CIVIS Course
Ocean Sciences: Lagrangian methods in marine sciences in parallel with OPB309 course

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1. Introduction : why study vertical velocities in the ocean ?

- **Vertical velocities :**

→ Where ? Upwellings, fronts, eddies

→ Very hard to measure since often very small << compared to u and v

→ Studied with numerical models, ex estimated thanks to Omega equation

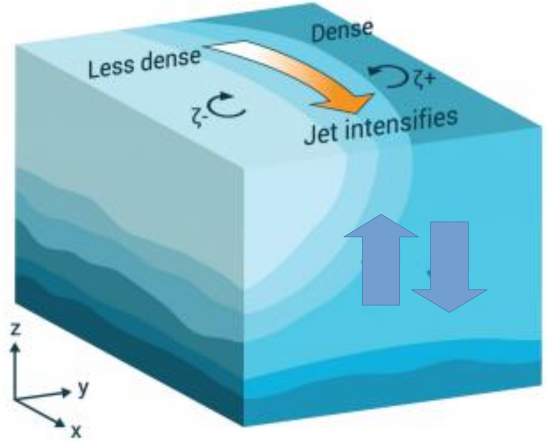

Figure extraite de Mercator Ocean.

- **Key role on biogeochemistry and biological processes:**

→ **Vertical dynamics ===>**
(Lévy et al., 2001 ; Mahadevan, 2016).

- **carbon sequestration at depth**
- advection of Organic Matter downward
- **advection of nutriments upward in the euphotic zone**

Phytoplankton developpement

Q-vector Omega equation (Hoskin et al., 1978)

$$f^2 \frac{\partial^2 w}{\partial z^2} + \nabla_h (N^2 . \nabla_h w) = \nabla . \boldsymbol{Q}$$

→ f : Coriolis parameter (s^{-1})
→ w : vertical velocity ($m\ s^{-1}$)
→ N^2 : Brünt Vaïsäla frequency
→ **Q** : **Q**-vector

Quasi-geostrophic solution

$$Q = \left(\frac{g}{\rho_0} \frac{\partial V_g}{\partial x} . \nabla \rho , \frac{g}{\rho_0} \frac{\partial V_g}{\partial y} . \nabla \rho \right)$$

Vg : geostrophic horizontal velocity

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$$Q_1 = \frac{g}{\rho_0} \left(\frac{\partial u_g}{\partial x} \frac{\partial \rho}{\partial x} + \frac{\partial v_g}{\partial x} \frac{\partial \rho}{\partial y} \right)$$

$$Q_2 = \frac{g}{\rho_0} \left(\frac{\partial u_g}{\partial y} \frac{\partial \rho}{\partial x} + \frac{\partial v_g}{\partial y} \frac{\partial \rho}{\partial y} \right)$$

Omega equation to obtain vertical velocity;
PROTEVSMED-SWOT 2018 cruise example

PROTEVSMED-SWOT 2018 cruise example from PhD work of Roxane Tzortzis;
collaboration with Andrea M. Doglioli, Stéphanie Barrillon, Anne A. Petrenko, Francesco d'Ovidio, Lloyd Izard, Melilotus Thyssen, Ananda Pascual, Bàrbara Barceló-Llull, Frédéric Cyr, Marc Tedetti, Nagib Bhairy, Caroline Comby, Pierre Garreau, Franck Dumas, Lucie Bordois and Gérald Gregori

Institut Pythéas Observatoire des Sciences de l'Univers Aix Marseille Université
MIO Mediterranean Institute of Oceanography
LOCEAN
IMEDEA CSIC Universitat de les Illes Balears
Fisheries and Oceans Canada
Ifremer
SHOM L'océan en référence

1. Introduction : Steps to estimate vertical velocities with the Omega solver

- **“Preliminary” step:**

→ Determine a structure of interet (front, eddy).

- **Step 1 : Determine the size of the interpolation grid.**
- **Step 2 : Objective mapping (interpolation of data ρ, u, v).**

Preparation inputs to the solver

- **Step 3 : Calculation of w with the Omega solver.**

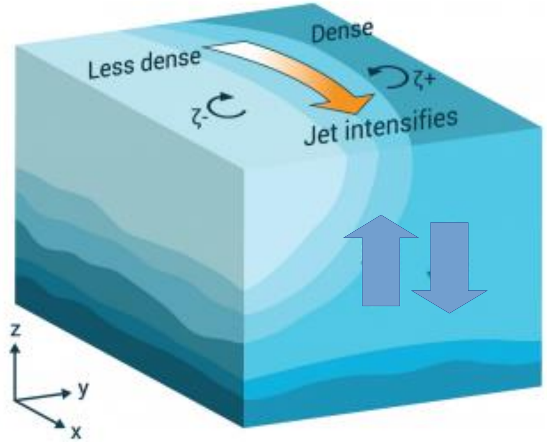
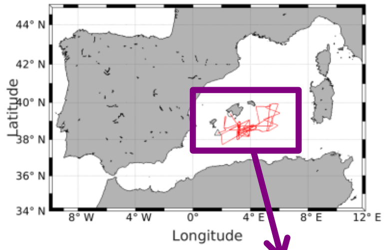
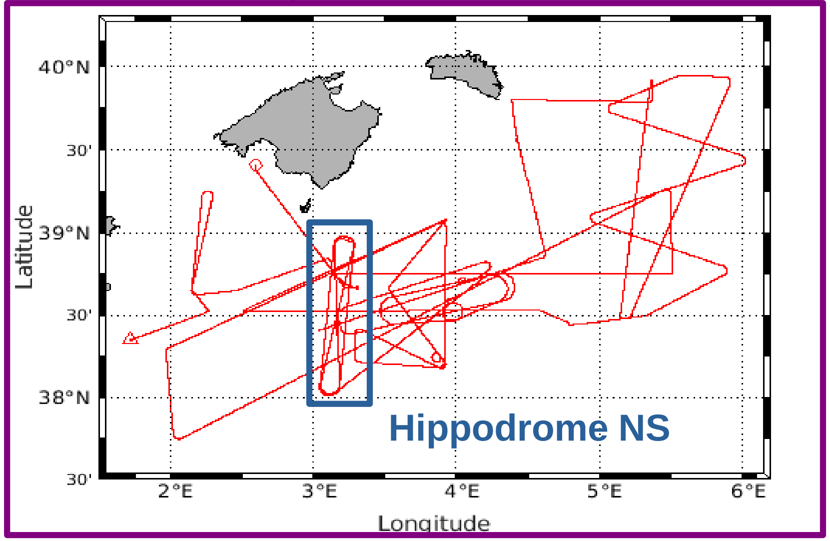


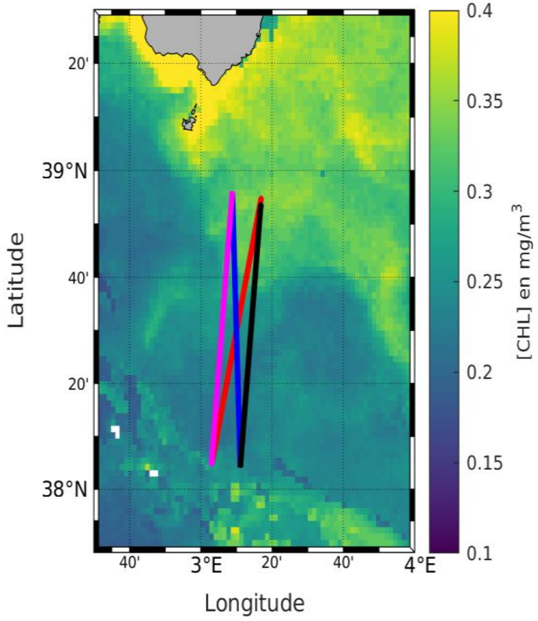
Figure extraite de Mercator Ocean.

Example Cruise PROTEVSMED-SWOT 2018 : selection of transects across a front





RV Beautemps-Beaupré trajectory



→ Identification of a front

→ Selection of transects across the front

Transect 1: 11 May 2:10 am – 8:40 am
Transect 2: 11 May 10:00 am – 4:40 pm
Transect 3: 11 May 6:00 pm – 12 May 00:45 am
Transect 4: 12 May 2:00 am – 8:20 am

2. Determination of the size of the interpolation grid

→ **Arakawa C grid**.

→ The **inputs** must have the following sizes :

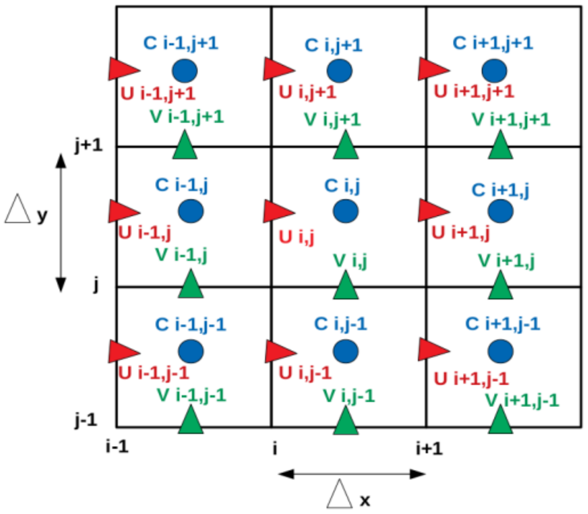
→ $\rho(L+1, M+1, N)$

→ $u(L, M+1, N)$

→ $v(L+1, M, N)$

L: nb of grid meshes in the x axis
M : nb of grid meshes in the y axis
N : nb of grid meshes in the z axis

==> **w(Lm, M, N) = w(L-1, M, N)**



→ L, M and N defined in the omega solver, according to :

→ $Lm = L-1 = ixp \times 2^{(iex-1)} + 1$

→ $M = jyq \times 2^{(jey-1)} + 1$

→ $N = kzr \times 2^{(kez-1)} + 1$

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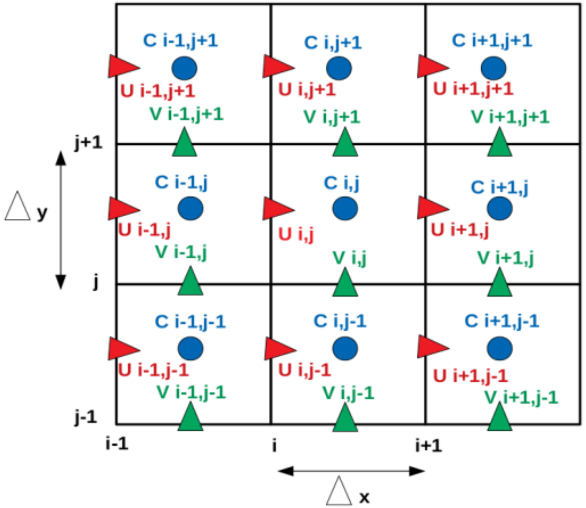

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→ The solver starts by an interative calculation on the grid of dimension **[ixp+1,jyq+1,kzr+1]** then it increments the resolution at each calculation.

→ To reduce the calculation time, the goal is for the coefficients ixp, jyq, kzr to be as small as possible, 2, 3 (worse case 5) but not more, otherwise it will slow down the convergence rate to the solution.

→ iex, jey et kez allow to adjust the size of the grid to the size of the studied domain.

→ **Aim:** get ixp,jyq, kzr as small as possible; but iex, jey, kez as large as possible.

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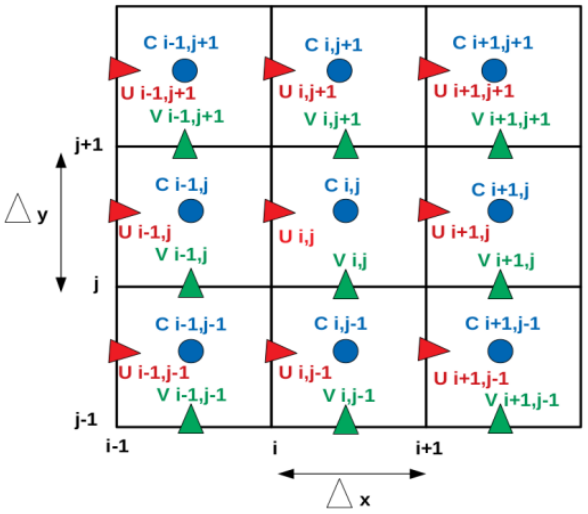

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Alice Pietri code:
grid_for_w.m
→ provides all the potential Lm, M, N of w with the associated coefficients, respecting the solver constraints.

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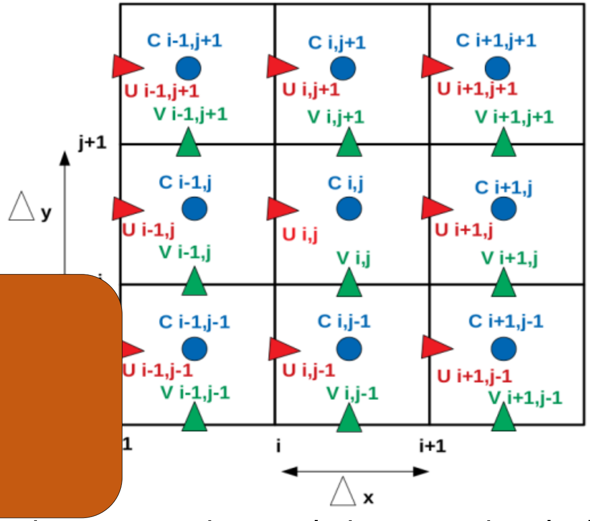
Attention to the unit of grid mesh !
(m)



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3. Objective mapping

→ Objective mapping consists in reconstructing the density and horizontal velocities fields

→ Advantage (rather than a simple interpolation): provides a confidence level (error grid < calculation of the data variance).

→ signal (u) = average ($\bar{u}$) + fluctuations (u') + noise (n) (caused by the fine scale variability and instrument measurement errors) [Le Traon, 1990]

$$\mathbf{u} = \bar{\mathbf{u}} + \mathbf{u}' + \mathbf{n}$$

→ Le Traon, 1990 hypothesis: the fluctuations (u') are anisotropic (i.e. direction dependant), with a Gaussian auto-covariance (C) :

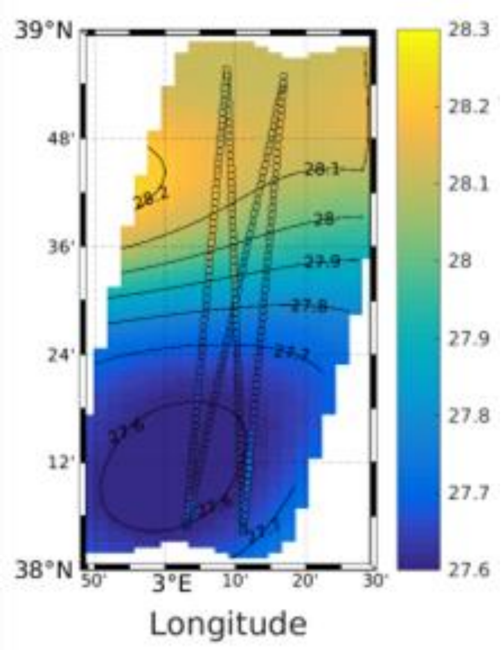
$$C(x,y) = A\exp\left[-\frac{(x\cos\theta - y\sin\theta)^2}{L_x^2} - \frac{(x\sin\theta + y\cos\theta)^2}{L_y^2}\right]$$

→ Lx, Ly : decorrelation scale
→ θ : angle orientation of the studied structure (/ North, clockwise)
→ E/A = 0,05 noise to signal ratio (Rudnick, 1996)
→ E : matrix of covariance of fluctuations u' and noise n

$$\mathbf{E} = \left\langle (\mathbf{u}' + \mathbf{n})(\mathbf{u}' + \mathbf{n})^T \right\rangle$$

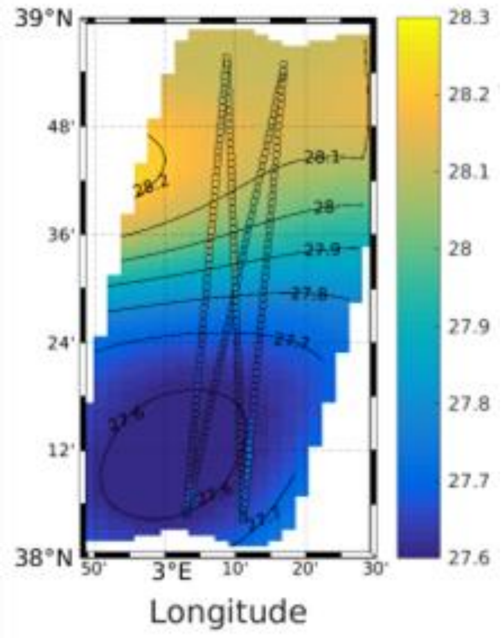
Rudnick site :
http://chowder.ucsd.edu/Rudnick/SIO_221B.html

At 25 m depth:



Potential density anomaly (after the objective mapping)

At 25 m depth:



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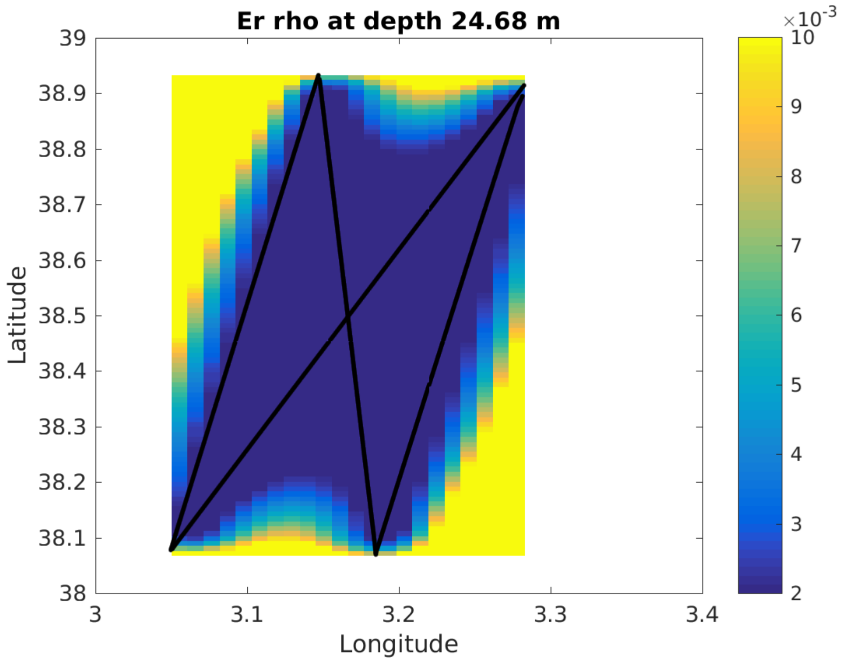

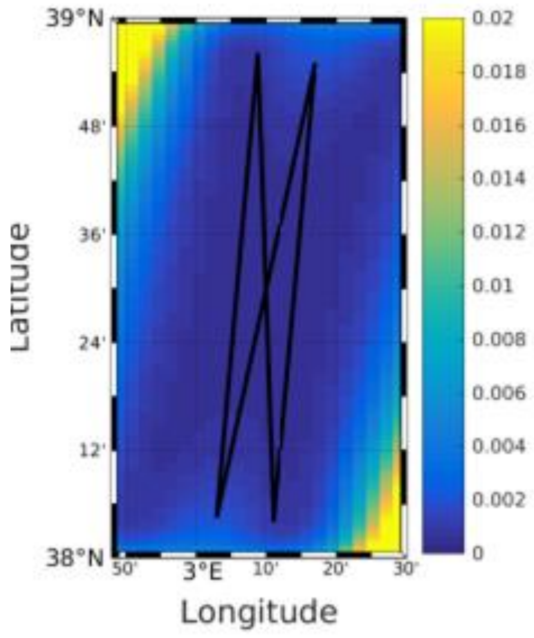

Error grid

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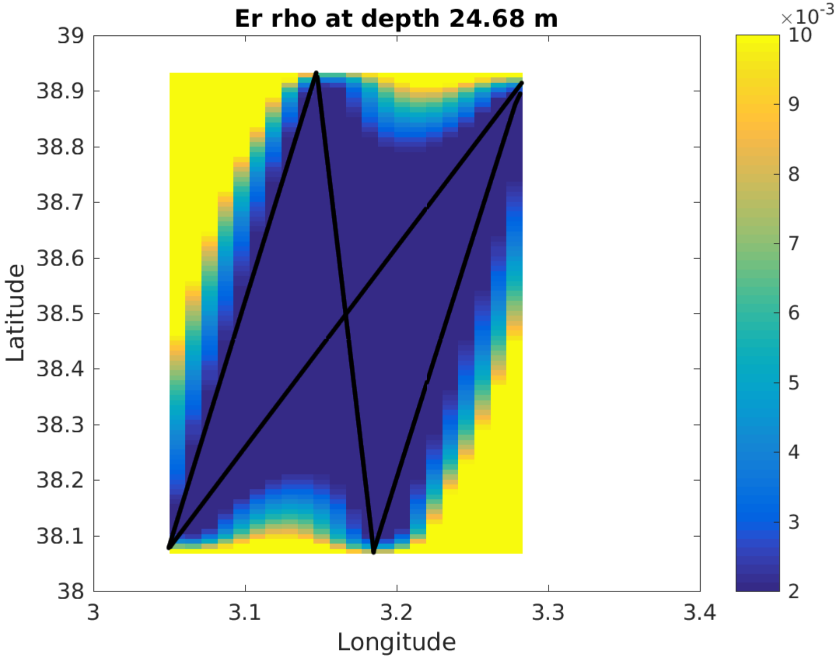
→ Le Traon, 19
with a Gaussia

In the objective mapping code:
→ User inputs: Lx, Ly, θ, E/A
→ Lx, Ly influences mainly the error matrix

$$C(x,y) = A\exp\left[-\frac{(x\cos\theta - y\sin(}{L_x^2}\right.$$



'SIO

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- Omega equation (Hoskin et al., 1978)

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- Different forcings can be considered (Giordani et al., 2006)

Q = **Q**th + **Q**dm + 2 **Q**tg + **Q**tag + **Q**dag + **Q**dr

Qtg : Buoyancy forcing +
Qdm : Momentum forcing
= Turbulent forcings

TWI = Thermal Wind Imbalance

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~~**Q**tg : Buoyancy forcing + **Q**dm : Momentum forcing = Turbulent forcings~~

Generally negligeable (as well as the forcing due to beta-effect)

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Qtg : Geostrophic

$$Q = \left(\frac{g}{\rho_0} \frac{\partial V_g}{\partial x} . \nabla \rho , \frac{g}{\rho_0} \frac{\partial V_g}{\partial y} . \nabla \rho \right)$$

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(2 **Q**tg + **Q**tag = **Q**tw)

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Qtw = 2 **Q**tg + **Q**tag = kinetic deformation vector

Also named the frontogenesis vector
= variability in the horizontal density gradient
induced by
the horizontal advection of the total velocity
(Qtg geostrophic velocity
Qtag ageostrophic velocity)

In QG theory, only Qtg remains

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Vag : ageostrophic horizontal velocity

Qdag : TWI deformation

$$\begin{cases} Q_{dagx} = f\left(\frac{\partial v}{\partial x} \frac{\partial u_{ag}}{\partial z} - \frac{\partial u}{\partial x} \frac{\partial v_{ag}}{\partial z} \right) \\ Q_{dagy} = f\left(\frac{\partial v}{\partial y} \frac{\partial u_{ag}}{\partial z} - \frac{\partial u}{\partial y} \frac{\partial v_{ag}}{\partial z} \right) \end{cases}$$

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with

$$\begin{cases} TWI(x) = f\frac{\partial u}{\partial z} - \frac{g}{\rho}\frac{\partial \rho}{\partial y} = f\frac{\partial u_{ag}}{\partial z} \\ TWI(y) = f\frac{\partial v}{\partial z} + \frac{g}{\rho}\frac{\partial \rho}{\partial x} = f\frac{\partial v_{ag}}{\partial z} \end{cases}$$

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Vg : geostrophic horizontal velocity

Qdag = TWI deformation

= stretching and re-orientation
by the total horizontal velocity
of the pre-existing ageostrophic horizontal
pseudo-vorticity (-TWI(y)/f, + TWI(x)/f)

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Qdr = TWI trend also generally neglected

TWI = Thermal Wind Imbalance

Q terms, generally neglected

Turbulent forcings

In the dynamic forcings:

Buoyancy: $\mathbf{Q}_{\text{th}}$

$$\begin{cases} Q_{thx} = -\dfrac{g}{\rho}\dfrac{\partial}{\partial x}\left(\dfrac{\partial F_{\rho\alpha_i}}{\partial_{\alpha_i}}\right) \\ Q_{thy} = -\dfrac{g}{\rho}\dfrac{\partial}{\partial y}\left(\dfrac{\partial F_{\rho\alpha_i}}{\partial_{\alpha_i}}\right) \end{cases}$$

$$\begin{cases} \dfrac{d\rho}{dt} = -\dfrac{\partial F_{\rho\alpha_i}}{\partial\alpha_i} \\ F_{\rho\alpha_i} = \left(\overline{u'\rho'}; \overline{v'\rho'}; \overline{w'\rho'}\right) \end{cases}$$

$F_{\rho\alpha_i}$ are the turbulent buoyancy fluxes

Momentum: $\mathbf{Q}_{\text{dm}}$

$$\begin{cases} Q_{dmx} = f\dfrac{\partial}{\partial z}\left(\dfrac{1}{\rho}\dfrac{\partial\tau_{y\alpha_i}}{\partial\alpha_i}\right) \\ Q_{dmy} = -f\dfrac{\partial}{\partial z}\left(\dfrac{1}{\rho}\dfrac{\partial\tau_{x\alpha_i}}{\partial\alpha_i}\right) \end{cases}$$

$$\tau_{x\alpha_i} = \rho\left(\overline{u'^2}; \overline{u'v'}; \overline{u'w'}\right)$$
$$\tau_{y\alpha_i} = \rho\left(\overline{v'u'}; \overline{v'^2}; \overline{v'w'}\right)$$

Turbulent momentum fluxes

TWI trend: $\mathbf{Q}_{\text{dr}}$

$$\begin{cases} Q_{drx} = \dfrac{d}{dt}\left(f\dfrac{\partial v_{ag}}{\partial z}\right) \\ Q_{dry} = -\dfrac{d}{dt}\left(f\dfrac{\partial u_{ag}}{\partial z}\right) \end{cases}$$

α_i : x, y, z for i=1,2,3

4. Omega equation

- Omega equation (Hoskin et al., 1978)

$$f^2\frac{\partial^2 w}{\partial z^2}+\nabla_h(N^2.\nabla_h w)=\nabla.\boldsymbol{Q}$$

→ f : Coriolis parameter (s^{-1})
→ w : vertical velocity ($m\ s^{-1}$)
→ N^2 : Brünt Vaïsäla frequency
→ **Q** : **Q**-vector

- Different forcings can be considered (Giordani et al., 2006)

Q = **Q**th + **Q**dm + 2 **Q**tg + **Q**tag + **Q**dag + **Q**dr

(2 **Q**tg + **Q**tag = **Q**tw)

→ Most studies only uses : **Q**tg (Quasi-geostrophy)
→ Omega solver uses : **Qtw** + **Qdag**
→ Inputs : TWI(x), TWI(y), uADCP, vADCP, ρ, N^2

Qtag : Ageostrophic

$$\boldsymbol{Q}=\left(\frac{g}{\rho_0}\frac{\partial \mathbf{V_{ag}}}{\partial x}\nabla\rho,\ \frac{g}{\rho_0}\frac{\partial \mathbf{V_{ag}}}{\partial y}\nabla\rho\right)$$

Vag : ageostrophic horizontal velocity

Qdag : TWI deformation

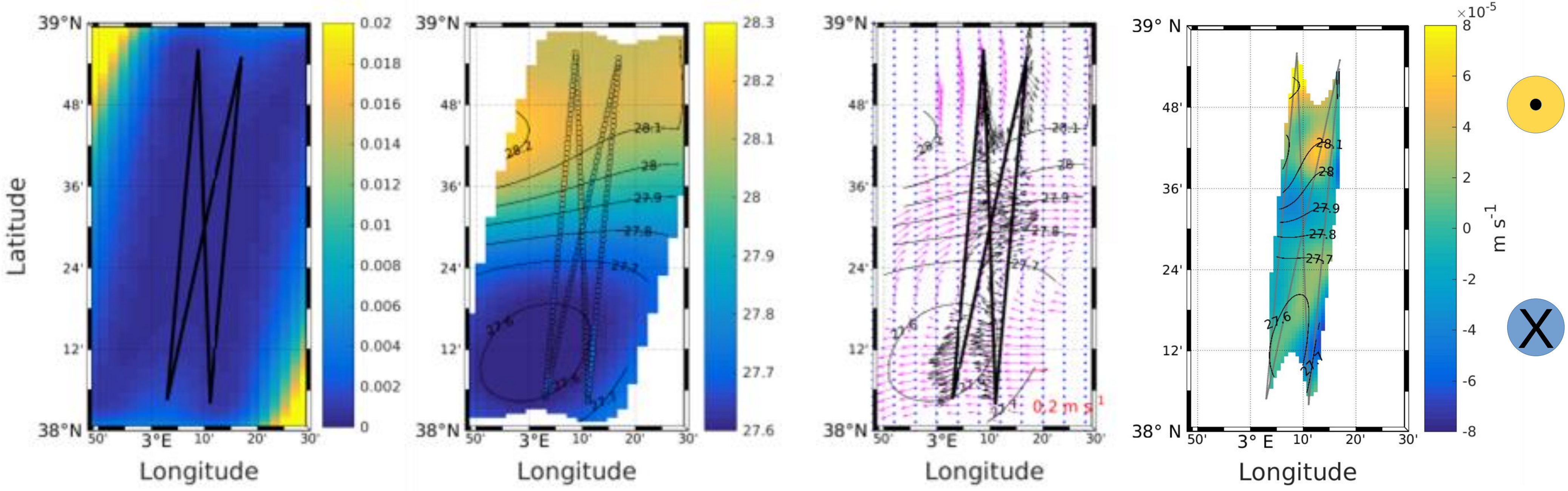
$$\begin{cases} Q_{dagx}=f\left(\frac{\partial v}{\partial x}\frac{\partial u_{ag}}{\partial z}-\frac{\partial u}{\partial x}\frac{\partial v_{ag}}{\partial z}\right)\\ Q_{dagy}=f\left(\frac{\partial v}{\partial y}\frac{\partial u_{ag}}{\partial z}-\frac{\partial u}{\partial y}\frac{\partial v_{ag}}{\partial z}\right)\end{cases}$$

with

$$\begin{cases} TWI(x)=f\frac{\partial u}{\partial z}-\frac{g}{\rho}\frac{\partial \rho}{\partial y}=f\frac{\partial u_{ag}}{\partial z}\\ TWI(y)=f\frac{\partial v}{\partial z}+\frac{g}{\rho}\frac{\partial \rho}{\partial x}=f\frac{\partial v_{ag}}{\partial z}\end{cases}$$

5. Results

At 25 m depth:



Error grid | Potential density anomaly | Horizontal velocities | Vertical velocities

$8\ 10^{-5}$ m/s = How many meters in one day ?

5. Results

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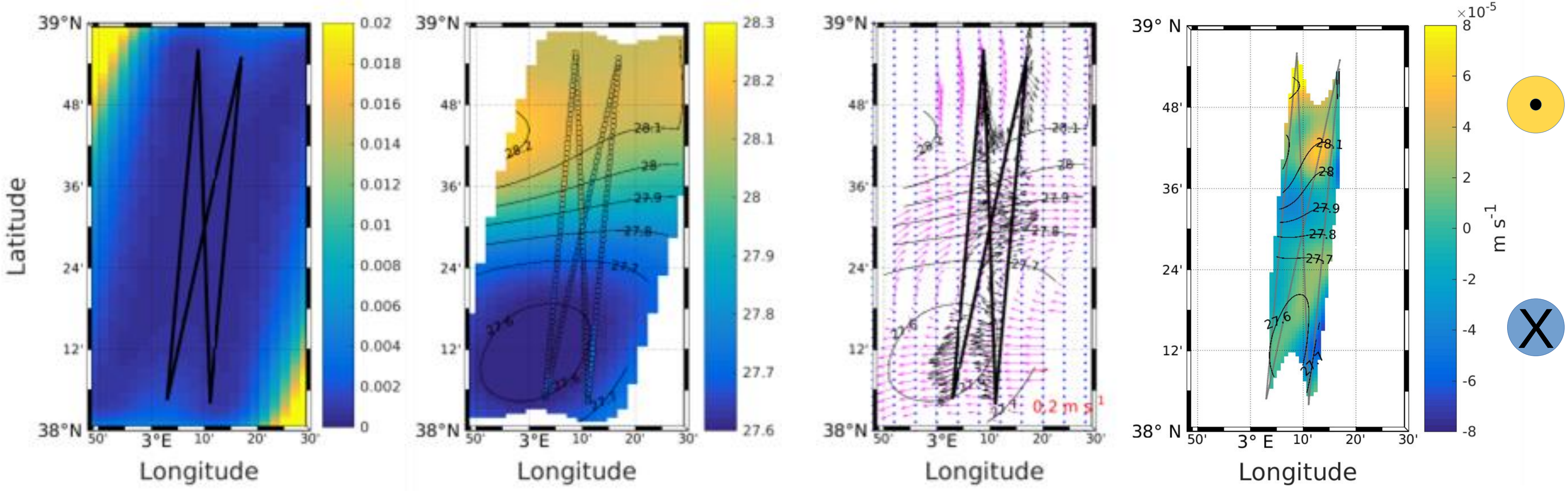
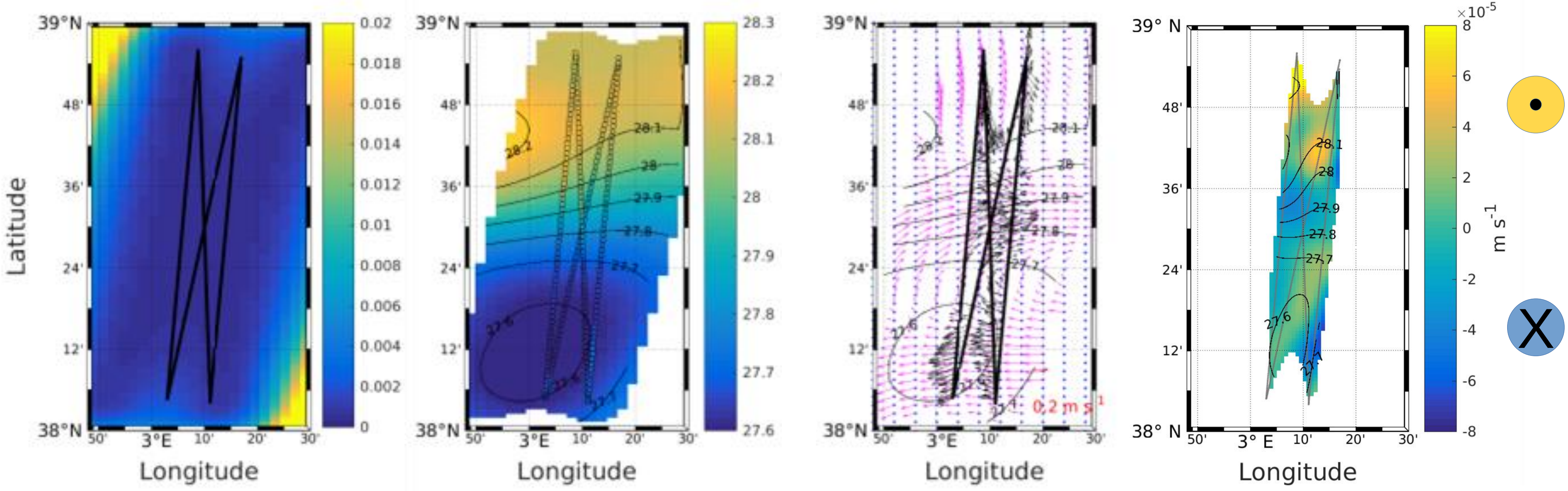


Error grid | Potential density anomaly | Horizontal velocities | Vertical velocities

8 10^{-5} m/s = 7 m/jour

5. Results

At 25 m depth:

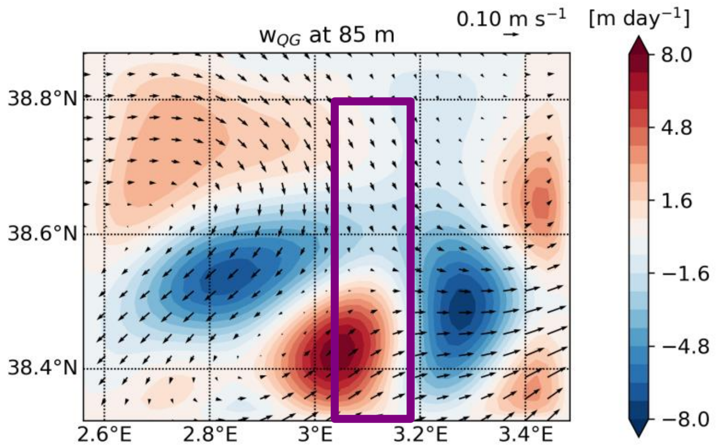


Error grid | Potential density anomaly | Horizontal velocities | Vertical velocities

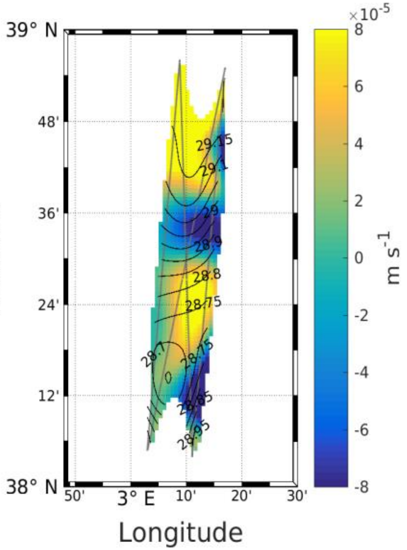
Max w intensity ~ $8\ 10^{-5}$ m s^{-1} ~ 7 m day^{-1}.

5. Results

Comparison with another study in the area, where w was obtained from the Omega equation using only the geostrophic component



Vertical velocities at 85 m
(Figure extracted from Barcelo et al., Front. Mar. Sci., 2021)



Vertical velocities at 85 m
[Tzortzis et al., 2021]

→ Intensity ~ **8 10^{-5} m s^{-1}** ~ **7 m day^{-1}**.

→ Results in agreement with Barcelo et al., 2021

7. Conclusions for Tzortzis study

→ PROTEVSMED-SWOT maximum of w intensity ~ **8 10^{-5} m s^{-1}** ~ **7 m day^{-1}**.

→ In agreement with other study in the area, where w was obtained from Omega equation using only geostrophic velocity components

→ Zone of PROTEVSMED-SWOT probably quasi-geostrophic (contains little ageostrophy)

General Conclusions on Omega equation (some adapted from Pietri et al., JPO 2021)

→ Quasi-Geostrophy QG hypothesis for omega equation, very well adapted for low Rossby number flows

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→ Generalized Omega equation necessary for high Rossby number flows
L < 10 km, down to 5 km (for the moment)

Pietri et al., abstract JPO 2021

The quasi-geostrophic and the generalized omega equations are the most widely used methods to reconstruct vertical velocity (w) from in-situ data.

As observational networks with much higher spatial and temporal resolutions are being designed, the question rises of identifying the approximations and scales at which an accurate estimation of w through the omega equation can be achieved and what are the critical scales and observables needed. In this paper we test different adiabatic omega reconstructions of w over several regions representative of main oceanic regimes of the global ocean in a fully eddy-resolving numerical simulation with a 1=60° horizontal resolution. We find that the best reconstructions are observed in conditions characterized by energetic turbulence and/or weak stratification where near-surface frontal processes are felt deep into the ocean interior. The quasi-geostrophic omega equation gives satisfactory results for scales larger than 10 km horizontally while the improvements using a generalized formulation are substantial only in conditions where frontal turbulent processes are important (providing improvements with satisfactory reconstruction skill down to 5 km in scale).

The main sources of uncertainties that could be identified are related to processes responsible for ocean thermal wind imbalance (TWI), which is particularly difficult to account for (especially in observation-based studies) and to the deep flow which is generally improperly accounted for in omega reconstructions through the bottom boundary condition. Nevertheless, the reconstruction of mesoscale vertical velocities may be sufficient to estimate vertical fluxes of oceanic properties in many cases of practical interest.

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(+ Giordani et Pietri papers):
https://people.mio.osupytheas.fr/~petrenko/TEACHING/OPB306/

Tzortzis and Rousselet papers on my publication web site:
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