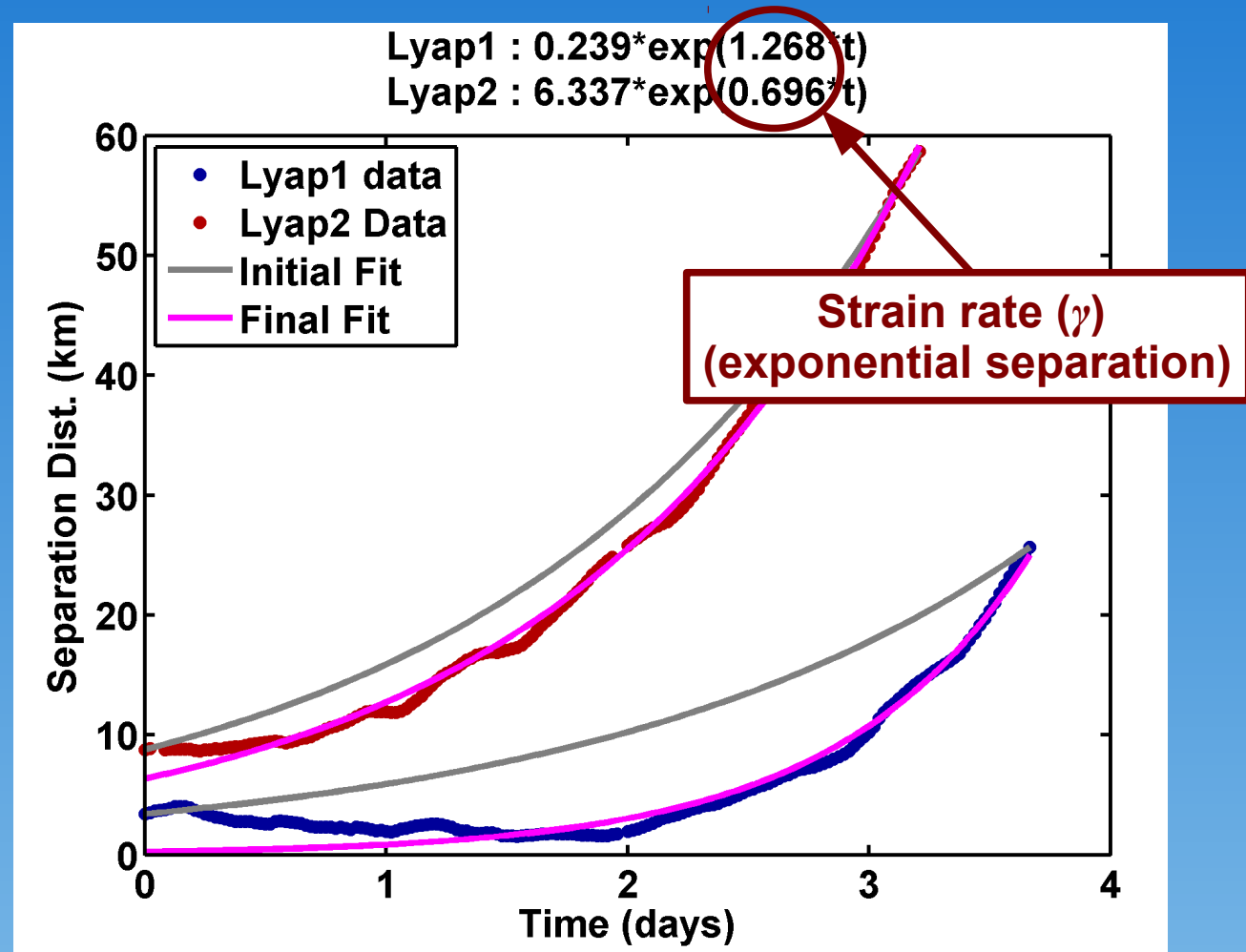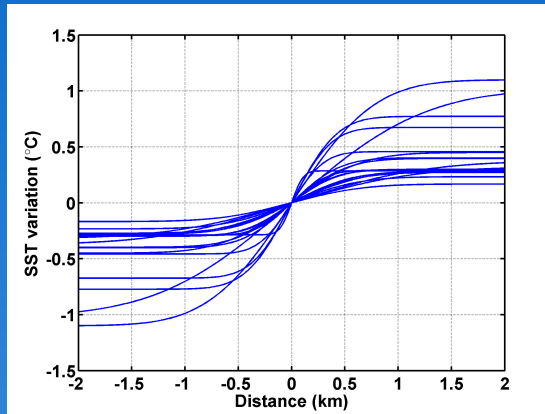(Sep 12-14, 2010)

Lyap02
(Sep 18-20, 2010)

Dispersion patterns of drifter arrays

- **For each deployment, computed fastest separation rate between buoy couples (analogous to Lyapunov exponent)**

Lyap1 : 0.239*exp(1.268*t)
Lyap2 : 6.337*exp(0.696*t)

- Lyap1 data
- Lyap2 Data
- Initial Fit
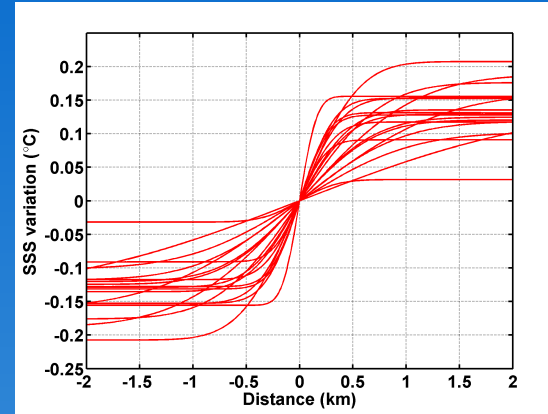- Final Fit

Strain rate

Dispersion patterns of drifter arrays

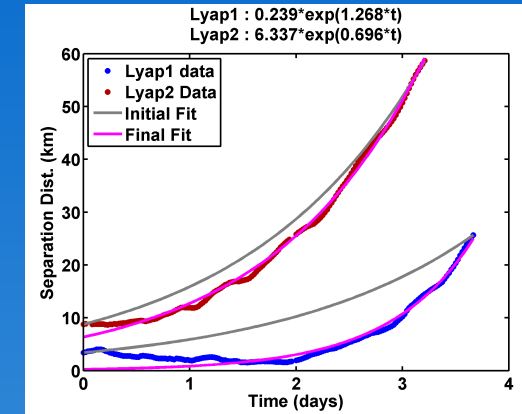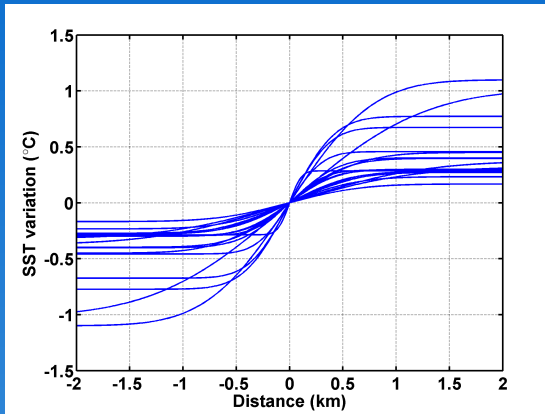- **For each deployment, computed fastest separation rate between buoy couples (analogous to Lyapunov exponent)**

Lyap1 : 0.239*exp(1.268*t)
Lyap2 : 6.337*exp(0.696*t)

Strain rate (γ)
(exponential separation)

Eddy diffusivity coefficients

T Front

S Front

Strain rate

Eddy diffusivity coefficients

T Front

S Front

Strain rate

$$K_H = \frac{\gamma}{\left(2\, C3^2\right)}$$

Eddy diffusivity coefficients

Eddy diffusivity coefficients
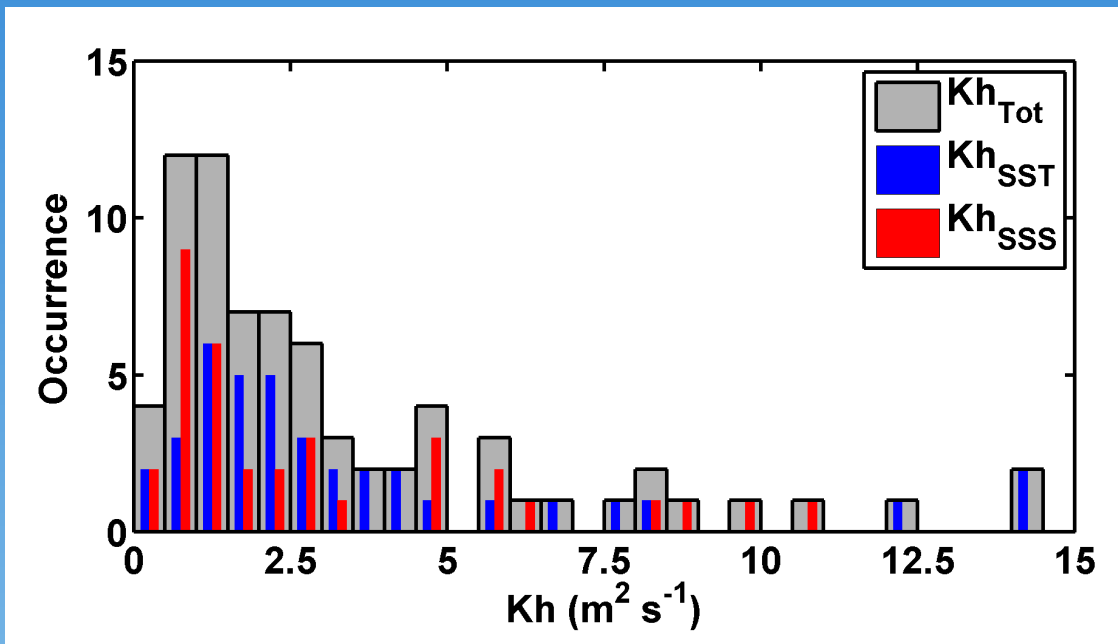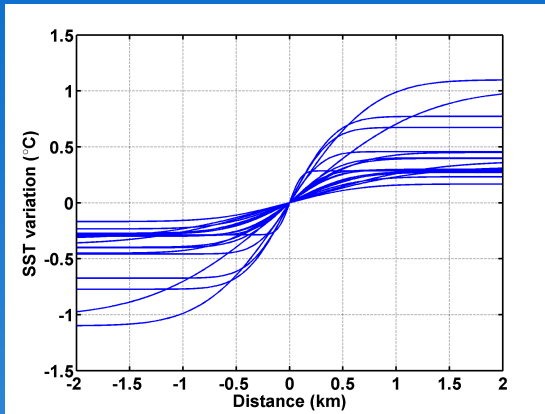
T Front

S Front

Strain rate
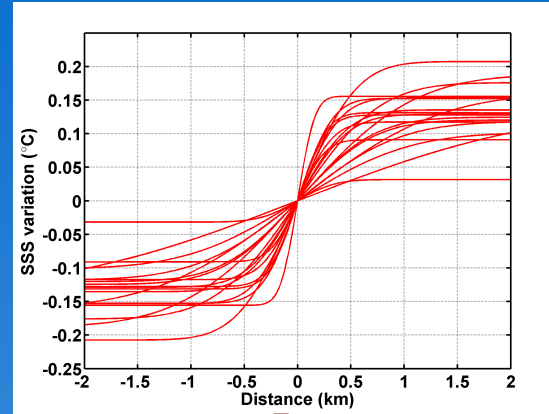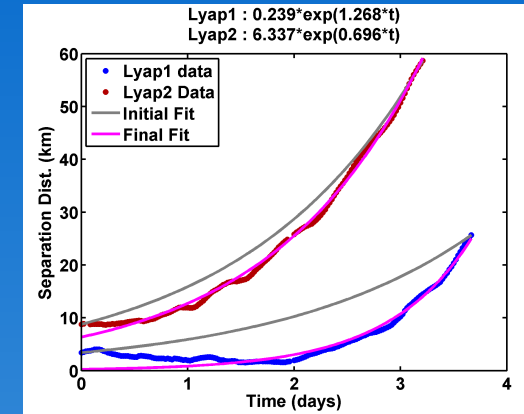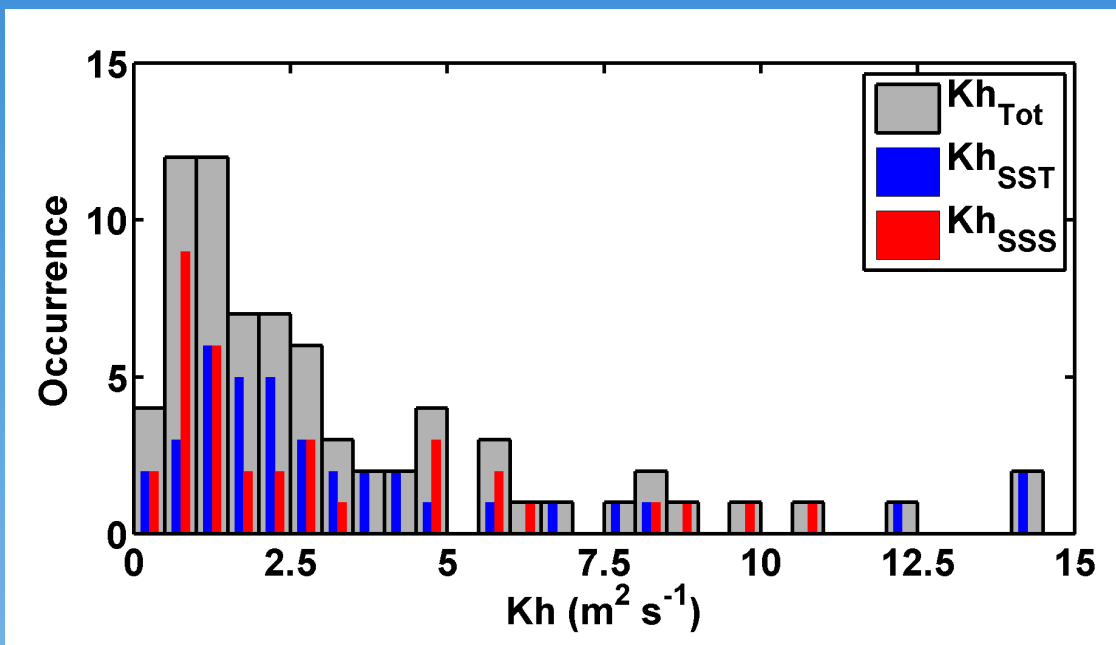

$$K_H = \frac{\gamma}{\left(2\,C3^2\right)}$$

Eddy diffusivity coefficients

- Kh_{SST} similar to Kh_{SSS}
- 75% of estimates between 0.5 - 5 m s^{-2}
- Front widths range from 250 m to 2km
- Long tail

➔ **New approach relatively simple and cheap**
(i.e. compared to passive tracer release experiments)

➔ **In-situ estimates of Kh at the submesoscale in line with values used in high-resolution numerical models**

➔ **Tail of high values affects Kh statistics (mean and standard deviation) => check starting assumptions:**

- **Steady state**
- **Uniform strain rate**
- **Vertical motions**
- **...**

Perspectives

➔ **Further dedicated in-situ experiments**

➔ **Test approach from high-resolution models**

➔ **Extend analysis of Kh over wider regions/the global ocean using remote sensed datasets**

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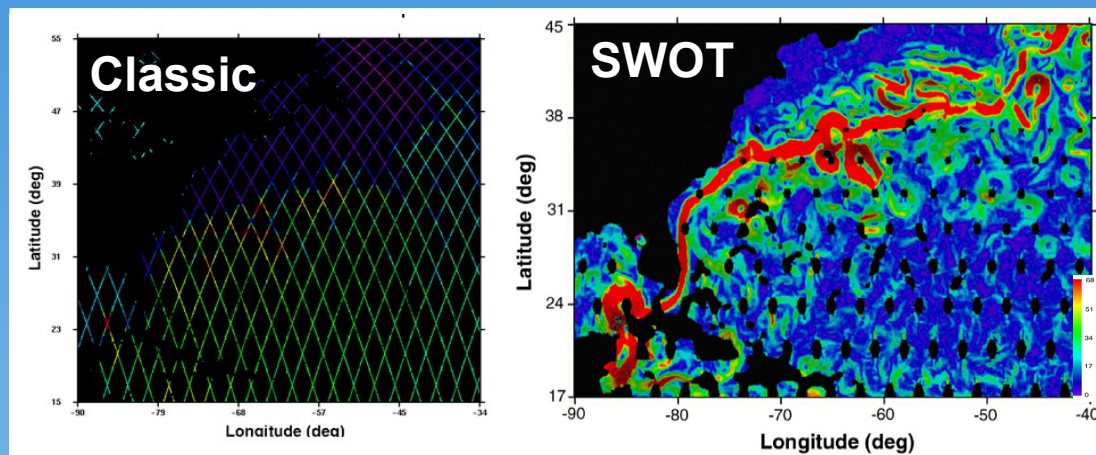
SWOT satellite

AirSWOT

Airborne SWOT sensor tested over key ocean regions (2013-2020)

Surface Water and Ocean Topography

NASA – CNES mission

• **New generation, high-resolution (1Km) altimeter**

• **Launch: Fall 2020**

Classic

SWOT

This work has been developed within the project:

Lyapunov Analysis in the CoaSTal Environment (LACOSTE)

Marie Curie Intra-European Fellowship
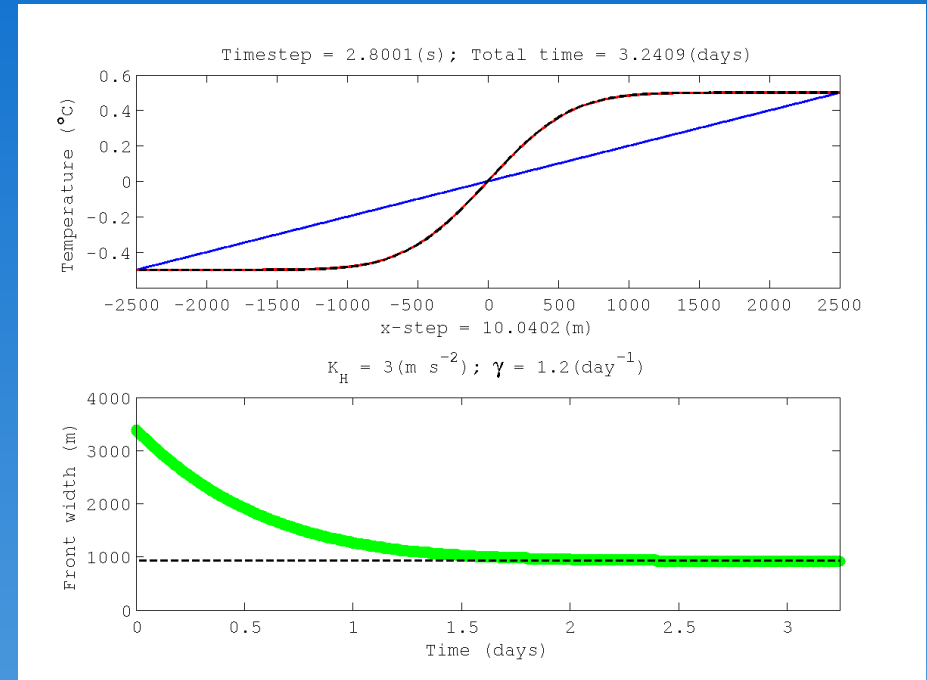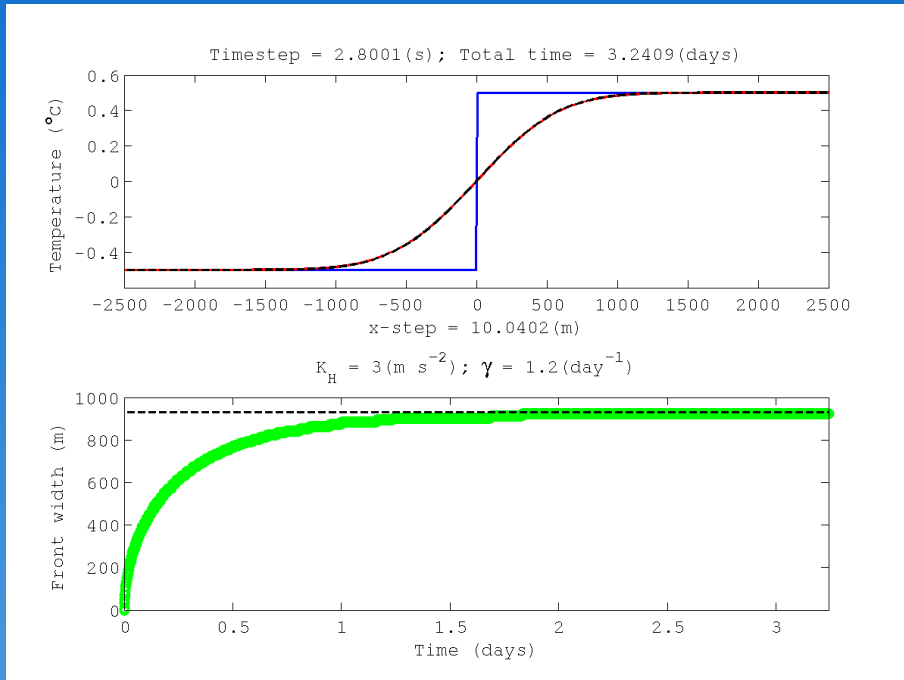*Call: **FP7 - PEOPLE - 2011 - IEF***
*Leading PI: **F. Nencioli***

F. Nencioli, F. d'Ovidio, A. Doglioli, A. Petrenko
Surface coastal circulation patterns by in-situ detection of Lagrangian Coherent Structures.
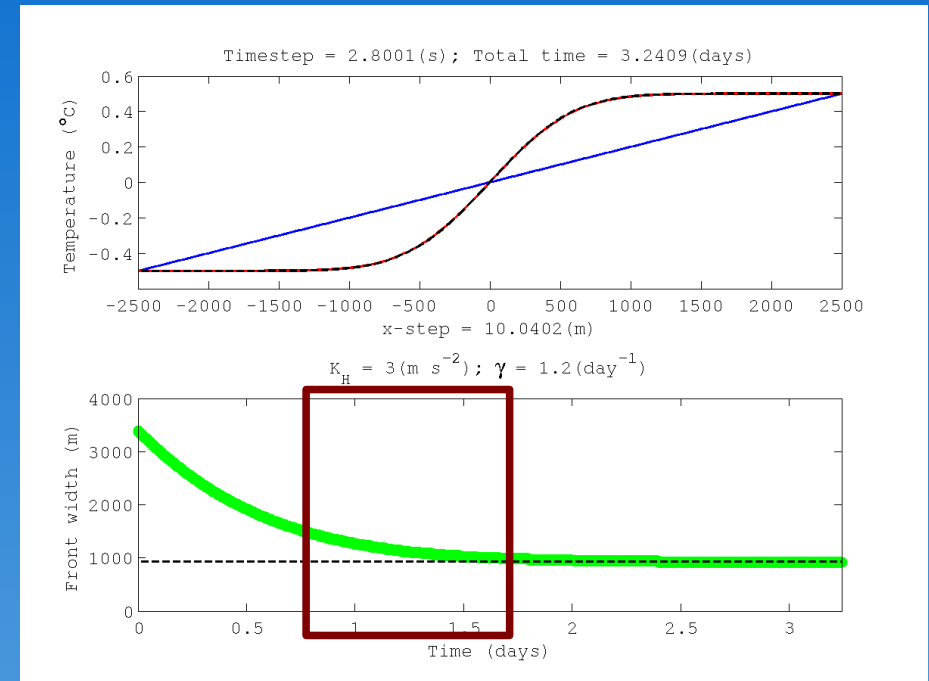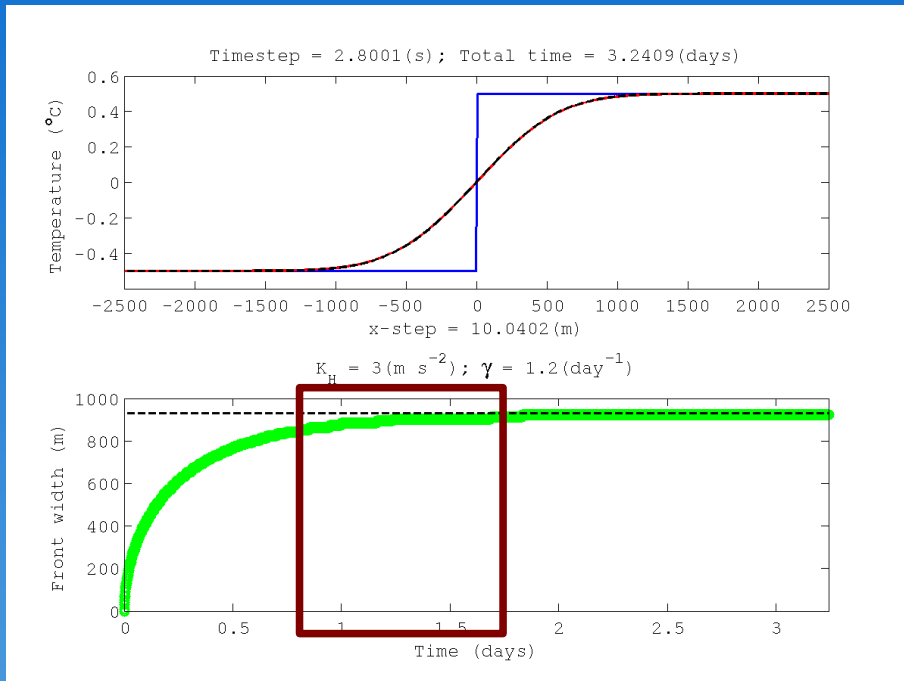*Geophysical Research Letters, 38, L17604,* doi:10.1029/2011GL048815

LATEX website: *www.com.univ-mrs.fr/LOPB/LATEX*

EXTRA SLIDES

Numerical solution

First order upwind scheme

First order upwind scheme

- **Fast adjustment to equilibrium (within 1 day)**

- **However Kh proportional to square of width**

- **Even small errors in width could affect estimate of Kh**