Chapter 6 – Velocity measurements

There are two main methods for measuring velocity: the Lagrangian and the Eulerian approach.

I) Lagrangian measurements of velocity

With this method we follow a fluid parcel along its trajectory.
See Lagrangian float in Chapter 5.

Joseph-Louis Lagrange (born Giuseppe Luigi Lagrangia or Giuseppe Ludovico De la Grange Tournier; 1736 – 1813), also reported as Giuseppe Luigi Lagrange or Lagrangia, was an Italian mathematician and astronomer, later naturalized French. He made significant contributions to the fields of analysis, number theory, and both classical and celestial mechanics.
Lagrange was one of the creators of the calculus of variations, deriving the Euler–Lagrange equations for extrema of functionals. He extended the method to include possible constraints, arriving at the method of Lagrange multipliers. Lagrange invented the method of solving differential equations known as variation of parameters, applied differential calculus to the theory of probabilities and worked on solutions for algebraic equations. He proved that every natural number is a sum of four squares. His treatise Theorie des fonctions analytiques laid some of the foundations of group theory, anticipating Galois. In calculus, Lagrange developed a novel approach to interpolation and Taylor theorem. He studied the three-body problem for the Earth, Sun and Moon (1764) and the movement of Jupiter's satellites (1766), and in 1772 found the special-case solutions to this problem that yield what are now known as Lagrangian points. Lagrange is best known for transforming Newtonian mechanics into a branch of analysis, Lagrangian mechanics, and presented the mechanical "principles" as simple results of the variational calculus. https://en.wikipedia.org/wiki/Joseph-Louis_Lagrange

II) Eulerian measurements of velocity

With this method we measure the currents from a fixed point in space.

Leonhard Euler (1707 – 1783) was a Swiss mathematician, physicist, astronomer, geographer, logician and engineer who founded the studies of graph theory and topology and made pioneering and influential discoveries in many other branches of mathematics such as analytic number theory, complex analysis, and infinitesimal calculus. He introduced much of modern mathematical terminology and notation, including the notion of a mathematical function. He is also known for his work in mechanics, fluid dynamics, optics, astronomy and music theory.
Euler is held to be one of the greatest mathematicians in history and the greatest of the 18th century. A statement attributed to Pierre-Simon Laplace expresses Euler's influence on

mathematics: "Read Euler, read Euler, he is the master of us all." Carl Friedrich Gauss remarked: "The study of Euler's works will remain the best school for the different fields of mathematics, and nothing else can replace it." Euler is widely considered to be the most prolific; his more than 850 publications are collected in 92 quarto volumes, (including his Opera Omnia) more than anyone else in the field. He spent most of his adult life in Saint Petersburg, Russia, and in Berlin, then the capital of Prussia.
Euler is credited for popularizing the Greek letter π to denote Archimedes' constant (the ratio of a circle's circumference to its diameter), as well as first employing the term f(x) to describe a function's y-axis, the letter i to express the imaginary unit $\sqrt{-1}$, and the Greek letter Σ (capital sigma) to express summations. He gave the current definition of the constant e, the base of the natural logarithm, now known as Euler's number.
Euler was also the first practitioner of graph theory (partly as a solution for the problem of the Seven Bridges of Königsberg). He became famous, among others, for solving the Basel Problem, after proving that the sum of the infinite series of squared integer reciprocals equalled exactly $\pi^2/6$, and for discovering that the sum of the numbers of vertices and faces minus edges of a polyhedron equals 2, a number now commonly known as the Euler characteristic. In the field of physics, Euler reformulated Newton's laws of physics into new laws in his two-volume work Mechanica to explain the motion of rigid bodies more easily. He also made substantial contributions to the study of elastic deformations of solid objects.
https://en.wikipedia.org/wiki/Leonhard_Euler

Velocity in three dimensions is usually expressed as:

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

The vertical coordinate, *w*, is often ignored and one typically measures U and θ

$$u\vec{i} + v\vec{j} = U(\cos\theta\,\vec{i} + \sin\theta\,\vec{j})$$

* Methods for measuring U:
  a.propellers
  b.rotors with cups and vanes (known as Savonius)
  c.acoustic sensors measuring sound propagation and Doppler shift
  d.electromagnetic sensors measuring the induced magnetic field
  e.variations in the resistance of a conductor (e.g., Platinum) due to cooling of a fluid flowing around a doncudtor
  f.radar

Mechanical current meters (a,b)
Non-mechanical current meters (c-f)

*The direction of the horizontal current, θ, is measured by an additional device (drift, etc) or by combining the measurements of the orthogonal current components.

You need to know the angle of the current/instrument (but also the angle of the intrument with respect to magnetic North):
This gives the current measurement relative to the instrument (sensor frame of reference).
In addition, an internal compass is needed to measure the orientation of the instrument with respect to the north (problem of compass connections; reliability; correction of the effects of declination of the magnetic North/geographic North). [note: not possible to use at the poles]

1) Brief History

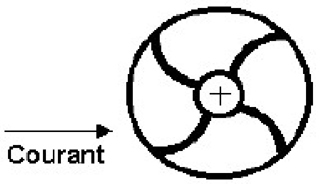
1930 first measurements using an Ekman current meter (Ekman 1932)
rotor with messenger (1 start, 1 end)
balls rolling in a cylinder, measuring the direction
Profile from 10 to 100 m done in 30 min
1960 US (Savonius rotor) and France (propeller torpedo)
1960 Norway Aanderaa (NATO contract)
1970 Japan acoustic

2) Current meters with rotors

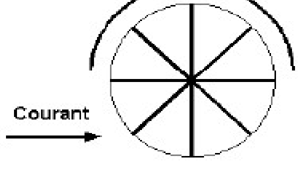
The rotor initially consisted of 6 curved blades in the shape of cups whose axis of rotation was oriented perpendicular to the main direction of the flow (see Figure "Savonius Rotor").
However, this design had some unwanted sensitivity to wave motions and led to a different design using flat-fined propellers (see Figure "Panemone").

The direction is measured by an additional vane that can move freely (see Figure "Aanderaa current meter").

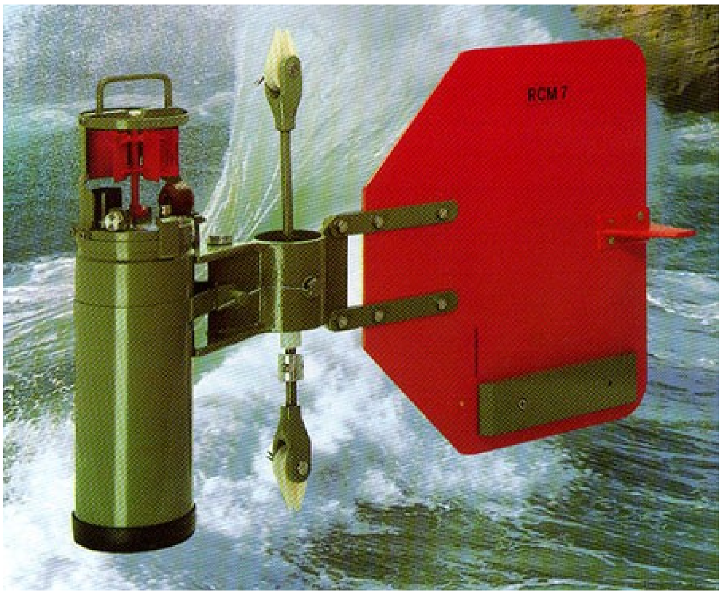
E.g., RCM = Rotor Current Meter



ROTOR DE SAVONIUS



Panémone



Aanderaa current meter

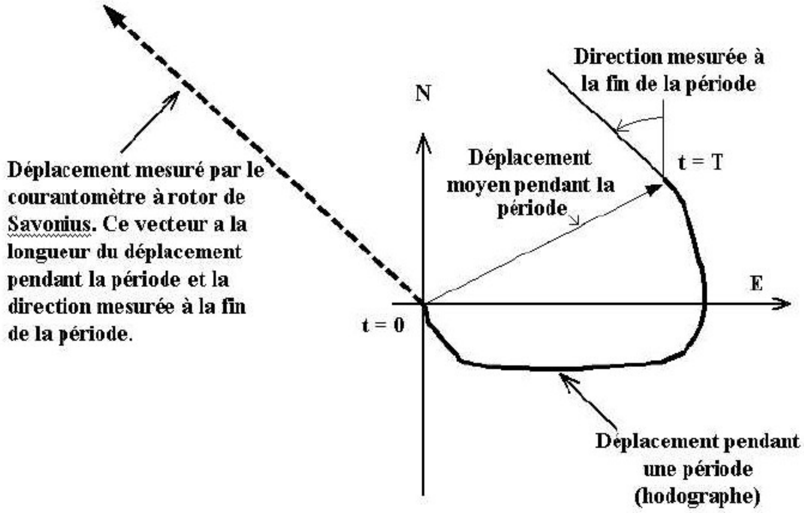
Introduction of burst sampling by Richardson 1963 (see Chapter 3)
+ internal data analysis

The fact that this instrument can internally create VECTOR averages of the measurements has revolutionized velocity measurements.

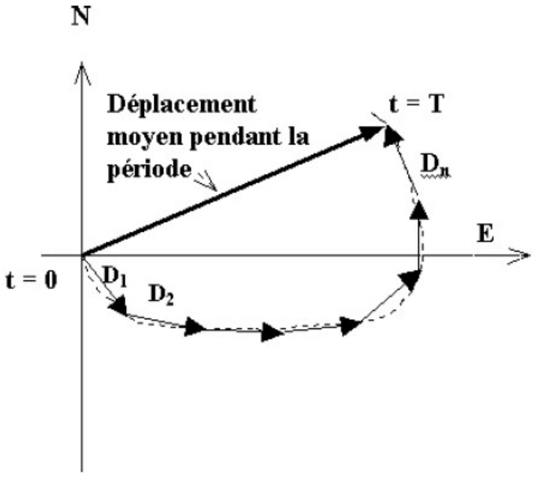
VACM = Vector Averaging Current Meter, commercially available since the 1970s

Example: measure the amplitude during 20 s
Measure the direction after 20 s
Internal data analysis: calculate the west and north component of the current
Storage in memory
Every Δt (e.g., 4 to 15 min), average n component measurements acquired during Δt

E.g., “Aanderaa” (name of brand) current meter; still in use today
burst sampling + component measurements



Déplacement réel, moyen et mesuré pendant une période de mesure.



Approche du déplacement moyen pendant une période (somme vectorielle)

(from Girardot 2003)

Note: It is also possible to internally calculate the standard deviation which provides information on the high frequency variability of the currents.

3) <u>Acoustic current meters</u>

3 a - <u>Acoustic non-Doppler current meters</u>
These current meters measure the speed along an axis by timing how long it takes a sound wave to travel between a transmitter (T) and a receiver (R).
The acoustic waves are emitted in the form of high frequency acoustic pulses (in the MHz range).

E.g., 1 T/R pair

V current velocity in the T/R direction

pulse velocity = c + V

The sound takes t1 to travel the distance L: (V+c) t1 = L

From this we obtain the current velocity: V = L/t1 – c

3b – Acoustic Doppler current meters

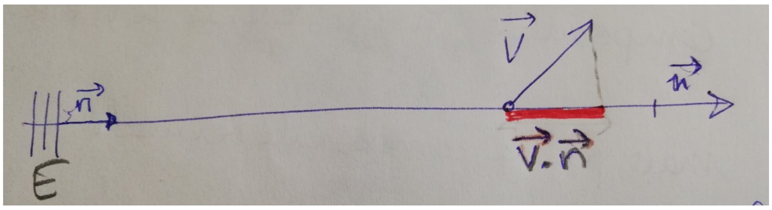
The Doppler effect refers to the frequency shift of a wave (mechanical, acoustic, electromagnetic, or any other wave) between its transmission and reception reception, when the source or receiver is moving relative to the other.
This effect was presented by Christian Doppler in 1842 in the article "Über das farbige Licht der Doppelsterne und einiger anderer Gestirne des Himmels (About the coloured light of binary stars and some other celestial bodies)", and observed in sound waves by the Dutch researcher Buys Ballot (using musicians playing a calibrated note on a train on the Utrecht-Amsterdam line), while in 1848 Hippolyte Fizeau suggested that it also applies to electromagnetic waves.

* The acoustic Doppler effect
Sound is produced by pressure variations in solid, liquid, or gaseous media. Sound waves are comparable to ocean waves. Imagine yourself on an anchored boat and waves are arriving perpendicular to the axis of the boat. Say you see x waves pass during dt seconds. Now if the boat was heading in the direction the waves are coming from, during the same time dt, you would see more waves, y>x and the frequency of the waves will seem higher to you. If you were travelling in the opposite direction, you would see fewer waves, y<x, and the frequency would seem lower to you. This is called the Doppler effect.

Instead of looking at sound waves, we listen to them. The Doppler effect manifests itself, for example, in the perception of sound waves and the change in pitch of a passing car or siren. The frequency is different depending on whether you are inside the vehicle (the transmitter being stationary relative to the receiver), or whether you are outside and the vehicle approaching you, the receiver, (causing you to perceive a higher frequency) or away (you perceive a lower frequency) (listen to the audio file of a passing car horn at https://en.wikipedia.org/wiki/Doppler_effect).
If c is the speed of sound, F the frequency emitted by the source, and V the relative velocity between the source and receiver, then the frequency shift is $\Delta F_1 = FV/c$

It should be noted that the frequency shift in this example is due to the change in position of the observer with respect to the trajectory of the moving car.

Indeed, the speed of the car perceived by the observer varies according to the angle, θ, between the line-of-sight (direction of unit vector $\vec{n}$) to the car and the car's velocity vector $\vec{V}$. Thus, the exact equation is: $\Delta F_1 = F\frac{-\vec{V}.\vec{n}}{c}$



There is thus a modulation by $\cos(\theta)$ if the velocity vector is not parallel to the source-receiver line-of-sight, thus $\vec{V}.\vec{n} = V\cos(\theta)$.
If the source moves perpendicular to the source-receiver line-of-sight, then there is usually no frequency shift.

If the observer moves directly away from the source, $\vec{V}.\vec{n} = V$, then $\Delta F_1 = -FV/c$ is negative (lower pitched sound).

If the observer moves directly toward the source, $\Delta F_1 = FV/c$ is positive (higher pitched sound).

<u>Measuring current speeds using the Doppler effect</u>
Initially, a transducer emits, along an axis, a certain train of acoustic waves of frequency F called “ping”. A short time later, this same transducer listens to the echo returning to it. This echo is due to the emitted wave being reflected by particles suspended in the water (or backscatter). If the particle is stationary, it sends back a signal of the same frequency as the original signal. If the particle moves away from the emitter, the frequency of the return signal is lower; if the ocean current moves the particle closer to the emitter, the frequency of the return signal is higher.

In both cases, the return signal is also affected by a Doppler effect, so the frequency is shifted twice, and the final formula becomes:

$$\Delta F_2 = 2F \frac{-\vec{V}.\vec{n}}{c}$$

which gives the radial velocity of the current in the direction of the beam:

$$\vec{V}.\vec{n} = \frac{-\Delta F_2 c}{2F}$$

A major limitation is that the result depends on the sound velocity, which is not necessarily known over the entire water column.

<u>Measuring current profiles using the Doppler effect</u>
The current meter using the Doppler effect for its measurements is usually abbreviated as **ADCP (Acoustic Doppler Current Profiler)** and can record profiles of both current speed and direction.

An ADCP contains piezoelectric transducers that act as both transmitter and receiver of acoustic signals. The travel time is used to estimate the distance of a data point from the transmitter. The frequency shift of the return signal is proportional to the water velocity along the axis of the acoustic beam. To obtain 3D velocities, we need at least three beams.

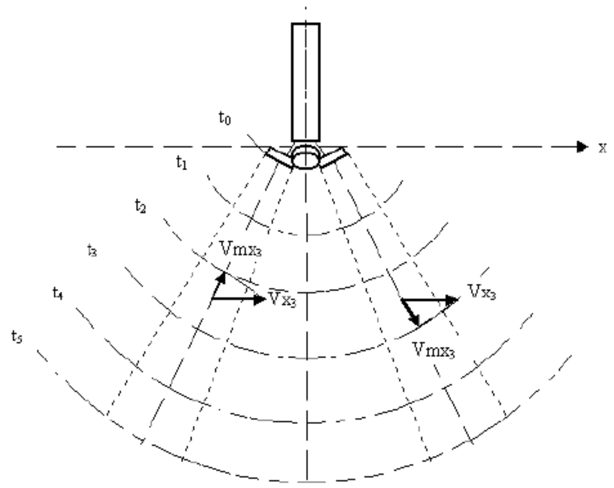
Often, these devices contain 4 beams in order to have some redundancy.

In recent years, new functionalities have been added to ADCPs (notably sensors to measure waves or turbulence) which means that some profilers may contain 2,3,4,5 or even 9 beams.

ADCPs can be used as a fixed bottom-mounted (using a frame or buoy) platform or aboard a ship. The ultrasonic signal is emitted either upwards (for bottom-mounted configurations) or downwards (if aboard a ship or suspended near the surface) and backscattered by suspended micro-particles which is detected by the transducers, with a cross-configuration as in the image below (or arranged in networks as for the latest generation ADCPs).

Vxi = actual in situ x-component of the current in layer I;
Vmxi is the measured x-component of the current in the layer I. It corresponds to the projection of Vxi on the source-receiver direction, that we commonly call "beam" (note that depending on the beam, Vmxi may be directed towards or away from the transducer)

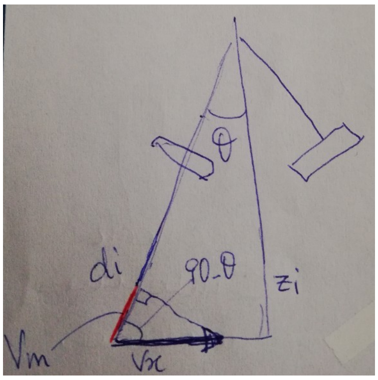
ADCP current meter



Principe de fonctionnement d'un ADCP
Vmx3 :composante de vitesse mesurée dans la 3e couche,

As an example, we will now lock at the operating principle of the RDI (Research Development Instruments) ADCP consisting of 4 beams. The profiler has four transducers (= acoustic tiles) inclined at an angle θ with respect to the vertical (generally θ= 30 ° for RDI ADCPs but other angles are found for other manufactureres, e.g., 20°).

The profiler simultaneously emits a ping on all 4 transducers and then listens for the signal echo, backscattered by suspended moving particles (and assuming that the particles are moving passively if we want to measure ocean currents).



From trigonometry, with $\alpha = 90° - \theta$ (with $\theta = 30°$, $\alpha = 60°$), we get $\cos(\alpha) = \frac{Vmx_i}{Vx_i}$

Vxi = (1/cos60°)*Vmxi

Numerical value (NV): Vxi = 2 Vmxi (cos 60°=0.5)

This measurement is made at an average distance "di" from the transducer, i.e. at a vertical depth of:
zi = (cos θ) di NV: zi= (cos30°)di = sqrt(3)/2di = 0.86di

The distance travelled is linked to the time that elapses between the signal emission and the reception of its echo as the sound travels this distance at velocity c (speed of sound). For the layer zi, we consider all signals returning between times t_i and t_{i+1}, hence the mean travel time of the returned signal is $(t_{i+1} - t_i)/2$
The distance di is travelled twice during this mean time:
2di = c $(t_{i+1} – ti)/2$
hence di = c $(t_{i+1} – t_i)/4$
where zi = (cos30°) di = (cos30°)c $(t_{i+1} – t_i)/4$

Three directions are needed to create a 3D velocity vector (u,v,w). ADCPs often have 4 beams which are used to measure the 3 velocity components and a quality flag for each measurement (or used as couple of opposite beams, each providing horizontal velocities and being compared afterwards)**.**

Be careful, the final velocity vector is not obtained by simply combining three perpendicular components but through a more complex trigonometric analysis.

The assumption was made that the current is homogeneous throughout a layer of thickness dz, despite the horizontal distance separating the beams issued by the 4 transducers. There may be potential problems for the most distant layers; but this problem is generally detected by a systematic analysis of the quality flags.

"Shadow/"contaminated" zones" (see Figure on following page on left)

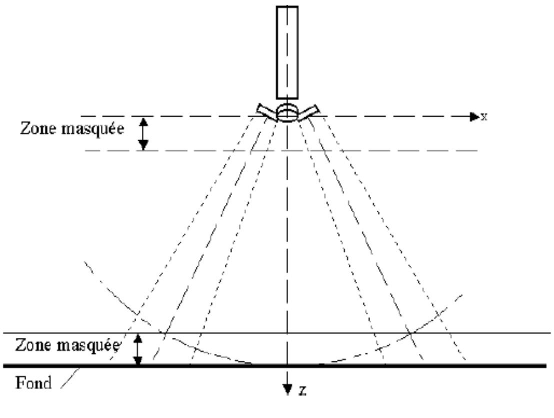
Reason a) After each ping, the transducers continue to resonate for a while ("ringing") so it is not possible to get measurements close to the device. The time it takes for the transducers to completely stop vibrating after sending the signal and before they can start listening for echoes determines the size of the "ringing zone" near the profiler, for which no current measurements can be made. This "ringing zone" is obviously close to the device.

Reason b) Although the transducers are directional, a small amount of energy gets emitted in other directions, in particular along the vertical axis of the ADCP. If the ADCP is at the

surface looking down, then the ping is reflected off the bottom and the return signal is strong enough to mask any signal due to near-bottom currents. In a bottom layer of height h (distance between the ADCP and the bottom), the shadow zone is usually about 0.15 × h.
These 15% correspond to 1-cos 30°, which is 1-0.86=0.14.

In this zone, the current cannot be measured. The same goes for the surface if the device is positioned at depth and looks upwards as a strong echo is produced by the surface.

For some devices, the size of these shadow areas can be smaller, around 6-12% of the water column height.



Zones masquées : résonance et écho sur le fond

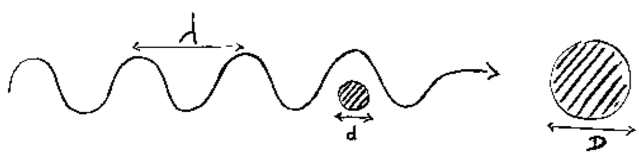
Fréquence en kHz	Portée en m	Diamètre transducteur
76.8	400	280 mm
153.6	240	165 mm
307.2	120	133 mm
614.4	60	101 mm
1228.8	25	54 mm

In fact, the size of the volume ensonified by each transducer is determined by the beam width emitted by the transducer. The ceramic assembly of each transducer has a certain diameter that depends on the frequency of the emitted wave (see table above, right column). **The higher the frequency, the smaller the transducer**. Also, **the higher the frequency, the shorter the range.** The low frequencies propagate much better in water compared to high frequencies as they are less absorbed.

Supplement:Backscatter

The amplitude of the backscattered signal can provide information beyond the current velocity. It can also provide information on the size of the backscattering particles.

The acoustic signal has a wavelength λ which determines the minimum size of the obstacles that can be detected. The following sketch illustrates how an obstacle of dimension d does not send back any signal while a larger object of dimension D does. **A wave of wavelength λ can only detect obstacles of dimensions ≥λ.**



This wavelength is related to the frequency by:
$\lambda = c/F$ where λ is the wavelength (m), c the speed of sound (m/s), and F the frequency (Hz).

The table below gives some examples:

c (m/s)	F (kHz)	λ (cm)
1500	12 38 120 200	12.5 3.9 1.25 0.75
1550	12 38 120 200	12.9 4.07 1.29 0.77

An object smaller than 2cm is “invisible” to a 38 kHz beam, but is detected by a 120 or 200 kHz beam. A school of sardines cannot be seen by the 38 kHz beam. On the other hand, it is also useless to try the 200 kHz beam to detect the sardines as the return signal will contain a lot of noise from all smaller fish as well.

Another manner to present this table is as follows:

TARGET SIZE	12 kHz λ = 12.5 cm	38 kHz λ = 3.9 cm	120 kHz λ = 1.25 cm	200 kHz λ = 0.75 cm
20 cm	YES	YES	YES	YES
10 cm	NO	YES	YES	YES
3 cm	NO	NO	YES	YES
1 cm	NO	NO	NO	YES
0.5 cm	NO	NO	NO	NO

In addition, another characteristic of the signal must be taken into account: its range. As mentioned earlier, low frequencies propagate much better in water than high frequencies. The choice of transmission frequency is thus a compromise between range and spatial resolution: high frequencies detect small organisms over short distances while low frequencies will only see large organisms but over long distances.

New ADCP technologies

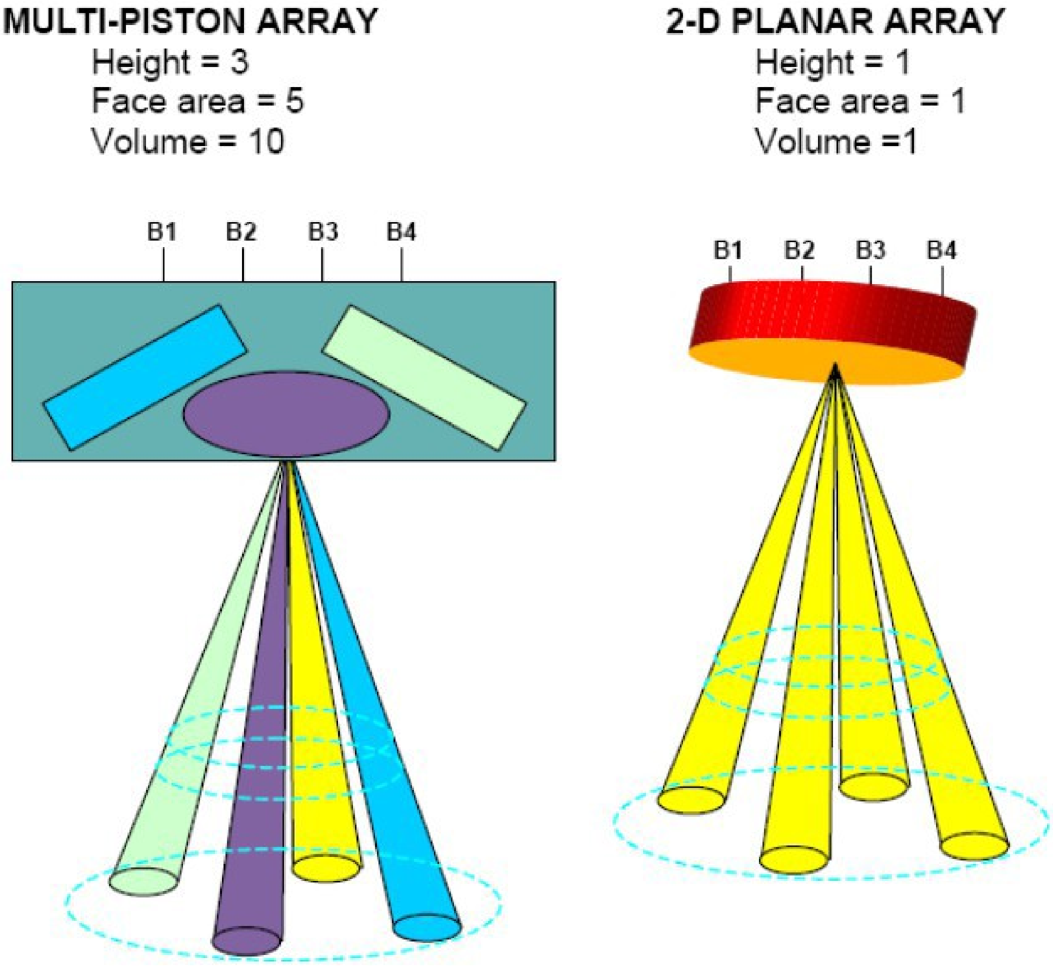
Phased array system ADCPs
ADCP transducers are designed and manufactured so that the beams are transmitted at the proper angle. This is necessary to be able to calculate the true horizontal and vertical velocity components properly. RDI manufactures two types of transducer assemblies for ADCPs: a piston transducer with 4 individual ceramic assemblies oriented at specific, fixed angles, and a

phased-array transducer with single ceramic assembly. Both transducer designs simultaneously produce 4 acoustic beams at specific, fixed angles, but the phased-array creates all of the beams electronically from a single aperture, instead of from 4 separate apertures. **One significant advantage is the greatly reduced overall size of the transducer at some particular frequency, employing the single-aperture phased array** as compared to four independent piston transducers. This size comparison is demonstrated in the figures below.

Moreover, it turns out that **the horizontal components of velocity measured by the phased-array transducer are independent of variations in the speed of sound in the water column.** This technology is therefore gradually replacing the old ADCP models.

Comparison of a 600kHz developmental phased array transducer with a standard four piston 600kHz WorkHorse transducer.

(Courtesy, RDI Primer, 2011)



Comparison of a multi-piston (left, with 4 beams from 4 transducers) and a 2-dimensional phased array transducer (right, a single transducer emits all 4 beams).

Other types of Doppler current meters

	Acoustic Doppler Velocimeter (type Vectrino, manufactured by Nortek; Nortek 2013) The basic principle is the same as outlined above except that **the 4 beams target the same sampling volume.**

Current measurements using radar

HF/VHF radar measurements are based on the interaction of the electromagnetic wave emitted by the radar and the sea surface according to Bragg's coherent diffraction mechanism (Barrick, 1972). The radar echo consists of two spectral lines arranged on either side of the transmission frequency of the radar, associated with those parts of the signal that get reflected on the waves moving towards (positive Doppler shift) and away from (negative Doppler shift) the radar.

The presence of currents modifies the position of these lines by introducing an additional Doppler shift (Barrick, 1972). This additional shift of the two Bragg lines of 1st order is used to determine the surface current strength while the spectral content at the other frequencies, the "2nd order spectrum", is used to determine the physical characteristics of the sea state: significant wave height, dominant period, etc. (Hasselmann 1971, Brooch et al, 1983).

Bref bibliographie :

Barrick, D.E.,1972: Remote Sensing of Sea State by Radar, Chapter 12 of Remote Sensing of the Troposphere. V.E. Derr, Editor, NOAA/Environmental Research Laboratories, 12.1-12.6.

Broche P., J. C. De Maistre and P. Forget, 1983: Mesure par radar decametrique coherent des courants superficiels engendres par le vent. Oceanologica Acta, 6, 43-53.

Hasselmann, K., 1971: Determination of Ocean-Wave Spectra from Doppler Radio Return from the Sea Surface. Nature, 229, 16-17.