Chapter VI: Fate of light in water – qualitative

A) Underlying hypotheses to formulate an equation

- Principle of interaction: based on the hypothesis of a linear theory of the interaction of light with matter at a phenomenological level. This principle of interaction has two parts:
- linearity of radiative phenomena for "low" energies (< 10^{10} Wm^{-2}). This is a necessary hypothesis for the implementation of the Maxwell equations, as well as those governing radiative transfer and the geometric laws of reflection, refraction, etc.
- phenomenological theory: the measurements and variables in the equations represent the variables at a macroscopic level (at which the geometrical optics approximation applies) and does not explain what happens at smaller scales. For instance, the VSF is used to describe light scattering without going into any detail about photon absorption and re-emission etc.
- Parallel planes: water is considered homogeneous and horizontally infinite. There are no boundary conditions at the lateral boundaries, only at the sea bed.

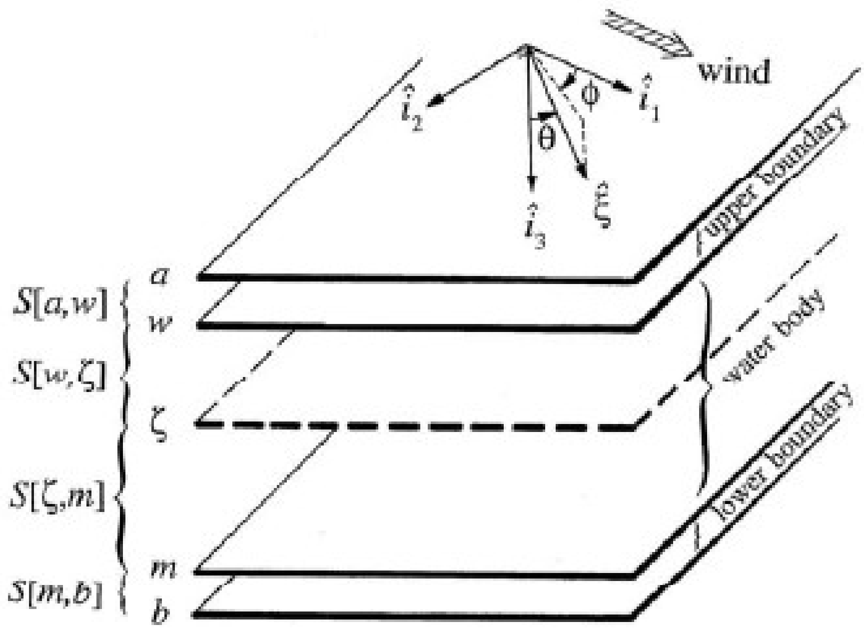


Fig. 4.1. Representation of a plane-parallel water body and the associated coordinate system. [redrawn from Mobley and Preisendorfer (1988)]

B) Radiative transfer equation

When photons cross the air/sea interface and start to propagate in the water column, they can be absorbed, with *a* the absorption coefficient; scattered in another direction without changing wavelength, with *b* the elastic scattering coefficient; or scattered by changing wavelength, with b^i the inelastic scattering coefficient. If we study the fate of the radiance, L, by focusing on photons of wavelength λ over a given distance *r* and in a specific direction ξ, the three aforementioned types of events can occur and are represented as the following loss terms: 1) –aL, 2) -bL, and 3) $-b^iL$. However, in the same way, three gain terms are to be taken into account describing: 4) a potential source of photons of this wavelength λ in the direction ξ (L^s), 5) elastic (L^e), or 6) inelastic (L^i) scattering resulting in one or more photons in direction ξ with wavelength λ.

This intuitive approach to describe these phenomena (Mobley, 1994) yields the following "**Radiative Transfer Equation"** (RTE):

$$\frac{dL}{dr} = -(a + b + b^i)L + L^s + L^e + L^i$$

1 2 3 4 5 6

The number 1 to 6 correspond to the 6 terms described in the previous paragraph. This RTE can be used to calculate the fate of light in water, provided we know *a* and *b* (b^i is often neglected).

This equation is in $Wm^{-3}\ sr^{-1}\ nm^{-1}$.

We can add some more detail to the equation, for example:

$$L_*^E(\vec{x};\xi;\lambda) = \int_{\xi' \in \Xi} L(\vec{x};\xi';\lambda)\, \beta(\vec{x};\xi' \rightarrow \xi;\lambda)\, d\Omega(\xi')$$

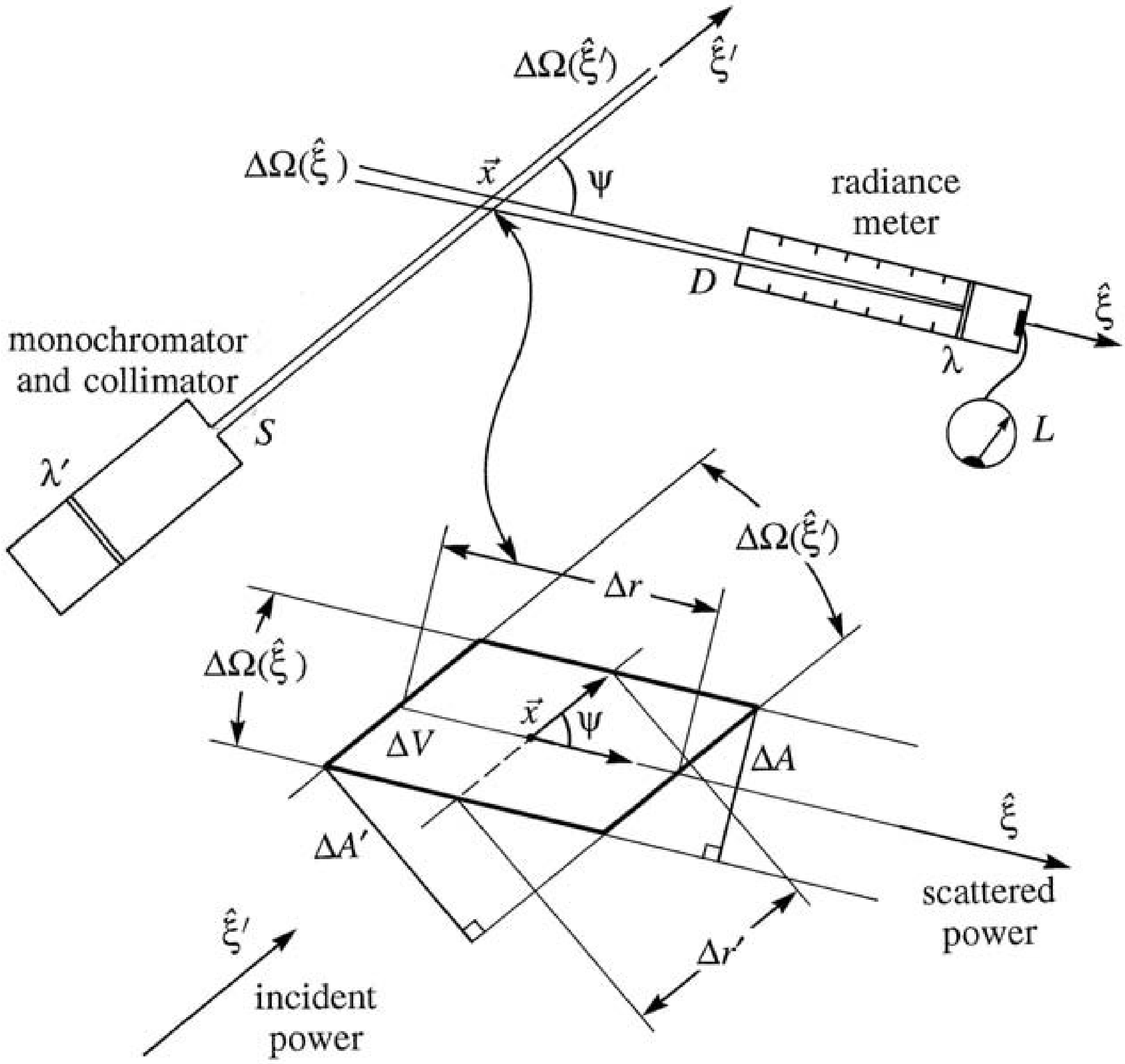


Fig. Geometry used in defining elastic and inelastic volume scattering functions. [redrawn from Preisendorfer (1987)] (with permission from Mobley, 1994)

Reminder: What does the operation 1/dr correspond to?

$$V = \frac{Dr}{Dt} = \frac{dr}{Dt}$$

$$\frac{1}{dr} = \frac{1}{V}\frac{1}{Dt}$$

or $\frac{1}{Dt}=\frac{1}{\partial t}+\vec{V}.\vec{\nabla}=\frac{1}{\partial t}+V\vec{\xi}.\vec{\nabla}$ thus

$$\frac{1}{dr}=\frac{1}{V}\frac{1}{\partial t}+\vec{\xi}.\vec{\nabla}$$

$$\frac{1}{dr}=\frac{1}{V}\frac{1}{\partial t}+\xi_1\frac{1}{\partial x}+\xi_2\frac{1}{\partial y}+\xi_3\frac{1}{\partial z}$$

The assumptions of conservation of energy and of a plane-parallel water body yield:

$\frac{1}{\partial t}=0$ and $\frac{1}{\partial x}=\frac{1}{\partial y}=0$ thus $\frac{1}{dr}=\xi_3\frac{1}{\partial z}=\cos\theta\frac{1}{\partial z}=\mu\frac{1}{\partial z}$

hence $\frac{dL}{dr}=\mu\frac{dL}{\partial z}$ corresponds to the preceding equation.

The Beer-Lambert law (see the end of Chapter 5) describes and idealised case:

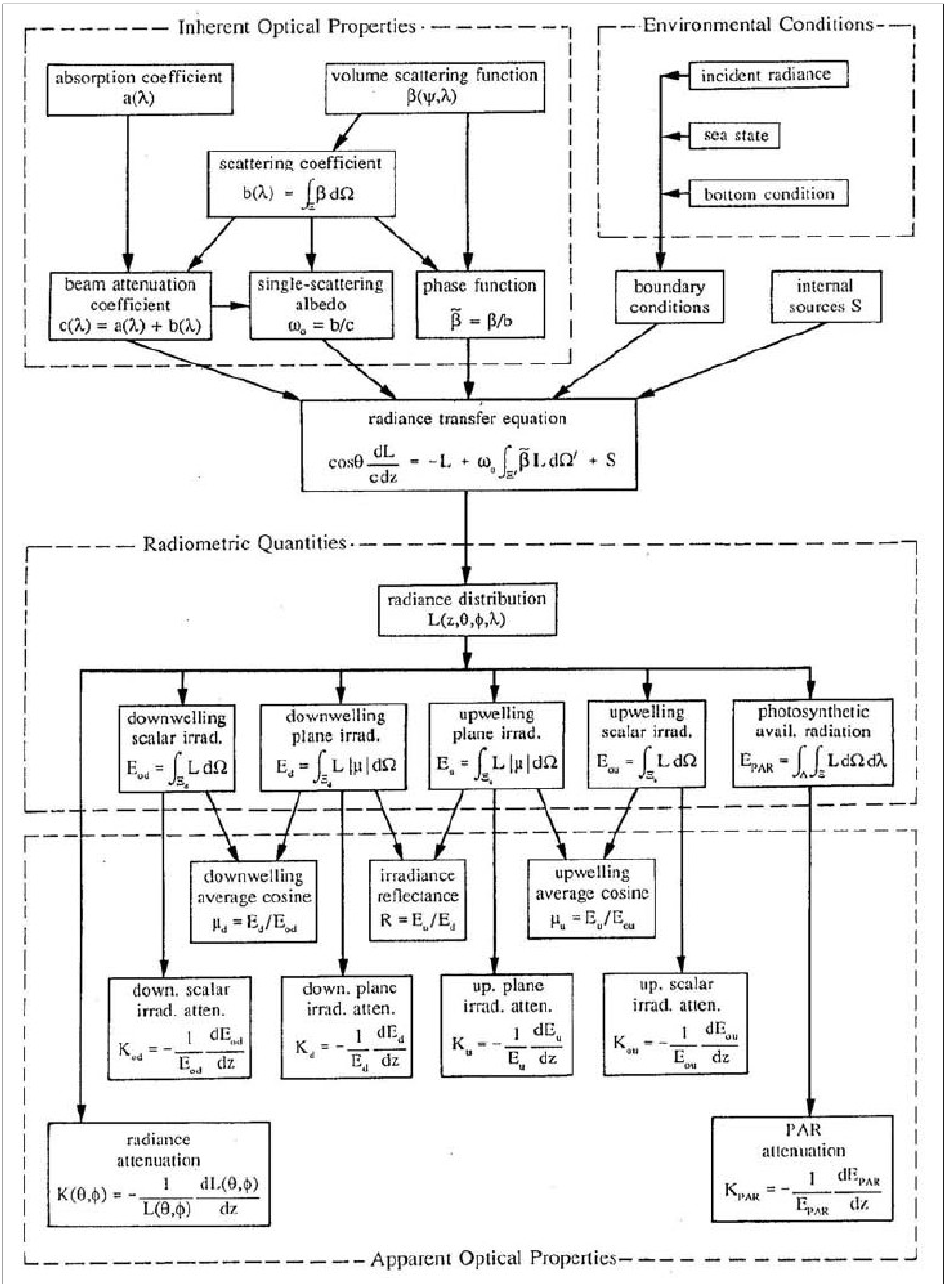
without source L^s

without inelastic scattering L^i (and $a^i = 0$)

without elastic scattering L^e (and $b = 0$)

Which yields:

$\frac{dL}{\partial z}=-\frac{a}{\mu}L$ or $L=L(z=0)e^{-\frac{a}{\mu}z}$



Inherent Optical Properties
absorption coefficient a(λ)
volume scattering function β(ψ,λ)
scattering coefficient b(λ) = ∫Ξ β dΩ
beam attenuation coefficient c(λ) = a(λ) + b(λ)
single-scattering albedo ω0 = b/c
phase function β̃ = β/b
Environmental Conditions
incident radiance
sea state
bottom condition
boundary conditions
internal sources S
radiance transfer equation
cosθ dL/dz = -L + ω0 ∫Ξ' β̃ L dΩ' + S
Radiometric Quantities
radiance distribution L(z,θ,φ,λ)
downwelling scalar irrad. Eod = ∫Ξd L dΩ
downwelling plane irrad. Ed = ∫Ξd L |μ| dΩ
upwelling plane irrad. Eu = ∫Ξu L |μ| dΩ
upwelling scalar irrad. Eou = ∫Ξu L dΩ
photosynthetic avail. radiation EPAR = ∫Λ ∫Ξ L dΩ dλ
downwelling average cosine μd = Ed/Eod
irradiance reflectance R = Eu/Ed
upwelling average cosine μu = Eu/Eou
down. scalar irrad. atten. Kod = - 1/Eod dEod/dz
down. plane irrad. atten. Kd = - 1/Ed dEd/dz
up. plane irrad. atten. Ku = - 1/Eu dEu/dz
up. scalar irrad. atten. Kou = - 1/Eou dEou/dz
radiance attenuation K(θ,φ) = - 1/L(θ,φ) dL(θ,φ)/dz
PAR attenuation KPAR = - 1/EPAR dEPAR/dz
Apparent Optical Properties

C) **Methods for solving the RTE**

The RTE has a unique solution, i.e., there is only one radiance value that satisfies the RTE equation for a particular location, wavelength, and direction of propagation.

Introducing different types of approaches

1) The Monte-Carlo approach

e.g., Morel A, Gentili B., Diffuse reflectance of oceanic waters: its dependence on sun angle as influenced by the molecular scattering contribution., Appl Opt. 1991 Oct 20;30(30):4427-38. doi: 10.1364/AO.30.004427.

2) The approach using invariant embedding

Preisendorfer, Hydrologic Optics, 1976 ; Preisendorfer and Mobley, 1988 (Theory of fluorescent irradiance fields in natural water, JGR 93(D0),10831-10855); and the model “Hydrolight”, Mobley, 1994.

3) Using eigenmatrices

Stamnes K, Tsay SC, Wiscombe W, Jayaweera K., Numerically stable algorithm for discrete-ordinate-method radiative transfer in multiple scattering and emitting layered media, Appl Opt. 1988 Jun 15;27(12):2502-9. doi: 10.1364/AO.27.002502.

There exist other methods that have not been mentioned here (e.g., iterative, spherical harmonics, etc.; see Mobley 1994 for details).

Classification of models

Predictive (predict something we don't know from somthing we do know, e.g., radiance from IOP's, etc.)
vs.
Diagnostic (analyze or transform known information, e.g., curve-fitting to data)

Direct (e.g., predict radiance given IOP's)
vs.
Inverse (e.g., deduce IOP's given the radiance)

Approximate analytical (e.g., single-scattering approximation)
vs.
"Exact" numerical (e.g., Hydrolight and Monte Carlo)

Deterministic (no statistical noise, e.g., Hydrolight)
vs.
Probabilistic (statistical noise, e.g., Monte Carlo)

1) The Monte-Carlo approach (probabilistic statistics)

The Monte Carlo techniques were developed in the 1940s to study neutron transport which was used to design nuclear weapons (Metropolis, 1949, Eckhardt, 1987). While the name "Monte Carlo" was initially just the code name for this classified research, it was chosen well because probability and statistics lie at the heart of both the simulation technique and the gambling that takes place at the legendary Monte-Carlo Casino in Monaco. Monte-Carlo techniques are now fairly advanced and are used to solve many types of problems in the physical and biological sciences, in finance, economy and business, in engineering, in computer animations for movie production, and in pure mathematics.

In the context of solving the RTE, the Monte-Carlo approach employs algorithms based on probability theory and random numbers to simulate the fate of many photons as they propagate trough a medium. Averages of large ensembles of photon trajectories yield statistical estimates of radiance, irradiance, and other quantities of interest.

It is the most general approach to solve the RTE and can be used with any type of boundary condition, incident radiance level (L), or IOPs. It is based on the **ray tracing method** which is based on the underlying assumption that if we know the probability of each individual event we can determine the probability of a series of events.

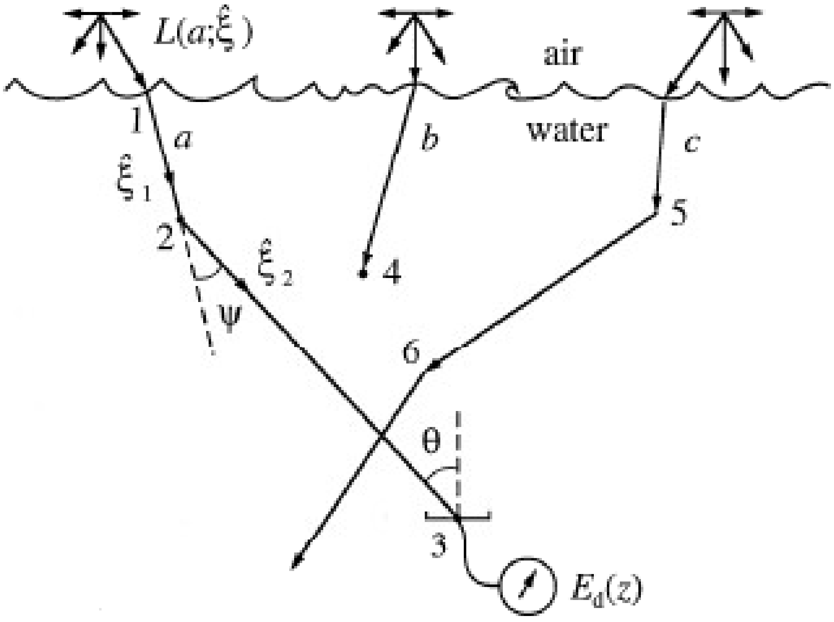


Fig. 6.1. Illustration of three photon trajectories and of the computation of E_d.

With permission by Mobley, 1994

Specifically, the direct Monte-Carlo approach involves the following steps:
- simulate the directions of photon propagation for given sun and sky conditions
- simulate the air/sea interface
- trace all photons
- count the photons arriving at the receiver (correction factor $\cos\theta$)
- calculate Ed(z)

<u>Highly simplified example to trace a photon</u>
What is the distance, *r*, travelled by a photon? (the same question could be asked for the optical path $l = cr$ instead of r)

for simplicity, we assume we remove the gains from the RTE:
dL/dr= -cL r? l? reminder: c = spectral attenuation coefficient

The amount by which the radiance, L, decreases in the direction of propagation, ξ , depends on the probabilities of a photon being absorbed or scattered while travelling between l and l+dl: $p_l(l)$
By definition, this probability needs to equal 1 if integrated between 0 and ∞ and

$$p_l(l)dl = e^{-l}\, dl$$

The cumulative probability distribution is thus $C_l(l) = \int_0^l p_l(l')dl' = \int_0^l e^{-l'}dl' = 1 - e^{-l}$

where y is a random number between 0 and 1 (uniform distribution):

$y = C_l(l) = 1 - e^{-l}$

Let $l = -\ln(1-y)$

hence $cr = -\ln(1-y)$

$r = -1/c \ln(1-y)$

Since y is a random number between 0 and 1 taken from a uniform distribution, 1-y is also a random number between 0 and 1 with a uniform distribution. To simplify, r is often taken as: $r = -1/c \ln(y)$

$<r>$ corresponds to the geometric mean distance, also called "free mean path": $<r> = <l/c> = <l>/<c>$

Once the distance travelled, r, has been calculated, the same approach can be used to determine the types of interaction that may or may not have taken place along the way.

Let y again be a random number between 0 and 1, then the simple backscattering albedo $w_o=b/c$ could be calculated as:

If $y > w_o$ absorption (ratio a/c)

If $y < w_o$ scattering (ratio b/c)

(Remember that w_o is also called the probability of photon survival.)

In case of scattering, the same approach can be used to determine the scattering angle (from between 0 and pi).

Properties of a forward Monte-Carlo approach (= photons are tracked in the sense of increasing time)

+ analogue simulation: i.e., analogous to physical processes
+ conceptually simple
+ instructive
+ very general
+ simple to code (computer //)
- no straightforward connection with the mathematical structure of the RTE
- can be very computationally inefficient (high CPU times)

Example for calculating the efficiency of this method. How many photons leaving a circular departure area with radius R (R=100m) arrive at a circular detector with radius r (r=1cm) at the level of the euphotic layer?

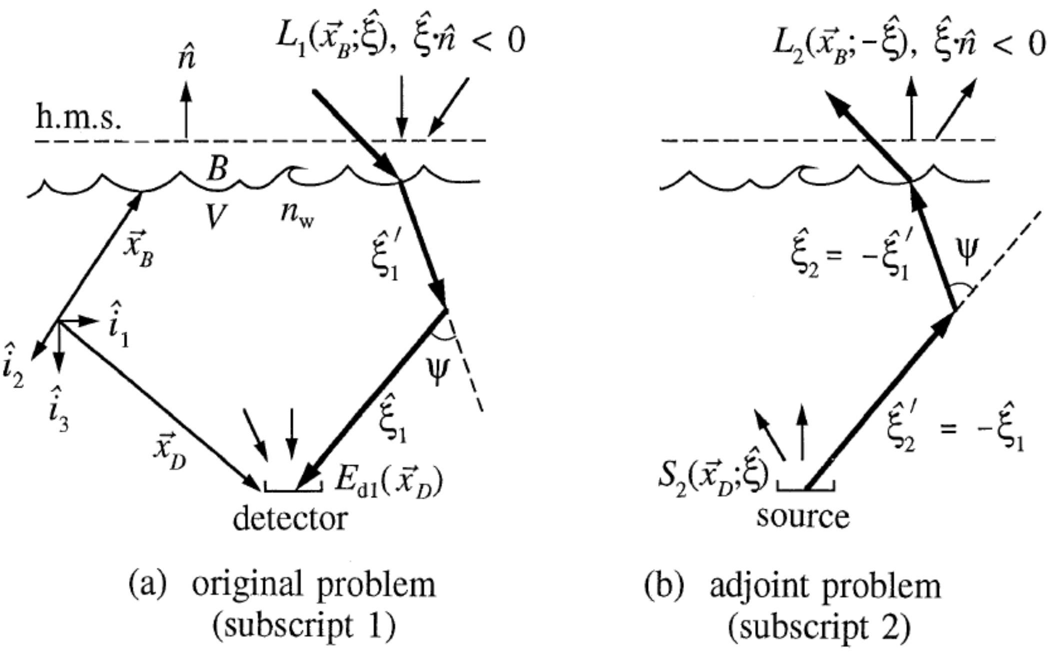
Answer: just 1 out of 10^{10} photons leaving the departure area arrive at the detector!

However, this calculation is more pessimistic than the reality. Due to the plane-parallel water body assumption all photons arriving at the euphotic depth can be counted and we therefore only have a loss of 99%: 1 photon out of 100 arrives at the bottom of the euphotic layer.

In general, to obtain good estimates with this method, the associated error needs to be smaller than the measurement error (e.g., of the order of 2-5 % for Ed).

The size of the error is linked to the number of photons, n, arriving at the detector and is given by: error = $n^{-1/2}$
If n = 10^4 photon detected at the base of the euphotic layer, then the error is 0.01 and Ed is determined to within 1% which is sufficient. This means that we need to release and trace 10^6 photons ($10^4 \times 10^2 = 10^6$) from the surface (and not $10^4 \times 10^6 = 10^{10}$ as would be suggested by the first calculation above).

However, we can increase the efficiency using a **reverse Monte-Carlo approach** where only those photons arriving at the detector need to be simulated. This can be achieved by running the simulation backward (in inverse time), i.e., tracing the photons from the detector (now the source) toward the sea surface (Gordon, 1985).



Monte-Carlo approach. a) forward and b) reverse (time runs backward) (Courtesy Mobley, 2014; Fig 6.2)

2) Method of invariant embedding
The is an analytical method (deterministic and not probabilistic) invented by Ambarzumian (1943) while working in astrophysics.

Characteristics:
+ can be applied to any situation
+ no statistical noise in the results
+ mathematically elegant

+ computationally efficient
- mathematically very complex

Mobley and Preisendorfer have worked for 20 years on the implementation and numerical modelling of the RTE; see the 6 volumes of Preisendorfer "Hydrologic Optics" (1976), and the equations in Preisendorfer and Mobley, 1988; and Chapter 8 of Mobley 1994 for a lighter version.

The approach is briefly explained here for a simplified RTE, consisting of "two-flow" irradiance equations. For this we need to transform:

- a linear problem (see Table 7.1) with 2 equations, 2 unknowns Ed and Eu, and 2 boundary conditions at the air/sea interface and at the sea bed
- into a non-linear problem with only 1 boundary condition that can "easily" be solved numerically

This is the basis of invariant embedding (very complex process).

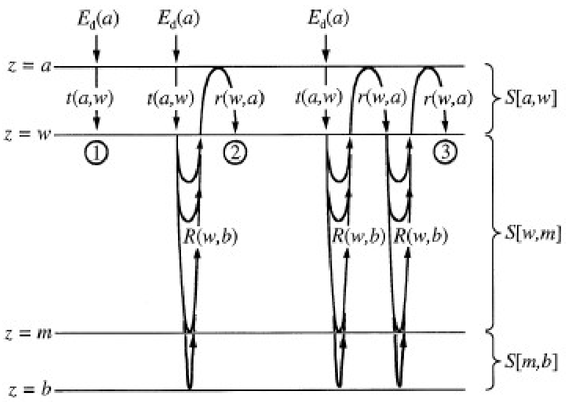


Figure 7.1
Illustration of the physical significance of the two flow equations. The circled numbers correspond to the first three terms of the series expansion in Eq. (7.19) of Mobley (Courtesy Mobley, 1994)

Table 7.1. The two-flow irradiance equations and associated boundary conditions. The underlined quantities are assumed known.

water layer	equations to be satisfied	equation number
$S[a,w]$	$E_u(a) = E_u(w)\,\underline{t(w,a)} + \underline{E_d(a)}\,\underline{r(a,w)}$	(7.1)
	$E_d(w) = E_u(w)\,\underline{r(w,a)} + \underline{E_d(a)}\,\underline{t(a,w)}$	(7.2)
$S[w,m]$	$\frac{dE_d(z)}{dz} = E_d(z)\,\underline{\tau_{dd}(z)} + E_u(z)\,\underline{\rho_{ud}(z)} + \underline{E^S_{od}(z)}$	(7.3)
	$-\frac{dE_u(z)}{dz} = E_u(z)\,\underline{\tau_{uu}(z)} + E_d(z)\,\underline{\rho_{du}(z)} + \underline{E^S_{ou}(z)}$	(7.4)
$S[m,b]$	$E_u(m) = E_d(m)\,\underline{r(m,b)}$	(7.5)

with permission from Mobley 1994

See Chapters 7 and 8 of the book by Mobley (1994) for more details about the approach used in the Hydrolight model.

3) Approach using eigenmatrices (or discrete ordinates)

This is an analytical method. The scattering phase function is approximated as a series of Legendre polynomials.

This powerful solution method is based on *approximating the scattering phase function as a series of Legendre polynomials, truncated to a finite number 2n of terms:*

$$\tilde{\beta}(\psi) \approx \sum_{k=0}^{2n-1} g_k P_k(\cos\psi) \,. \tag{9.1}$$

Here the g_k are the *expansion coefficients*, and the P_k are *Legendre polynomials*. It will prove convenient [see Eqs. (9.6) and (9.11)] to have an even number of terms in Eq. (9.1), hence the upper limit of $2n-1$ in the sum. For the moment, we consider n to be an arbitrary integer; we shall discuss below how to determine its value.

Legendre polynomials are treated in textbooks on mathematical methods of physics, for example Boas (1983) or Mathews and Walker (1965). They can be defined in general by

$$P_k(x) \equiv \frac{1}{2^k k!} \frac{d^k}{dx^k}(x^2 - 1)^k,$$

where $x = \cos\psi$ in the context of Eq. (9.1). The first few Legendre polynomials are

$$\begin{aligned} P_0(x) &= 1 & P_3(x) &= \frac{1}{2}(5x^3 - 3x) \\ P_1(x) &= x & P_4(x) &= \frac{1}{8}(35x^4 - 30x^2 + 3) \\ P_2(x) &= \frac{1}{2}(3x^2 - 1) & P_5(x) &= \frac{1}{8}(63x^5 - 70x^3 + 15x). \end{aligned} \tag{9.2}$$

The P_k form a complete set of orthogonal functions on the interval $-1 \le x \le 1$. They satisfy the orthogonality relation

$$\int_{-1}^{1} P_k(x)\, P_m(x)\, dx = \frac{2}{2m+1}\,\delta_{k,m}\,, \tag{9.3}$$

where $\delta_{k,m}$ is the Kronecker delta function of Eq. (1.19).

For additional details see Stamnes (1988).

Characteristics:
+ very efficient for solving the RTE
+ L is obtained at all depths in any direction ξ
- inefficient for a very “sharp” scattering phase function
- inefficient if the IOPs vary with depth
- surface wind effects are not well captured

See Mobley (1994) for more details about these 3 approaches presented here, as well as for other methods to solve the RTE.

D) **Inverse method**

IOPs → RTE → AOPs   Direct problem
←   Inverse problem

Solving the direct problem is not trivial.

http://www.oceanopticsbook.info/view/remote_sensing/inverse_problems Fig 1

But, at least, as indicated at the beginning of Section C, the solution of the RTE is unique, i.e., there is only one radiance that satisfies the equation at a particular point and for a particular wavelength and given direction.

However, the inverse is not true: knowing the radiance does not necessarily mean that we can determine the IOPs that produced this radiance nor that the set of IOPs we find is the only possible set to deliver this radiance.

There are two specific (and major) problems with inverse methods:
- the uniqueness of the solution
- error sensitivity (small errors in Lw can lead to completely wrong IOPs)

http://www.oceanopticsbook.info/view/remote_sensing/inverse_problems Fig 2

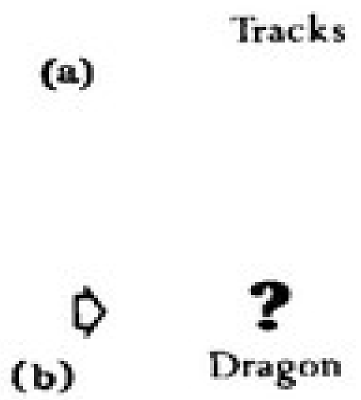
For example, it is necessary to know at least dL/dz; and yet this is not always sufficient. It can be in situations where the scattering is isotropic (certain atmospheric cases or in neutron scattering; but not in the oceanic environment).

The inverse problem has not yet been resolved, although it is a problem of major interest because it is what we would like to do with satellite data.

L sat = L atm + t Lw with Lw = 2% of Lsat

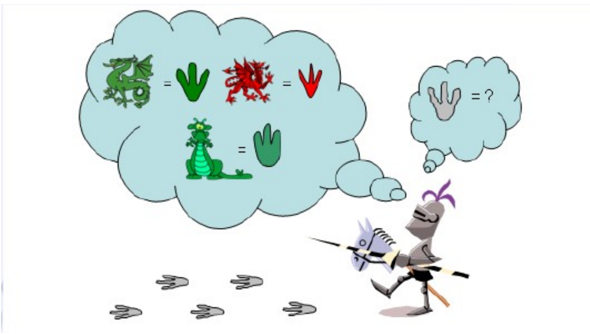
once L atm has been subtracted and the atmospheric corrections made.

If we know or can model Ed, then we have Rrs = Lw/Ed from where we want to derive the seawater constituents.



Bohren and Huffman, 1987, Absorption and Scattering of Light by Small Particles, book, ed Wiley, new edition in 2008

Direct problem: describe the footprints left by a dragon
Inverse problem: describe the dragon based on its footprints



ftp://ftp.ingv.it/pub/antonio.montuori/Corso_Telerilevamento_SAR_2014/Remote_Sensing/Materiale_Didattico_Migliaccio_2014/8.Inverse Problems.pdf

Classification of inverse problems (adapted from Mobley's Ocean Optics Book on the web)

There are many types of inverse problems. For example, let us assume that we are trying to characterize a medium by obtaining information about the IOPs of the medium, which in our case is a body of water with all its constituents. This is the type of problem we are considering here. There are also problems to try and characterize "hidden objects", the purpose of which is to detect or obtain information on an object embedded in the environment, such as a submerged submarine for isntance. Inverse problems can use optical measurements taken *in situ*, e.g., using the Gershun equation (see below) to obtain the absorption coefficient. Remote sensing uses measurements taken outside the medium, usually from a satellite or aircraft.

Another type of inverse problem lies in trying to determine the properties of individual photons scattered by simple particles. These problems usually start with us already possessing a certain knowledge about scattering particles (e.g., the particles are spherical and have a certain radius) and we then seek to obtain other specific information (such as the refractive index of the particle). The associated inverse algorithms generally assume that the detected light has undergone a single scattering event. Even these fairly constrained problems can be very difficult to solve. In the ocean, it is also impossible for light to undergo multiple scattering which greatly complicates our problem and we usually do not possess the necessary *a priori* knowledge regarding the properties of the different scatterers needed to constrain the inverse problem.

Techniques to solve these inverse problems fall into two categories: explicit and implicit. The **explicit solutions** are essentially formulas that give the desired IOPs as a function of the measured radiometric quantities. A simple example is Gershun's law which gives the absorption as a function of irradiance. The **implicit solutions** are obtained by solving a series of direct problems or in a forward approach. Roughly speaking, we can imagine having a remotely sensed reflectance (or a set of *in situ* measurements of radiance and irradiance). We then solve a set of direct problems to be able to predict the reflectance for a range of different sets of IOPs. Each predicted reflectance is compared to the measured value. The set of IOPs associated with the predicted reflectance that provides the best match with the measured reflectance is taken as the solution of the inverse problem. Such a plan of attack can only be effective if we have a rational way of moving from one set of IOPs to the next, i.e., if there is a logical way to proceed in this iterative process of testing different sets of IOPs such that the solution converges in the end to the measured reflectance or radiance.

A simplified inverse method: Gershun's law

the RTE can be written as:

$\mu \; dL/dz = -c\,L + L^e + L^i + L^s$

If there is no inelastic scattering nor any internal sources, this reduces to:

$\mu \; dL/dz = -c\,L + L^e$

which, if integrated over the entire sphere, gives:

d/dz(Ed-Eu) = -cEo +bEo = -aEo

a = -1/Eo d/dz(Ed-Eu)

Having used relatively few assumptions, we can thus obtain an IOP, namely the absorption coefficient as a function of Ed, EU, and Eo.
These assumptions apply to water columns with only low amounts of scattering.

E) **Bio-optical modelling (primary production...)**

Behrenfeld, M., and P. Falkowski (1997). A consumer's guide to phytoplankton primary productivity models. Limnology and Oceanography 42 (7), 1479–1491.

Classification system for primary production models based on 4 levels of mathematical integration. In fact, at least one new model of depth-integrated PP has appeared every 2 years in the literature for the past 40 years. All of these patterns can be written just by equating ΣPP, the depth-integrated primary production with factors that include
Csurf, the phytoplankton biomasse at the surface,
Φ, φ, , P^bopt, P^b , photo-acclimative variables
Zeu euphotic depth

F function depending on irradiance
DL day length
R phytoplankton respiration rate

The main difference between different models is how they implement F (although the conclusions will show that the irradiance level only has a minor influence on ΣPP)

I. Wavelength-resolved models (WRMs)

$$\sum PP = \int_{\lambda=400}^{700} \int_{t=\text{sunrise}}^{\text{sunset}} \int_{z=0}^{Z_{eu}} \Phi(\lambda, t, z) \times \text{PAR}(\lambda, t, z) \times a^*(\lambda, z) \times \text{Chl}(z)\, d\lambda\, dt\, dz - R$$

II. Wavelength-integrated models (WIMs)

$$\sum PP = \int_{t=\text{sunrise}}^{\text{sunset}} \int_{z=0}^{Z_{eu}} \varphi(t, z) \times \text{PAR}(t, z) \times \text{Chl}(z)\, dt\, dz - R$$

II. Time-integrated models (TIMs)

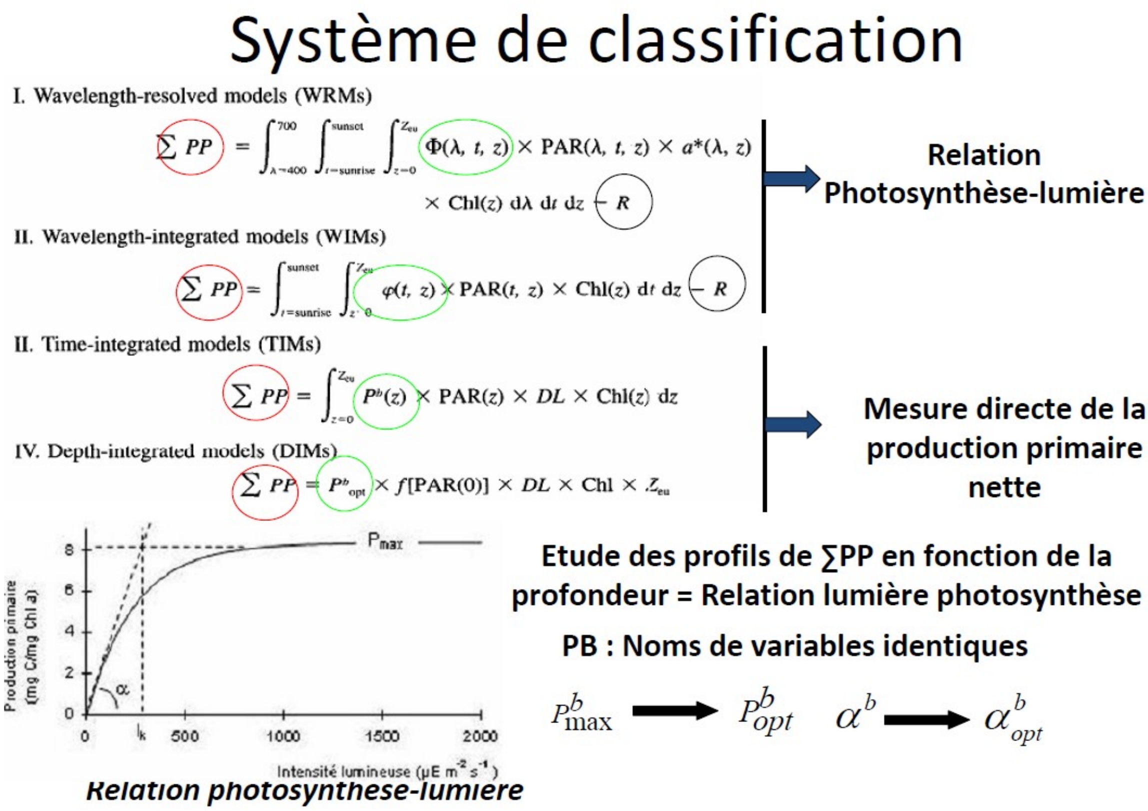
$$\sum PP = \int_{z=0}^{Z_{eu}} P^b(z) \times \text{PAR}(z) \times DL \times \text{Chl}(z)\, dz$$

IV. Depth-integrated models (DIMs)

$$\sum PP = P^b_{opt} \times f[\text{PAR}(0)] \times DL \times \text{Chl} \times Z_{eu}$$

The following slides are from a presentation of the Behrenfeld and Falkowski, 1997 paper by Marion Kersale (M2, 2009).

[More more info read
Behrenfeld, M. and E. Boss, 2014. Resurrecting the ecological underpinnings of ocean plankton blooms. Annual Review of Marine Science, 6, 167-194, DOI: 10.1146/annurev-marine-052913-021325
(can be downloaded from
http://misclab.umeoce.maine.edu/publications/scientific_articles.php)]



note: a* is typically known (from Mobley, 1994):

Table 3.7. Absorption by pure sea water, a_w, and the nondimensional chlorophyll-specific absorption coefficient, $a_c^{*\prime}$, for use in Eq. (3.27).[a]

λ (nm)	a_w (m^{-1})	$a_c^{*\prime}$	λ (nm)	a_w (m^{-1})	$a_c^{*\prime}$	λ (nm)	a_w (m^{-1})	$a_c^{*\prime}$
400	0.018	0.687	500	0.026	0.668	600	0.245	0.236
410	0.017	0.828	510	0.036	0.618	610	0.290	0.252
420	0.016	0.913	520	0.048	0.528	620	0.310	0.276
430	0.015	0.973	530	0.051	0.474	630	0.320	0.317
440	0.015	1.000	540	0.056	0.416	640	0.330	0.334
450	0.015	0.944	550	0.064	0.357	650	0.350	0.356
460	0.016	0.917	560	0.071	0.294	660	0.410	0.441
470	0.016	0.870	570	0.080	0.276	670	0.430	0.595
480	0.018	0.798	580	0.108	0.291	680	0.450	0.502
490	0.020	0.750	590	0.157	0.282	690	0.500	0.329
						700	0.650	0.215

[a] Condensed with permission from Prieur and Sathyendranath (1981), who give values every 5 nm.

Conclusion of the article

2/ Synthèse

- **Variabilité de ∑PP** a été divisée en l'associant à chaque variable d'un DIM standard $\Sigma PP := C_{surf} \times Z_{eu} \times P^b_{opt} \times DL \times F,$

➡ **Amélioration des évaluations de ∑ PP entre les catégories est négligeable**
Si un paramétrage équivalent est fait pour la variabilité horizontale de P^b_{opt} et Ek*
+ une dépendance linéaire de Eo n'est pas supposée

➡ La **variabilité de Eo** est responsable d'une partie mineure de la variabilité de ∑ PP

EVOLUTION DES MODELES → AMELIORATION RESTREINTE DES ESTIMATIONS DE ∑ PP

Effort sur la compréhension des causes de variabilité des facteurs physiologiques les + influents sur la variabilité de la productivité du phytoplancton

Excerpt from the article:
"We partitioned variability in Σ PP into that associated with each variable in the standard DIM (Eq. 11) and found that nearly all (-85%) could be attributed to changes in depth-integrated biomass (i.e. Csat * Zeu) and spatial (i.e. horizontal) variability in the photoadaptive variable P^b_{opt}. Only a small fraction (< 15%) of variability in Σ PP can be attributed to the cumulative effect of Eo-dependent changes in the depth of light saturation (F), spatial variability in E^x_k, and vertical variability in Cz and E^x_k. Because it is the variable description of the vertically resolved factors that distinguishes different categories of Σ PP models, it appears that the potential for improvements in Σ PP estimates between categories is negligible, so long as equivalent parametrizations are used between models for the horizontal variability in P^b_{opt} and Eo and a linear dependence on E, is not assumed.

That variability in Eo explains a relatively minor portion of the variability in Σ PP is perhaps the most counter-intuitive result of our investigation, because the effect of Eo on Pz is so clear that any biological oceanographer or limnologist could differentiate between Pz profiles from low-light and high-light conditions without any additional information.
However, this unavoidable conclusion is a consequence of the exponential attenuation of Ez restricting the effect of the full range in Eo on variability in Σ PP to a small fraction of that attributable to variability in P^b_{opt} and Σ C.

Specifically, changes in Eo typically contribute a factor of ~2 to variability in Σ PP, which is a small fraction of the three orders of magnitude variability observed in Σ PP.

The widespread use of light as the principal forcing component in Σ PP models is understandable, because physical processes governing the wavelength-specific distribution of light in the world's oceans are well characterized and easily rendered into mathematical formulations and computer code.
Consequently, models have been developed with the capacity to relate production at any depth to the spectrally dependent absorption of time-dependent irradiance.
Conceivably, this reductionist approach could be continued ad infinitum but with negligible benefits, toward improving model estimates of Σ PP.
The intent of this report was not to diminish the important advances made in the development of WRMs, for such models provide a sound foundation for developing mechanistic productivity models once a better understanding of algal physiology has been achieved. Rather, we hope this analysis demonstrates the fundamental synonymy between models and will help us.her the productivity modeling community into abandoning a long history of parallel and redundant modeling efforts By doing so, a more focused effort can be made in the future on understanding the underlying causes of variability in physiological factors most influential on variability in depth-integrated phytoplankton productivity."